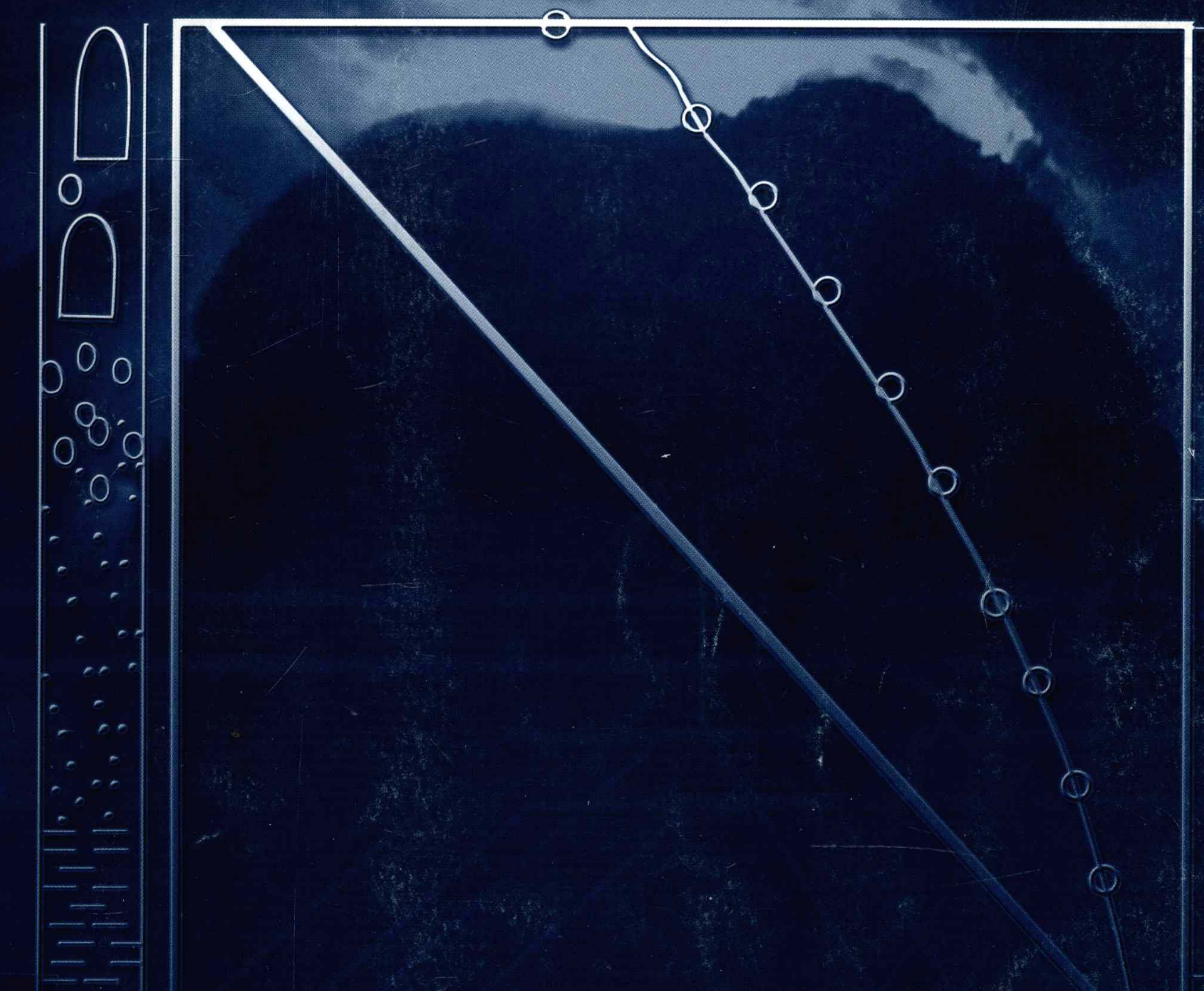
Fluid Flow and Heat Transfer in Wellbores



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FLUID FLOW AND HEAT TRANSFER IN WELLBORES

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Dedication

To Neelufar and Kumkum

Rashid Hasan is a professor at the U. of Minnesota-Duluth. Previously, he was the Chair of Chemical Engineering at U. of North Dakota and was named the Olson Professor of Engineering. He has also served as a visiting professor at the U. of Texas and was a research fellow at the NASA Glenn Laboratory and the Idaho National Engineering Laboratory. Momentum and energy transport, especially coupled transient transport processes, is Hasan's major research interest. Other interests include the rheology of non-Newtonian fluids, pressure-transient analysis, and cryogenic fluids. He has performed contract research for various organizations and has been sought as a consultant by major oil and service companies. Hasan has published extensively and has served on various SPE committees. He earned a PhD degree in chemical engineering from the U. of Waterloo, Canada in 1979.

Shah Kabir has over 25 years experience in the oil industry, with the last 12 years at ChevronTexaco. Currently, he is with ChevronTexaco's Mid-Africa Business Unit in Bellaire, Texas. He has lived and/or worked in many producing provinces of the world and contributed to various facets of petroleum engineering. His experience includes transient testing, production logging, production operations, and reservoir/reservoir-simulation engineering. Kabir has published extensively and has served various SPE committees, including editorial review for *SPEPF*, *SPEREE*, and *SPEJ*. He has received commendation as an outstanding technical editor four times for two different journals. In 2002, he received the SPE Western North America Region's Service Award. He holds a Master's degree in chemical engineering from the U. of Calgary.

Preface

Fluid flow in wellbores occurs during various phases of a well's life. Our ability to optimize each flow process depends largely on grasping the underlying physics so that we can mathematically describe the process involved. At a well's inception, drilling operations require mud circulation causing a considerable heat exchange between the colder fluid and the warmer formation. In the event of an unfortunate blowout, because of lost circulation or an unexpected overpressured zone, we encounter transient two-phase flow as the formation interacts with the wellbore prematurely. When we initiate flow from the formation by design, such as in a drillstem or production test, flow of either a single- or multi-phase fluid occurs. As the fluids ascend the wellbore, the warm formation fluid begins to exchange heat with the colder formation above it. Therefore, heat flow is always coupled with fluid flow in actual wellbores.

In this book, we attempt to address the coupled fluid and heat flow issue as encountered in many practical production-operation problems, including drilling. Both steady and unsteady-state transport problems are considered. Even when steady fluid flow is maintained during circulation, injection, or production modes, unsteady heat transport in the formation occurs nonetheless. Fluid circulation during drilling and workover operations and injection of annular gas in a gas-lift operation are cases in point. We also examine fully transient processes of fluid and heat flows, such as those in drillstem or production testing.

Before we undertake a detailed treatment of each operational problem, we introduce the reader to some basic concepts, starting with the rudiments of single-phase flow (Chap. 1) to more complex issues of two-phase flow modeling (Chaps. 2, 3, and 4). Thereafter, the principles of heat conduction in the formation, and the elements of fluid flow and the associated heat flow are discussed in Chaps. 5, 6, and 7. These chapters provide the ingredients for solving various flow problems that we consider subsequently, termed collectively as application chapters. Field examples are used to illustrate the principles learned in Chaps. 2 through 7, wherever feasible.

In the application chapters, we present the working equations and simple worked-out examples to illustrate their use. Details of the derivation of models are shown in appendices. We endeavored to facilitate application of a piece of technology learned along with its underlying physics and the assumptions involved in developing the model, and therefore its limitations. Overall, we have presented eight different topics in application (Chaps. 8 through 10).

The application chapters vary in scope. Some deal with parameter estimation while others lead to the understanding of a flow process. For example, a section in the drilling chapter shows how the proper heat-transfer modeling can lead to reliable static-formation temperature estimation. By contrast, the production-logging chapter leads one to improved understanding of individual layer contribution, by invoking the principles of flow modeling. In all cases, we attempt to show the linkage between theory, as developed in Chaps. 2 through 7, and practice.

While discussing transient aspects of mass and fluid flow in production testing, we introduced the notion of seamless Nodal or systems analysis. In this context, the significance of non-Darcy flow, often a forgotten entity in oil-well testing, is shown. Because inflow performance is a key parameter that a production engineer uses to seek or to evaluate well remediation, we felt that this treatment was justified. Here, the idea is not to discuss the intricacies of transient-pressure analysis but rather to stress the importance of combining transient testing, production logging, and well-performance analysis needed for reservoir management from a production engineer's viewpoint.

The book is envisioned to serve a variety of readers, from advanced senior and graduate students to practicing engineers. The overall philosophy is to show not just how to solve a given problem but also why the recommended approach is superior. In other words, we attempted to avoid the proverbial cookbook approach. Instead, we strove to strike a balance between theory and practice. Illustrative examples are used to reinforce the principles learned at the end of each major section.

Although hundreds of papers have been written on both topics of multiphase flow and heat transfer over the last five decades, we attempted to present only those that pertain to solving the wellbore flow problems. Thus, this book is not designed to treat either topic in great depth but to acquaint the reader with enough information so that practical oilfield problems can be tackled. In presenting various approaches to solving a problem, we favored physical models, which have been verified with either laboratory and/or field data, over purely empirical correlations. However, in choosing mechanistic models we have leaned toward a simpler approach, rather than delving into complex but rigorous solutions. Here, the motivation was to retain simplicity and engineering accuracy. In this context, we must point out that we have drawn heavily from our experiences, both academic and applied, to present this material. In this compilation effort, our familiarity of material, which is our own, took precedence even though we tried to be objective.

We did not attempt to solve all the coupled fluid and heat flow problems in production operations. This book is simply an attempt to capture a few. Our ardent hope is that the presented material gives both the foundation and examples to tackle other problems.

A.R. Hasan

C.S. Kabir

Acknowledgments

In our quest to grasp various aspects of production operations as practiced in the oilfield, we learned from others, be it through personal interaction or through published work. Our colleagues, in both academia and the industry, enriched our knowledge over the years. We are indebted to them all.

Here, we recognize a few who made the real difference in our learning and the eventual compilation of this book. Dr. Xiaowei Wang, a former graduate student at U. of North Dakota, contributed a great deal to our cause. He was instrumental in developing many pieces of the models presented in Chaps. 6 through 9. In this respect, the contributions of another UND graduate, Dr. Dongqing Lin, are worthy of note. Former graduate student Dr. Mahbub Ameen also contributed significantly to solving the fluid-circulation problems. Many field examples were drawn from hands-on experience that one of us, C.S. Kabir, had in Kuwait. We thank Kuwait Oil Co. for allowing us to present those examples in various SPE papers.

We are grateful to Professor Khalid Aziz of Stanford U. for giving us the much-needed impetus to launch this project. Professor Emeritus James P. Brill of the U. of Tulsa helped broaden our horizons. Professor Cem Sarica of the U. of Tulsa reviewed the manuscript. His timely reviews and insightful comments helped enrich the content, for which we will remain indebted forever. We express our gratitude to our respective organizations, U. of North Dakota and ChevronTexaco, for aiding our pursuit. SPE's Books Committee deserves special recognition for entrusting us with this project. We owe Shelley Nash for her diligence and attention to meticulous details while reviewing the manuscript. Her extraordinary patience was instrumental in minimizing endless imperfections in the manuscript. We are thankful to SPE staff members Jennifer Wegman, for managing and reviewing the manuscript, and Fran Kennedy-Ellis, for overseeing the book's publication.

A few academicians helped shape our understanding of this technology through their exemplary leadership. Professor Graham Wallis of Dartmouth College formalized the drift-flux approach for two-phase flow modeling. Late professors Abraham Dukler of the U. of Houston and Hank Ramey Jr. of Stanford U. laid the foundation for mechanistic modeling of two-phase flow and wellbore heat transfer, respectively.

Last, but not least, our family members, especially our spouses, deserve particular mention for their encouragement and fortitude. Their extraordinary understanding allowed us to steal countless hours from the family time so that we could complete this task. Finally, our parents, who taught us values, inspired us to compile this material. To this end, we hope the reader finds this text stimulating and useful.

Table of Contents

Chapter 1—Overview 1

1.1 Single-Phase Flow 1
1.2 Flow in Nonisothermal Systems 3
1.3 Flow in Annulus 3
1.4 Flow in Horizontal Wells 4

Chapter 2—Multiphase Flow: Introduction 7

2.1 Introduction 7
2.2 Concepts and Definitions 7
2.3 Method of Analysis 9
2.4 Homogeneous Models 10
2.5 Separated Flow Models 11

Chapter 3—Multiphase Flow: Mechanistic Models for Vertical Wells 18

3.1 Introduction 18
3.2 Flow-Pattern Delineation 19
3.3 Liquid Holdup and Pressure Drop 24
3.4 Discussion 30

Chapter 4—Multiphase Flow: Deviated and Horizontal Wells and Other Systems 37

4.1 Introduction 37
4.2 Flow in Deviated Wells 37
4.3 Flow in Horizontal Pipes and Wells 47
4.4 Modeling Flow in Nonconventional Situations 55

Chapter 5—Wellbore Heat Transport 64

5.1 Introduction 64
5.2 Formation Temperature Distribution 64
5.3 Energy Balance for Wellbore Fluid 66
5.4 Wellbore Fluid Temperature 68

Chapter 6—Wellbore Temperature Profiles in Multiple Strings 74

6.1 Introduction 74
6.2 Production Through Tubing and Annulus 74
6.3 Production Through Two Tubing Strings 76
6.4 Production and Injection Through Common Wellbore 77

Chapter 7—Transient Fluid and Heat Flows in Wellbores 79

7.1 Introduction 79
7.2 Modeling Single-Phase Gas Flow 79
7.3 Modeling Single-Phase Oil Flow 83
7.4 Modeling Two-Phase Gas/Oil Flow 86

Chapter 8—Drilling Operations 90
8.1 Fluid Circulation Temperature 90
8.2 Static Formation Temperature 97
8.3 Oil Well Blowout 105

Chapter 9—Production Operations 113
9.1 Systems Analysis 113
9.2 Temperature Profiles In Gas-Lift Wells 117
9.3 Wellbore Steam/Water Flow and Surface Metering 124
9.4 Combating Wellbore Solids Deposition 130

Chapter 10—Production Logging 140
10.1 Introduction 140
10.2 PL Interpretation 140
10.3 Flow-Pattern Maps For PL Analysis 147
10.4 Discussion 149

Appendix A—Overall Heat Transfer Coefficients for Wellbores 151

Appendix B—Working Equations for Multiple String Completions 154

Appendix C—Finite-Difference Equations for Wellbore Fluid Temperature 157

Appendix D—Models Related to Drilling Operations 159

Appendix E—Models Related to Production Operations 162

Appendix F—Estimating Flow from Spinner Response 169

Author Index 171

Subject Index 174

Chapter 1
Overview

1.1 Single-Phase Flow

Fluid flow, in a variety of forms and complexities, is a basic entity that must be dealt with in the production of hydrocarbons. In its rudiments, single-phase gas or oil production and water injection form the core of all flow problems. Therefore, Chap. 1 discusses the mechanical energy balance equation, which relates pressure drop to its various components for single-phase flow. Next, the components of total pressure drop—static, kinetic, and frictional—are discussed. Also, flows in tubing-casing annuli and horizontal wells, which are of particular interest to petroleum engineers, are briefly discussed.

1.1.1 Mechanical Energy Balance. A simple one-dimensional (1D) analysis of single-phase gas or liquid flow is best made with the aid of a schematic, as shown in **Fig. 1.1.** The channel, inclined at an arbitrary angle (α) with the horizontal, shows upward flow of the fluid. For the present, we consider only the steady-state case and assume that pressure, at any point in the cross-sectional plane normal to flow, remains the same. With these simplifications, we derive the momentum balance equation.

Conservation of Momentum. The sum of forces acting on the fluid element, shown in Fig. 1.1, equals the change of momentum of the fluid. The forces acting on the fluid element are those owing to pressure, p, friction, F, and gravity. Referring to the differential length, dz, of Fig. 1.1, we write $pA-(p+\mathrm{d}p)-\mathrm{d}F-A(\mathrm{d}z)g\rho\sin\theta=$ change of momentum.

If the fluid mass flow rate is w and its velocity is v, then its momentum equals wv. For the general case of transient flow, when both flow rate and velocity change along the flow direction, fluid momentum change is given by $(w+\mathrm{d}w)(v+\mathrm{d}v)-wv$. Therefore,

$$pA-(p+\mathrm{d}p)A-\mathrm{d}F-A(\mathrm{d}z)g\rho\sin\theta$$
$$=(w+\mathrm{d}w)(v+\mathrm{d}v)-wv \quad \text{(1.1)}$$

Simplifying, we obtain

$$-A\mathrm{d}p-\mathrm{d}F-A(\mathrm{d}z)g\rho\sin\theta=w\mathrm{d}v+v\mathrm{d}w \quad \text{(1.2)}$$

Usually, the mass flow rate is invariant; that is, dw=0, leading to

$$-A\mathrm{d}p-\mathrm{d}F-A(\mathrm{d}z)g\rho\sin\theta=w\mathrm{d}v \quad \text{(1.3)}$$

Dividing both sides of Eq. 1.3 by $A\mathrm{d}z$, we obtain

$$-(\mathrm{d}p/\mathrm{d}z)+(\mathrm{d}p/\mathrm{d}z)_F-g\sin\theta-(w/A)\mathrm{d}v/\mathrm{d}z=0 \quad \text{(1.4)}$$

$$\text{or } (\mathrm{d}p/\mathrm{d}z)=(\mathrm{d}p/\mathrm{d}z)_F+(\mathrm{d}p/\mathrm{d}z)_H+(\mathrm{d}p/\mathrm{d}z)_A, \quad \text{(1.5)}$$

$$\text{where } (\mathrm{d}p/\mathrm{d}z)_H=g\rho\sin\theta, \quad \text{(1.6)}$$

$$\text{and } -(\mathrm{d}p/\mathrm{d}z)_A=(w/A)\mathrm{d}v/\mathrm{d}z=\rho v\mathrm{d}v/\mathrm{d}z \quad \text{(1.7)}$$

1.1.2 Components of Pressure Gradient. Eq. 1.5 shows the total pressure gradient is the sum of the frictional gradient $(\mathrm{d}p/\mathrm{d}z)_F$, the hydrostatic gradient $(\mathrm{d}p/\mathrm{d}z)_H$, and the accelerational gradient $(\mathrm{d}p/\mathrm{d}z)_A$. Of these three terms, perhaps the static gradient is the easiest to estimate because it only requires knowledge of the fluid density and well-deviation angle. Because gas density depends on pressure, the static term will vary along the well for gas wells. Usually such variation is small, and relatively simple equations of state can be used to account for it. To some extent, even for single-phase oil production, oil-density variation with well depth, owing to temperature and dissolved gases, must be taken into account. The same comments apply to the estimation procedure for the kinetic head (Eq. 1.7).

For incompressible flow in a straight pipe with no change in cross-sectional area (gases at very high pressures and liquids), the change in fluid velocity with axial distance (dv/dz) is generally negligible. However, for gases at moderate and low pressures, and especially at high velocities, the kinetic energy loss can be a significant portion of the total pressure loss and must be accounted for properly. Computational complications that arise for gas flow have led to a number of correlations for calculating pressure drop in a wellbore. We recommend the widely used Cullender and Smith[1] method for computing pressure drop in a gas well.

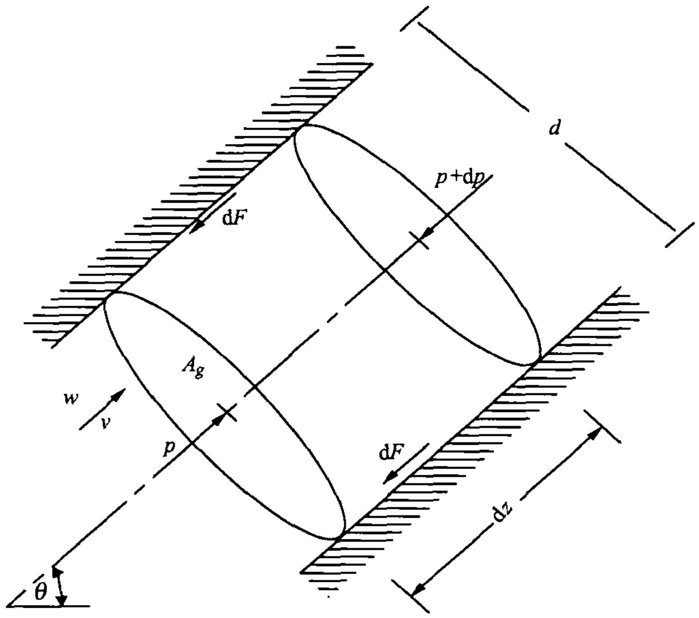

Fig. 1.1—Momentum balance for a fluid element.

The frictional pressure gradient is generally represented by

$$(\mathrm{d}p/\mathrm{d}z)_F = -fv^2\rho/2g_c d \text{ , } \qquad (1.8)$$

where the Moody friction factor, f, depends on the turbulence of the fluid and also on the pipe roughness. The friction factor is usually expressed as a function of Reynolds number

$$\mathrm{Re} = dv\rho/\mu \text{ , } \qquad (1.9)$$

and roughness factor ε/d. The chart for friction factor as a function of Reynolds number with pipe roughness as a parameter is shown in **Fig. 1.2;** whereas, **Fig. 1.3** presents the chart for estimating relative roughness. Note, k/d represents the relative roughness or ε/d in both figures, and in Fig. 1.3, the units of measure for pipe diameter (d) are ft. We point out that Fig. 1.2 is the Moody friction factor chart. The Fanning friction factor is simply one-fourth the Moody friction factor. Because of its popularity in the oil industry, we use the Moody friction factor throughout this book.

At low-Reynolds numbers (R<2,100), the flowing fluid elements do not interact with each other, and the flow is called laminar. For laminar flow in either rough or smooth pipes, friction factor is inversely related to Reynolds number

$$f = 64/\mathrm{Re} = 64dv\rho/\mu \text{ , } \qquad (1.10)$$

when Re<2,100.

At high-Reynolds numbers (Re>4,000), the flow is termed turbulent. During turbulent flow, the friction factor depends on both the Reynolds number and pipe roughness. For smooth pipes, such as plastic pipes and tubulars coated with PVC lining, friction factor can be estimated reliably from the Blassius equation,

$$f = 0.32(\mathrm{Re})^{-0.25} \text{ , } \qquad (1.11)$$

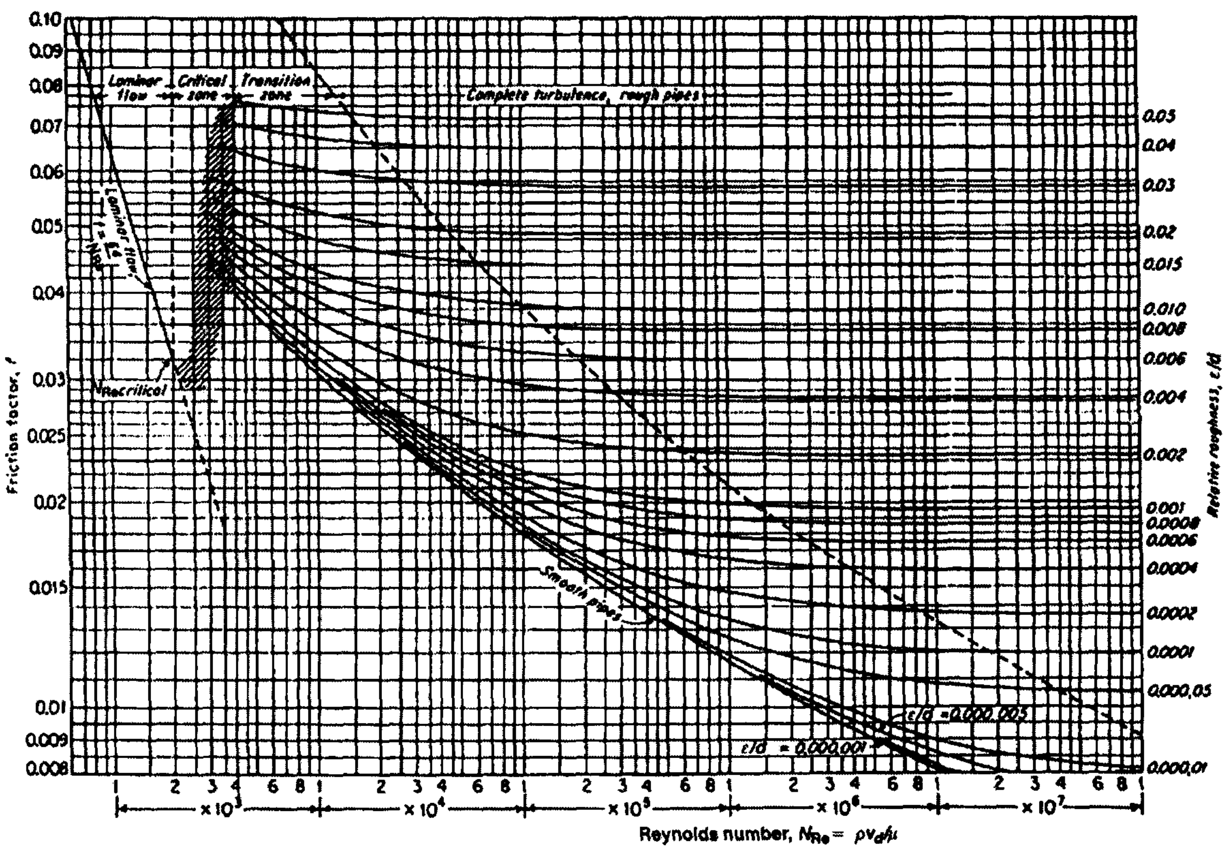

Fig. 1.2—Moody friction factor chart for turbulent flow (from Ref. 3; courtesy of ASME).

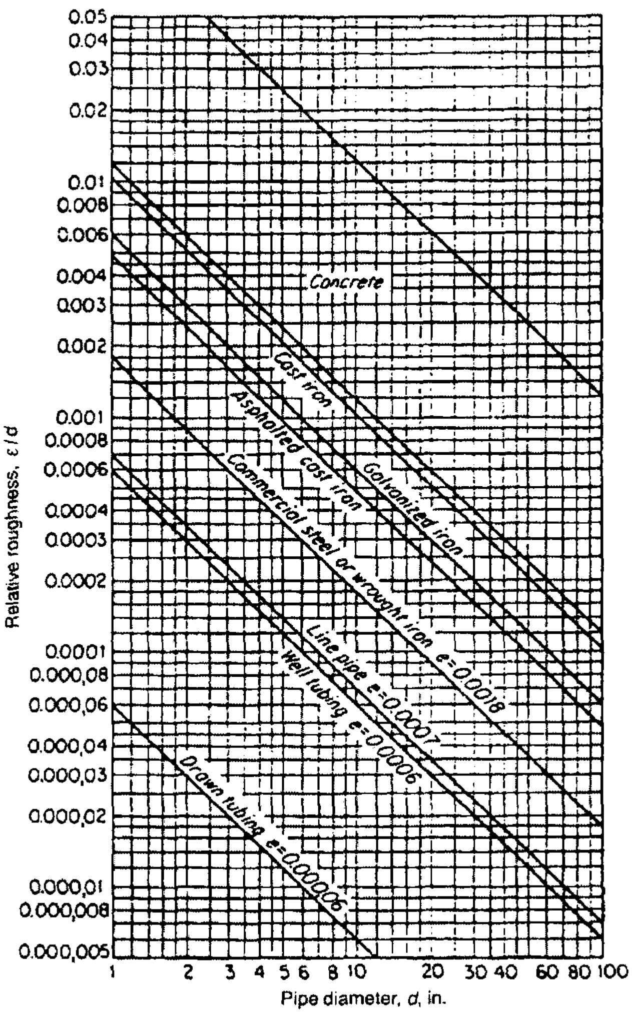

Fig. 1.3—Relative roughness of pipes (from Ref. 3; courtesy of ASME).

when Re>4,000.

For very high Reynolds numbers (Re>50,000), Eq. 1.11 is slightly modified as $f = 0.184\,(\mathrm{Re})^{-0.2}$.

Eq. 1.11, of course, is invalid for rough pipes. Although a chart is useful for all types of pipe roughness, chart reading is tedious and is not easily amenable to computer calculations. A number of equations, relating friction factor to Reynolds number and pipe roughness, has been proposed over the years and are in fair agreement with the original friction-factor charts. We recommend the following expression proposed by Chen,[2] which yields Fanning friction factor and is given by

$$f = \frac{1}{\left[4\log\left(\frac{\varepsilon/d}{3.7065} - \frac{5.0452}{\mathrm{Re}}\log\Lambda\right)\right]^2}, \quad \ldots\ldots (1.12)$$

where ε is pipe roughness, and the dimensionless parameter, Λ, is given by

$$\Lambda = \frac{(\varepsilon/d)^{1.1098}}{2.8257} + \left(\frac{7.149}{\mathrm{Re}}\right)^{0.8981} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1.13)$$

Unlike many other expressions, which require iterative solutions for the friction factor, Eq. 1.12 is explicit and, therefore, computationally efficient.

The evaluation of various terms in Eq. 1.12 is relatively easier for flow of single-phase fluids, even for gases, than for two-phase mixtures. In the latter case, estimating the average density and friction factor can be challenging because these are complex functions of fluid properties and flow conditions. Chap. 2 discusses various approaches taken to evaluate these entities in two-phase flow.

1.2 Flow in Nonisothermal Systems

Fluid temperature in the wellbore often varies significantly with depth, and sometimes with time. Many of the fluid properties that influence pressure drop, such as density and viscosity, are greatly influenced by the fluid temperature. Therefore, we cannot overemphasize the importance of accurate fluid temperature estimation as a function of well depth and production or injection time. This calculation can be done by a proper energy balance on the fluid-wellbore system, as shown in Chap. 5. For single-phase flow, the expression for fluid temperature, T_f, simplifies to

$$T_f = T_{ei} + \left[1 - e^{-zL_R}\right] g_G \sin\theta, \quad \ldots\ldots\ldots\ldots\ldots (1.14)$$

where the parameter, L_R, which is a function of wellbore heat-transfer coefficient U_{to} and formation heat conductivity k_e, is defined by

$$L_R = \frac{2\pi}{c_p w}\left[\frac{r_{to}U_{to}k_e}{k_e + (r_{to}U_{to}T_D)}\right]. \quad \ldots\ldots\ldots\ldots\ldots\ldots (1.15)$$

In Eq. 1.15, T_D represents dimensionless temperature, which is a function of dimensionless time, $t_D = k_e c_e t/\rho_e r_{wb}^2$.

$$T_D = \ln\left[e^{-0.3t_D} + \left(1.5 - 0.3719e^{-t_D}\right)\sqrt{t_D}\right]. \quad \ldots\ldots (1.16)$$

For a complete discussion of Eqs. 1.14 through 1.16, please refer to Chap. 5.

1.3 Flow in Annulus

Although flow through a tubing string is the most common configuration, many completions dictate modeling for flow up the tubing-casing annulus. The presence of two walls makes flow through an annulus different from that through ordinary circular strings. The classical work of Bird *et al.*[4] shows Eq. 1.8 is also applicable for such geometry, although the correlation for friction factor must be modified to reflect greater wall shear. For laminar flow in a concentric annulus, the Moody friction factor, f_{CA}, is given by[4]

$$f_{CA} = \frac{64}{\mathrm{Re}}\frac{(1-K)^2}{\left\{\frac{1-K^4}{1-K^2} - \frac{1-K^2}{\ln(1/K)}\right\}}, \quad \ldots\ldots\ldots\ldots\ldots (1.17)$$

where K is the diameter ratio, d_t/d_c. Following the studies of Gunn and Darling[5] and Caetano *et al.*,[6] we recommend expressing turbulent flow in a concentric annulus as

$$\frac{1}{\left\{f_{CA}\left(\dfrac{F_p}{F_{CA}}\right)^{0.45\exp\{-(\mathrm{Re}-3000)/10^6\}}\right\}^{0.5}}$$

$$= 4\log\left[\mathrm{Re}\left\{f_{CA}\left(\frac{F_p}{F_{CA}}\right)^{0.45\exp\{-(\mathrm{Re}-3000)/10^6\}}\right\}^{0.5}\right] - 0.4, \quad \text{(1.18)}$$

where F_p is the laminar-flow friction factor geometry parameter and F_{CA} is the ratio of friction factor for the annulus to that of a circular channel with the same d_c. Thus, from Eq. 1.18, F_p, for a concentric annulus, is given by

$$F_P = \frac{(1-K)^2}{\left[\dfrac{1-K^4}{1-K^2} - \dfrac{1-K^2}{\ln(1/K)}\right]} \cdot \quad \text{(1.19)}$$

For eccentric annuli, eccentricity (E) is defined as

$$E = D/(d_c - d_t), \quad \text{(1.20)}$$

where D is the distance between the pipe centers. The values of F_p, as a function of K and E, are shown in **Fig. 1.4.** For an eccentric annulus, the friction factor equation is similar to that of Eq. 1.18,

$$f_{ECA} = \frac{4}{\mathrm{Re}}\frac{4}{\xi}\frac{(1-K)^2\left(1-K^2\right)}{\sinh^4 \eta_o}, \quad \text{(1.21)}$$

where η_o and ξ incorporate the effect of eccentricity factor E. A complete treatment of flow through eccentric annuli is beyond the scope of this text; for further details, the reader is referred to the work of Caetano *et al.*[6] Two-phase flow in an annular geometry is treated in Chap. 4.

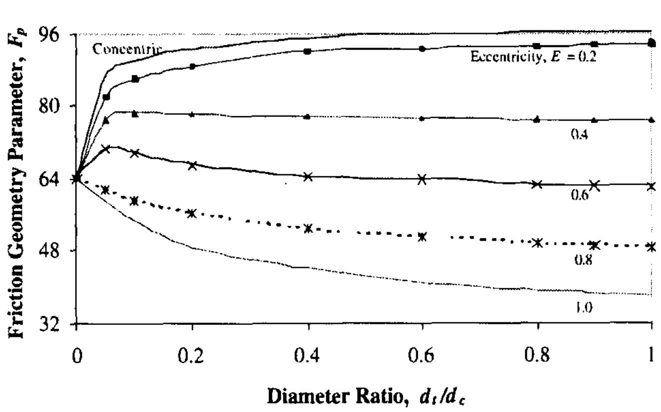

Fig. 1.4—Friction geometry parameter for concentric and eccentric annuli.

1.4 Flow in Horizontal Wells

The recent interest in horizontal wells stems from significant increases in productivity and ultimate recovery in certain cases. Initial efforts[7,8] to couple the wellbore with reservoir using analytic approaches considered frictional effects only. In other words, fluid ingress along the well length leading to momentum and related effects was ignored in those formulations.

Estimating pressure drop in horizontal wells presents a number of difficulties. First, pipe-surface roughness is a difficult entity to discern because of perforations along the well length in a cased borehole. Because most completions occur openhole, complexity increases significantly to ascribe a friction factor for an ill-defined surface—that is, the formation. The second factor revolves around fluid influx or changes in momentum that occur along the well length.

Recent experimental studies[9-11] in perforated horizontal pipes, allowing fluid ingress along the well length, led to the development of several friction-factor correlations. Of these, the results of Ouyang *et al.*[10] and Yuan *et al.*[11] are noteworthy.

Ouyang *et al.*[10] presented the following Moody friction-factor correlations for laminar and turbulent flows, respectively.

$$f = \frac{64}{\mathrm{Re}}\left(1 + 0.04304\mathrm{Re}_w^{\,0.6142}\right), \quad \text{(1.22)}$$

$$\text{and } f = f_o\left(1 - 0.0153\mathrm{Re}_w^{\,0.3978}\right), \quad \text{(1.23)}$$

where f_o is the no-wall-flow friction factor, which can be estimated from Eq. 1.12. Note, Re_w represents the wall Reynolds number, which is based on the pipe ID and equivalent inflow velocity per unit wellbore length.

A somewhat different approach led Yuan *et al.*[11] to obtain the following expression for the total or apparent friction factor, f_T (Moody friction factor), for fluid ingress along the borehole.

$$f_T = a\mathrm{Re}^{-b} + C_n 2d\varphi\frac{q_i}{q_a}, \quad \text{(1.24)}$$

$$\text{where } a = 10219.5\varphi - 3.25\frac{q_i}{q_a} - 8.87\times10^{-4}\varphi^2$$

$$+5.37\times10^{-2}\varphi - 0.075, \quad \text{(1.25)}$$

$$\text{and } b = \left(-1.24\times10^5\varphi^{-3.075} + 42.4\right)\left(\frac{q_i}{q_a}\right)^2$$

$$+1.577\times10^3\varphi^{-2.63}\frac{q_i}{q_a} - 5\times10^{-4}\varphi^2$$

$$+2.31\times10^{-2}\varphi + 0.085 \cdot \quad \text{(1.26)}$$

For $(q_i/q_a)<0.02$, C_n=2.3, and for $(q_i/q_a)>0.02$, C_n is given by

$$C_n = 4.25\left(\frac{q_i}{q_a}\right)^{-0.099} \qquad (1.27)$$

Experiences show that pressure drop in horizontal wells becomes important in high-transmissivity reservoirs, where the pressure drop in the wellbore becomes comparable to that in the formation. When the wellbore pressure drop becomes important, in most cases, the frictional component becomes the dominant mechanism. Chap. 4 discusses two-phase flow in horizontal wells.

Summary

The objective of this introductory chapter is to acquaint the reader with the rudiments of single-phase flow, which forms the backbone for understanding the mechanics of two-phase flow. Here, we attempted to capture some elements of fluid flow through conduits of various complexities, such as annulus and horizontal wells, and when fluid flow is accompanied by heat flow. Subsequent chapters discuss these elements in detail.

Nomenclature

a = parameter defined by Eq. 1.25, dimensionless
A = cross-sectional area for fluid flow, ft^2
A_g, A_l = cross-sectional area available for gas or liquid to flow, ft^2
b = parameter defined by Eq. 1.26, dimensionless
c_e = heat capacity of earth or formation, Btu/(lbm-°F)
c_p = heat capacity of fluid, Btu/(lbm-°F)
C_n = parameter defined by Eq. 1.27, dimensionless
d = pipe or well diameter, in.
d_c, d_t = casing or tubing diameter, in.
D = distance between pipe centers in Eq. 1.20, ft
E = eccentricity factor, dimensionless
f = friction factor, dimensionless
f_o = no-wall friction factor, dimensionless
f_{CA} = friction factor of concentric annulus, dimensionless
f_{ECA} = friction factor of eccentric annulus, dimensionless
f_T = apparent friction factor, dimensionless
F = force, lbm
F_p = friction geometry parameter, dimensionless
g = acceleration due to gravity, ft/sec^2
g_c = conversion factor, 32.17 lbm-ft/lbf-s^2
g_G = geothermal gradient, °F/ft
H = fluid enthalpy, Btu/lbm
k = formation permeability, md
k_e = earth conductivity, Btu/(hr-ft-°F)
K = diameter ratio of annulus to tubing, dimensionless
L_R = relaxation distance parameter, ft^{-1}
p = pressure, psi
(dp/dz) = pressure gradient, psi/ft
$(dp/dz)_A$ = accelerational (kinetic) pressure gradient, psi/ft
$(dp/dz)_F$ = frictional pressure gradient, psi/ft
$(dp/dz)_H$ = static pressure gradient, psi/ft
q_a = average flow rate over incremental length, ft^3/hr
q_i = influx rate from each perforation, ft^3/hr
r_{wb} = wellbore radius, ft
r_{to} = outside tubing radius, ft
Re = Reynolds number $[=dv\rho/\mu]$, dimensionless
Re_g, Re_L = Reynolds number for the gas $(=\rho_g v_g d/\mu_g)$ or liquid phase $(=\rho_L v_L d/\mu_L)$, dimensionless
Re_m = Reynolds number for the mixture $[=\rho_m v_m d/\mu_m]$, dimensionless
Re_w = wall Reynolds number, dimensionless
t = producing, injecting, or circulation (mud) time, hr
t_D = dimensionless time, $k_e c_e t/\rho_e r_{wb}^2$
T_{ei}, T_e = formation temperature at initial condition or at any radial distance, °F
T_D = dimensionless temperature = $(2\pi k_e)(T_{wb}-T_{ei})/Q$
T_f = fluid temperature, °F
T_{wb} = wellbore fluid temperature, °F
U = overall heat transfer coefficient, Btu/(hr-°F-ft)
v = fluid velocity, ft/hr
w = mass flow rate of fluid, lbm/hr
z = any vertical well depth, ft
Z = gas-law deviation factor, dimensionless
α = wellbore inclination with horizontal, deg
Λ = parameter given by Eq. 1.13, dimensionless
μ = oil viscosity, cp
ε = pipe roughness factor, ft
ϕ = parameter used in Eq. 1.21
φ = perforation density, 1/ft
ρ = density, lbm/ft^3
η = parameter used in Eq. 1.21

Subscripts

c = casing
e = earth or formation
o = oil
t = tubing
to = tubing outside
wb = wellbore

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SI Metric Conversion Factors

Btu × 1.055 056	E + 00 = kJ
Btu/lbm × 2.326*	E + 03 = J/kg
Btu/(lbm-°F) × 4.186 8*	E + 03 = J/(kg·K)
cp × 1.0*	E − 03 = Pa s
ft × 3.048*	E − 01 = m
ft² × 9.290 304*	E − 02 = m²
ft³ × 2.831 685	E − 02 = m³
°F (°F − 32)/1.8	= °C
in. × 2.54*	E + 00 = cm
lbf × 4.448 222	E + 00 = N
lbm × 4.535 924	E − 01 = kg
lbm/ft³ × 1.601 846	E + 01 = kg/m³
lbm/hr × 1.259 979	E − 04 = kg/s
psi × 6.894 757	E + 00 = kPa

*Conversion factor is exact.

Chapter 2
Multiphase Flow: Introduction

2.1 Introduction

Most installations for petroleum production and testing involve concurrent flow of gas and liquid. A simplified schematic representation of the overall production system is shown in **Fig. 2.1.** The reservoir fluid, entering the wellbore, may contain all three phases (gas, oil, and water), in which case, multiphase flow starts at the perforations. However, gas often enters the wellbore in solution with the oil. Gas comes out as a separate phase, only when fluid moves up enough for the pressure to drop below the bubblepoint pressure, thus leading to gas/liquid, two-phase flow. In a favorable system, the entire fluid mixture may flow freely to the wellhead through a vertical or directional wellbore and then onto the separator through a horizontal or near-horizontal flowline.

Artificial lift is required in low-energy environments and/or when oil production is accompanied by water. When the artificial stimulus is provided by gas injection (gas-lift) through the annulus, the entire tubing production becomes two- or three-phase flow. Similarly, when a sucker-rod or an electrical submersible pump is used for the lift, only liquid is produced through the tubing to sustain high-pump efficiency, while gas is vented through the annulus. Even in this case, a substantial portion of the tubing may experience gas/liquid flow, as well as, three-phase flow. Some production facilities have to contend with the simultaneous flow of solids (sand), liquids (water and oil), and gases (injected and in solution).

Interest in multiphase flow is not restricted to the oil industry. Nuclear, geothermal, and chemical processing plants routinely encounter two-phase flow problems. The diverse interest explains the large number of publications in this area. At the same time, the plethora of publications indicates that the basics of multiphase flow are not completely understood. Often, correlations are published that have no general applicability to any situation other than specific conditions under which those were developed.

One of the reasons multiphase flow is more complicated than single-phase flow is that two or more fluids compete for the available flow area. To model flow behavior, one needs to know how the flow cross section is occupied by each fluid phase. Therefore, understanding physics of multiphase flow demands grasping important concepts, such as flow patterns, in-situ velocity, and volume fractions. In these and subsequent discussions, we focus on systems containing only two phases—gas and liquid. Many of these concepts may be extended to systems experiencing three-phase flow.

2.2 Concepts and Definitions

2.2.1 Flow Pattern. During flow of two or more immiscible fluids, deformable interfaces present complications. Shape and distribution of these interfaces greatly influence flow characteristics. These interfaces tend to be spherical, especially at low relative velocities, owing to surface tension effects. However, at higher relative velocities of the lighter fluid, the bubbles begin to elongate and coalesce, gradually changing into a different flow pattern or flow regime. Thus, bubbly flow, with small bubbles that distribute uniformly across the flow channel, changes to slug flow, with large bubbles that fill the entire channel cross section with slugs of liquid between them. At extremely high-gas rates, all of the gas may flow through the core of the channel, while the liquid flows through the annulus formed by the gas core and pipe wall. This flow pattern is called annular flow because of the dominance of the gas core. Therefore, multiphase flow can be categorized into a number of flow patterns or regimes, whose dynamics differ from each other. The existence of these flow patterns is influenced by many parameters, including fluid velocities, fluid properties, channel geometry, and channel orientation. These flow patterns, their transition from one to another, and their effect on pressure drop will be discussed later in more detail.

2.2.2 Superficial and In-Situ Velocities. Superficial velocity of any phase is the volumetric flow rate of that phase, divided by the total cross-sectional area of the channel. Thus, the superficial liquid velocity, v_{sL}, is given in terms of the volumetric liquid flow rate, q_L, and cross-sectional area, A, of the pipe by

$$v_{sL} = q_L / A \cdot \quad \text{(2.1)}$$

Similarly, superficial gas velocity, v_{sg}, is defined in terms of q_g. Note, superficial velocity is a quantity averaged over the flow cross section. Even for single-phase flow, fluid velocity across the channel varies; elements of fluid flowing