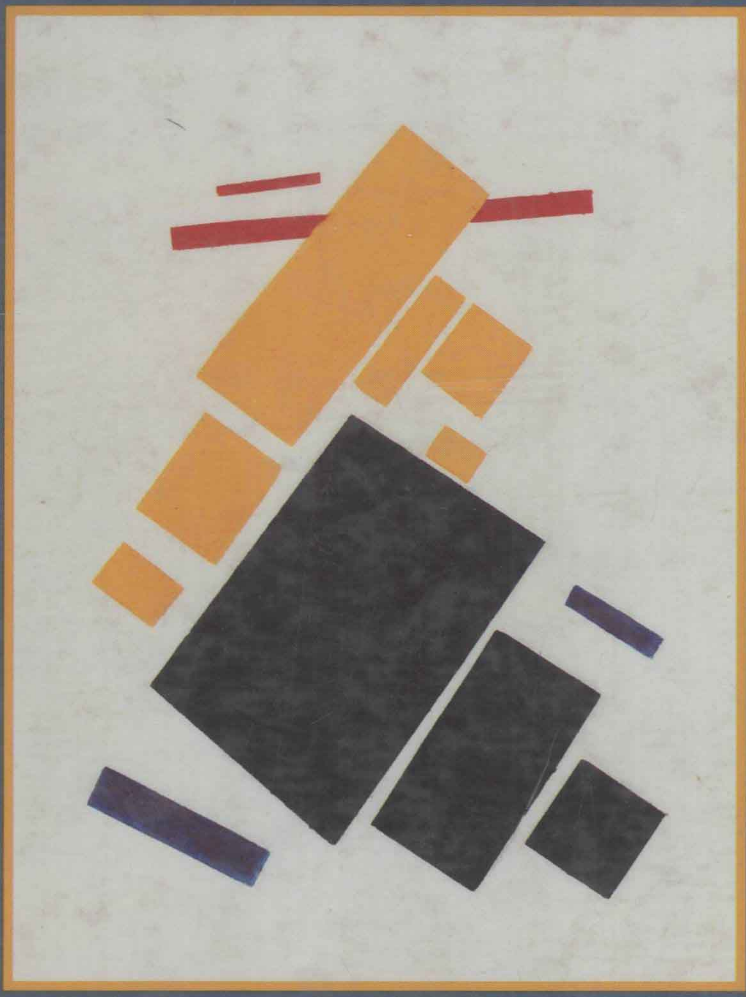
Introduction to

MATHEMATICAL PROGRAMMING

APPLICATIONS *and* ALGORITHMS



Wayne L. Winston

INTRODUCTION TO

Mathematical Programming

Applications and Algorithms

WAYNE L. WINSTON
Indiana University

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To my daughter Jennifer

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Preface

In recent years, operations research software for mainframes and microcomputers has become widely available. Like most tools, however, it is useless unless the user understands its application and purpose. Users must ensure that mathematical input accurately reflects the real-life problems to be solved and that the numerical results are correctly applied to solve them. With this in mind, this book emphasizes model-formulation and model-building skills as well as the interpretation of software output.

This book is intended to be used as an advanced beginning or an intermediate text in linear or mathematical programming. The following groups of students can benefit from using it:

- Undergraduate majors in quantitative methods in business, operations research, management science, or industrial engineering.
- MBA students enrolled in applications-oriented linear or mathematical programming courses.
- Graduate students who need an overview of the major topics in linear or mathematical programming.

The book contains enough material for a two semester-course; this allows instructors ample flexibility in adapting the text to their individual course plans.

This book is completely self-contained, with all necessary mathematical background given in Chapter 2 and Section 12.1. Students who are familiar with matrix multiplication should have no problems with the book. Chapter 12 is the only chapter that requires differential calculus. All topics in calculus needed in Chapter 12 are reviewed in Section 12.1.

The following features help to make this book reader-friendly:

- To provide immediate feedback to students, problems have been placed at the end of each section, and most chapters conclude with review problems. There are more than 750 problems, grouped according to difficulty: Group A for practice of basic techniques, Group B for underlying concepts, and Group C for mastering the theory independently.
- The book avoids excessive theoretical formulas in favor of word problems and interesting problem applications. Many problems are based on published applications of operations research and management science. The exposition takes

great pains, by means of several examples in each chapter, to guide the student step-by-step through even complex topics. Mathematical programming algorithms still receive comprehensive treatment; for instance, Karmarkar's method for solving linear programming problems is explained in detail.

- To help students review for tests, each chapter has a summary of concepts and formulas. An instructor's manual with complete solutions to all problems and advice for instructors is available to adopters of the text.
- Each section is written to be as self-contained as possible. This allows the instructor to be extremely flexible in designing a course. The instructor's manual identifies which portions of the book must be covered as prerequisites to each section.
- The book includes guidance in using the popular LINDO and GINO software packages and interpreting their output. If desired, PC versions of LINDO and GINO may be purchased with the book. The LINDO and GINO sections are completely self-contained, however, and classes that do not use them can simply skip those sections. Starred (*) sections cover less important topics and may be omitted with no loss of continuity.

Since not all students need a full-blown theoretical treatment of sensitivity analysis, there are two chapters on the topic. Chapter 5 is an applied approach to sensitivity analysis, emphasizing the interpretation of computer output. Chapter 6 contains a full discussion of sensitivity analysis, duality, and the dual simplex method. The instructor should cover Chapter 5 *or* Chapter 6, but not both. Classes emphasizing model-building and model-formulation skills should cover Chapter 5. Those paying close attention to the algorithms of mathematical programming (particularly classes in which many students will go on to further operations research courses) should study Chapter 6. If Chapter 5 rather than Chapter 6 is covered, then Chapter 2 may be omitted.

Mathematical Programming: Applications and Algorithms seeks to cover the important topics in mathematical programming at a beginning-to-intermediate level. It is hoped that after reading the book's discussions of particular topics, students will be able to (and will want to) delve into the more specialized references at the end of the chapters, which take up the topics in more detail.

◆ ACKNOWLEDGMENTS

Many people have played significant roles in the development of this text. My views on teaching mathematical programming were greatly influenced by the many excellent teachers I have had, including Gordon Bradley, Eric Denardo, John Little, Robert Mifflin, Matthew Sobel, and Harvey Wagner. The LINDO and GINO printouts throughout the book appear courtesy of Professor Linus Schrage.

Kathy Wilson diligently proofread the manuscript, and Nyla Fulk and Teena Albright did a great job of getting it ready for publication. Rob Easley checked the page proofs. Special thanks go to the helpful people at PWS-KENT: Senior Editor Michael Payne, Production Supervisor Eve Mendelsohn, and Assistant Editor Marcia Cole, as well as David Hoyt of Hoyt Publishing Services, who coordinated the production of the text.

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I retain responsibility for all errors and would greatly appreciate any comments on the book.

Wayne L. Winston

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*Starred sections may be omitted with no loss of continuity.

**Note: Instructors should cover Chapter 5 or Chapter 6, but not both.

CHAPTER 1

Introduction to Mathematical Programming

IN A MATHEMATICAL programming problem, the decision maker wishes to choose decision variables to maximize or minimize an objective function, subject to the requirement that the decision variables satisfy certain constraints. Some examples will illustrate the basic ideas.

1-1 EXAMPLES OF MATHEMATICAL PROGRAMMING

EXAMPLE 1

The ADV Advertising Agency has been hired by an auto company. ADV has been told how many high-income men, high-income women, low-income men, and low-income women should see the auto company's ads. ADV must determine the number of ads that should be placed in each available medium in order to minimize the automaker's advertising cost.

Under certain assumptions, this problem may be formulated as a **linear programming** problem. The decision variables represent the number of ads placed in each medium. The constraints ensure that sufficient numbers of each type of potential customer see the ads. The objective function represents the cost of all ads placed. In Example 2 of Chapter 3, we explain how to develop a mathematical model of this situation and determine cost-minimizing values of the decision variables. Chapters 3 through 10 discuss the solution of linear programming problems. In order to understand the methods used to solve linear programming problems, some knowledge of linear or matrix algebra is needed; Chapter 2 reviews the necessary linear algebra material.

EXAMPLE 2

Money Management Incorporated (MMI) has 100 possible investments. The company's goal is to maximize the expected return on its investment portfolio, subject to the constraint that the risk (or variance) of the portfolio not exceed a given level.

This problem may be presented as a **nonlinear programming** model (see Chapter 12). The decision variables represent the amount of money placed in each investment; the objective function represents the expected return of the portfolio; and the constraint ensures that the risk of the portfolio will not exceed a given level. The formulation and solution of portfolio models are discussed in Section 12.10.

In addition to linear and nonlinear programming, we also cover game theory (Chapter 11) and dynamic programming (Chapter 13). Situations in which a decision maker may have more than one objective are discussed in Section 4.14.

Our primary emphasis in this book is to teach the skills of model formulation and interpretation of the model's "solution." That is, we want the reader to be able to take a verbal description of a situation and formalize the decision variables, objective function, and constraints. In most cases, interpretation of a model's solution means interpreting computer output. This book covers the LINDO and GINO computer packages in considerable detail but does not neglect the algorithms used to solve mathematical programming problems; most of the standard algorithms are extensively discussed. However, the book has been written in such a way that the instructor who so desires may emphasize model-building skills and virtually ignore the discussion of algorithms.

1-2 SUCCESSFUL APPLICATIONS OF MATHEMATICAL PROGRAMMING

In this section, we list several applications of mathematical programming. In many of them, the business or government agency involved saved millions of dollars by successfully applying mathematical programming.

1. *Police Patrol Officer Scheduling in San Francisco.* Using linear programming (see Chapter 3), goal programming (see Chapter 4), and integer programming (see Chapter 9), Taylor and Huxley (1989) devised a method to schedule patrol officers for the San Francisco Police Department. By using their method, the department has saved \$11 million per year, improved response times by 20%, and increased revenue from traffic citations by \$3 million per year.

2. *Reducing Fuel Costs in the Electric Power Industry.* By using dynamic programming (see Chapter 13), Chao et al. (1989) saved 79 electric utilities over \$125 million in purchasing, inventory, and shortage costs.

3. *Designing an Ingot Mold Stripping Facility at Bethlehem Steel.* Using integer programming (see Chapter 9), Vasko et al. (1989) helped Bethlehem Steel design an ingot mold stripping facility. The integer programming model has saved Bethlehem \$8 million per year in operating costs.

4. *Gasoline Blending at Texaco.* Using the blending models discussed in Section 3.8 and nonlinear programming (discussed in Chapter 12), Dewitt et al. (1989) devised a model that is used by Texaco's refineries to determine how to blend incoming crude oils into leaded regular, unleaded regular, unleaded plus, and super unleaded gasolines. It is estimated that this model saves Texaco over \$30 million annually. The model also allows Texaco to answer many what-if questions, such as what an increase of 0.01% in the sulfur content of regular gasoline will do to the cost of producing regular gasoline. The method used to answer such what-if questions is called **sensitivity analysis** and is discussed in Chapters 5 and 6.

5. *Scheduling Trucks at North American Van Lines.* Using network models (see Chapter 8) and dynamic programming (see Chapter 13), Powell et al. (1988) developed a model that is used to assign loads to North American Van Lines drivers. Use of this model has provided better service to customers and reduced costs by $2.5 million per year.

6. *Using Linear Programming to Determine Bond Portfolios.* Linear programming (see Chapter 3) has been used by several people (see Chandy and Kharabe (1986)) to determine bond portfolios that maximize expected return subject to constraints on the level of risk and diversification in the portfolio.

7. *Using Linear Programming to Plan Creamery Production.* Sullivan and Secrest (1985) used linear programming (see Chapter 3) to determine how a creamery should process buttermilk, raw milk, sweet whey, and cream into cream cheese, cottage cheese, sour cream, and whey cream. Use of the model has increased the profitability of the creamery by $48,000 per year.

8. *Equipment Replacement at Phillips Petroleum.* How old should a car or truck be before a company should replace it? Phillips Petroleum (see Waddell (1983)) used equipment replacement models (discussed in Sections 8.2 and 13.5) to answer this question. These equipment replacement models are estimated to save Phillips $90,000 per year.

1-3 WHERE TO READ MORE ABOUT MATHEMATICAL PROGRAMMING

Many journals publish articles involving the theory and applications of mathematical programming: *Operations Research, Management Science, Interfaces, Mathematics of Operations Research, Marketing Science, AIIE Transactions, Decision Sciences, Mathematical Programming, European Journal of Operations Research,* and *Naval Research Logistics*, among others. For the reader who is particularly interested in present-day applications of mathematical programming, we heartily recommend *Interfaces.*

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CHAPTER 2

Basic Linear Algebra

IN THIS CHAPTER, we study the topics in linear algebra that will be needed in the rest of the book. We begin by discussing the building blocks of linear algebra: matrices and vectors. Then we use our knowledge of matrices and vectors to develop a systematic procedure (the Gauss-Jordan method) for solving linear equations, which we then use to invert matrices. We close the chapter with an introduction to determinants.

The material covered in this chapter will be used in our study of linear programming and nonlinear programming.

2-1 MATRICES AND VECTORS

MATRICES

DEFINITION A **matrix** is any rectangular array of numbers.

For example,

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \quad [2 \quad 1]$$

are all matrices.

If a matrix A has m rows and n columns, we call A an $m \times n$ matrix. We refer to $m \times n$ as the **order** of the matrix. A typical $m \times n$ matrix A may be written as

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

DEFINITION The number in the ith row and jth column of A is called the ***ij*th element** of A and is written a_{ij}.