STRENGTH OF MATERIALS

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PREFACE TO THE FIFTH EDITION

In this revision a conscious attempt has been made to emphasize the fundamentals of the theory and accent the process of analysis in the application of mathematics to strength of materials. The objective has been to inculcate sound methods by utilizing fundamental ideas, physical constants, and reasonable stresses with a minimum of memory formulas. A comprehensive grasp and working knowledge of the subject can be obtained only by a thoughtful application of principles. While this may require more time in the early development of the student, it enhances his growth later.

Several chapters have been rewritten. More emphasis has been placed on the area-moment method. The chapter on deflection by double integration has been retained, but the direction has been focused toward a logical development of the topic rather than the mechanical manipulation of equations. Integration between limits has been omitted. By the early introduction of Mohr's circle for the solution of simple stresses, it is hoped that the student will gain confidence before he is confronted with combined stress. Considerable new material has been added.

Some changes have been made in the order of topics. Shearing stresses in beams now immediately follows bending stresses. Special beam topics, including reinforced-concrete beams, have been delayed until after combined stress. References to both steel and aluminum rolled sections are made in the text and in some problems which require the use of the AISC Manual of Steel Construction and the Alcoa Structural Handbook. Many new problems have been added, making the total well over a thousand. The more difficult problems have been placed in miscellaneous lists at the ends of the chapters.

The writer acknowledges his indebtedness to Prof. J. E. Boyd, who pioneered this work and furnished much enthusiasm. His colleagues, Profs. P. W. Ott, R. W. Powell, E. C. Clark, and C. T. West have been very generous with their ideas and inspiration. Prof. M. G. Fontana of the Department of Metallurgy and Dr. John Zambrow of the Engineering Experiment Station furnished new data. Richard I. Hang of the Department of Engineering Drawing made the drawings, which uphold the tradition of the previous editions.

S. B. FOLK

COLUMBUS, OHIO
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PREFACE TO THE FIRST EDITION

This book is intended to give the student a grasp of the physical and mathematical ideas underlying the Mechanics of Materials, together with enough of the experimental facts and simple applications to sustain his interest, fix his theory, and prepare him for the technical subjects as given in works on Machine Design, Reinforced Concrete, or Stresses in Structures.

It is assumed that the reader has completed the Integral Calculus, and has taken a course in Theoretical Mechanics which includes statics and the moment of inertia of plane areas. Chapters XVI and XVII give a brief discussion of center of gravity and moment of inertia. Students who have not mastered these subjects should study these chapters before taking up Chapter V (preferably before beginning Chapter I).

The problems, which are given with nearly every article, form an essential part of the development of the subject. They were prepared with the twofold object of fixing the theory and enabling the student to discover for himself important facts and applications. The first problems of each set usually require the use of but one new principle—the one given in the text which immediately precedes; the later problems aim to combine this principle with others previously studied and with the fundamental operations of Mathematics and Mechanics. The constants given in the data or derived from the results of the problems fall within the range of the figures obtained from actual tests of materials. Many of the problems are taken directly from such measurements. Some of them are from tests made by the author or his colleagues at the Ohio State University; others are from bulletins of the University of Illinois Engineering Experiment Station, from "Test of Metals" at the Watertown Arsenal, and from the Transactions of the American Society of Civil Engineers.

This book is designed for use with "Cambria Steel," to which references are made by title instead of by page, so that they are adapted to any edition of the handbook.

The author acknowledges his indebtedness for suggestions and criticisms to Professors C. T. Morris, E. F. Coddington, Robert Meiklejohn, K. D. Swartzel, and many others of the Faculty of the

College of Engineering; and to Professor Horace Judd of the Department of Mechanical Engineering for the material for several of the half-tones. He also expresses his obligations to the books which have helped to mold his ideas of the subject,—Johnson's "Materials of Construction," Ewing's "Strength of Materials," and especially the textbooks which he has used with his classes,—Merriman's "Mechanics of Materials," Heller's "Stresses in Structures," and Goodman's "Mechanics Applied to Engineering."

The symbols used in the mathematical expressions are much the same as in Heller's "Stresses in Structures."

J. E. B.

Columbus, Ohio
November 6, 1911

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CHAPTER 1

STRESSES

1. Strength of Materials. That branch of mechanics which treats of the changes in form and dimensions of elastic solids and the forces which cause these changes is called *the mechanics of materials.* When the physical constants and the results of experimental tests upon the materials of construction are included with the theoretical discussion of the ideal elastic solid, the entire subject is called *the strength of materials* or *the resistance of materials.*

2. Tension. Figure 1 shows a rubber band which is suspended from a horizontal bar and carries a hook at the lower end. When a small weight is hung on the hook, the rubber band is stretched; its length is increased by an amount a, while its cross section is reduced. When a second weight is added, there is an additional elongation b. If the weights are equal, the elongation a caused by the first weight is equal to the elongation b caused by the second weight. When the weights are removed, the rubber band returns to its original length and cross section.

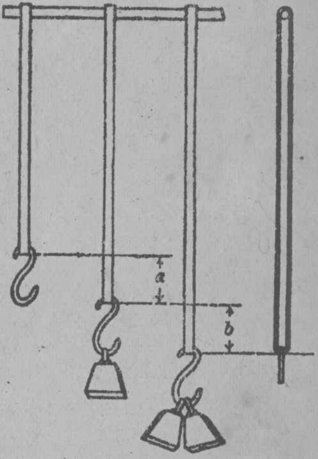

FIG. 1. Rubber bands in tension.

If steel, iron, wood, concrete, stone, or other solid material is used instead of rubber, the results are similar. There is this apparent difference: while the rubber may be stretched to twice or three times its original length and still return to its original size and shape after the load is removed, one of the other materials may be stretched only a very small amount (usually less than 0.002 of its length), without receiving a permanent change in its dimensions. Again, the force required to produce a relatively small increase in the length of a rod of wood or steel, for instance, is many times greater than that necessary to *double* the length of a soft rubber band of equal cross section. These differences between the behavior of soft rubber and other solid materials are differences of degree and not of kind. Essentially they are alike.

The rubber bands shown in **Fig. 1** are subjected to the action of two forces: the force of the weights pulling downward, and the reaction of the support pulling upward. The bands are in *tension*. A body is said to be in tension when it is subjected to two sets of forces whose resultants are in the same straight line, opposite in direction, and directed *away* from each other.

3. Compression. When a body is subjected to two sets of forces whose resultants are in the same straight line, opposite in direction, and directed *toward* each other, it is said to be in *compression*. In Fig. 2, the block B is in compression under the action of the 50 pounds pushing down and the reaction of the support pushing up. The effect of compression upon a body is to shorten it in the line of the forces and increase its dimensions in the plane perpendicular to this line.

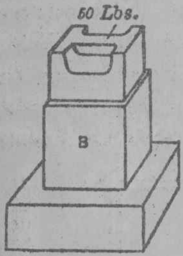

FIG. 2. Compression.

Tension and compression may be represented as in Fig. 3, in which the arrows represent the forces, and the small rectangles represent the bodies, or portions of a body, upon which the forces act. The rectangles are often omitted; a pair of arrows with their heads together indicates compression, and a pair with their heads in the opposite sense indicates tension.

4. Force. The force exerted by one body on another at their surface of contact produces a stress in the bodies. In Fig. 2 the total force is 50 pounds. The *stress* produced is compressive in block B. The support pushes up against the body with an equal force. The total load on a body will always be called the *force*, or *load*.

Figure 4 represents a bar subjected to a horizontal pull of P pounds. If the bar is supposed to be cut by an *imaginary plane* at C, the portion A to the left of this *plane section* is in equilibrium under the action of the external pull P_1 toward the left and an equal opposite pull P_3 at the section C. This force P_3 across the section is the pull exerted by the right portion B upon the left portion A. In like manner, the right portion B is in equilibrium under the external pull P_2 at the right end of the bar and the internal P_4, equal and opposite to P_3, exerted by the left portion A upon the right portion B across the section, as shown separately in Fig. 4,II.

FIG. 3.

Figure 4,II is a free-body diagram for body B. Equilibrium of a free body is maintained by the action of *total forces* on the body. These forces may be internal or external.

5. Unit Stress; Intensity of Stress. The average *unit compressive* or *tensile stress* at any section of a body is calculated by dividing the total force by the area of the cross section at right angles to the force. If a vertical force P is applied to the cylinder C of Fig. 5 by means of the plate B and the reaction of the support D, the unit stress at any section is given by the equation

$$s = \frac{P}{A} \qquad \text{Formula I}^{1}$$

in which s is the *unit stress*, P is the *external* force, and A is the area of cross section perpendicular to the direction of the stress. *Unit stress*

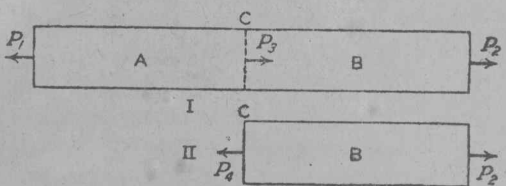

FIG. 4. Stress at section.

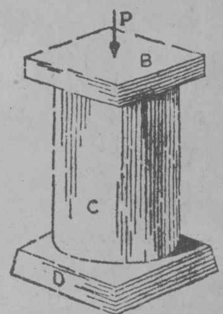

FIG. 5. Area under stress.

frequently is called *intensity* of *stress*. In American engineering practice, unit stresses generally are given in pounds per square inch or kips per square inch. (One *kip* or kilo pound is 1,000 pounds.) Frequently compressive stresses in large masonry structures are expressed in tons per square foot. It is the common practice to give bearing pressure of masonry on soils in this way. British engineers employ long tons per square inch as well as pounds per square inch to express the intensity of stress in steel and similar materials. Continental[2] engineers, of course, use kilograms per square centimeter. Physicists prefer dynes per square centimeter or dynes per square millimeter. Stress in pounds per square inch may be written psi.

In elementary mechanics the tensile or compressive stress exerted by a bar is usually assumed to lie in the axis of the member. In reality each longitudinal element exerts its portion of the stress. The force

[1] Important formulas, which should be understood and memorized, are designated by Roman numerals in this book.

[2] They sometimes use atmospheres. One atmosphere equals 14.7 pounds per square inch, or 1.033 kilograms per square centimeter.

assumed to act along the axis is the resultant of the forces exerted by all the elements. The unit stress obtained by dividing the total applied force by the area of the cross section is the *average unit stress* in the member.

Figure 6,I shows a bar under tensile stress which is uniform in all parts of the section. The arrows which represent the stress of different elements are all of equal length. Figure 6,II shows a bar under uniform compressive stress. Figure 6,III shows compressive stress which increases uniformly from left to right.

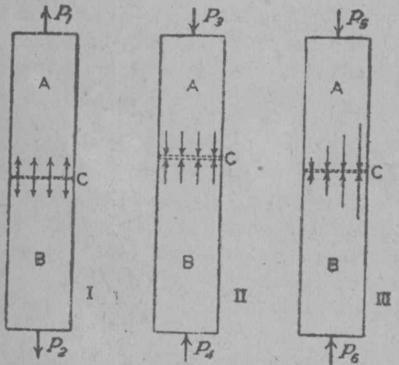

FIG. 6. Representation of stress.

When the stress is uniform, the resultant stress passes through the center of gravity of each cross section, which corresponds to the center of gravity of a short piece of uniform length cut from the bar. When the stress is not uniform, the location of the resultant may be found by calculating the sum of the moments with respect to some parallel plane of the force on each element of area and dividing this moment by the sum of the forces. In other words, the resultant passes through the center of gravity of a solid whose base is the section of the bar and whose altitude at any point is proportional to the unit stress at that point.

Problems

(Find the dimensions of rolled shapes in a steel or aluminum handbook.)

5-1. The cylinder of Fig. 5 is 2 in. in diameter and 5 in. long. Find the unit stress when a vertical load of 14,000 lb is applied by means of the plate B.

Ans. 4,456 psi.

5-2. A piece 6 in. long is cut from a 5- by 3- by ½-in. angle section by planes perpendicular to its length. The piece stands vertical and a load of 30,000 lb is applied at the top by means of a 5- by 3- by 1-in. steel plate. Find the unit stress in the angle.

Ans. $s = 8{,}000$ psi.

5-3. Two edges of the plate in Prob. 5-2 lie in the planes of the back of the legs of the angle section. The load is applied to the plate by means of a steel ball. Where must this ball be placed in order that the unit stress in the angle may be uniform?

Ans. 1.75 in. from one 3-in. edge, and 0.75 in. from one 5-in. edge.

5-4. A 10-in. 25.4-lb standard I beam 12 in. long stands on end and carries a total vertical load of 15,000 lb on top. Find the average unit stress.

Ans. 2,030 psi.

5-5. The beam in Prob. 5-4 rests on one flange and the load is applied on the top flange. Find the maximun unit stress and indicate the cross section where it occurs. *Ans.* 4,030 psi.

5-6. A 12-in. length of 6-in. standard pipe is placed between two steel plates and subjected to a force of 30,000 lb. Find the unit compressive stress in the cross section. *Ans.* 5,380 psi.

5-7. A 4- by 3-in. 2.84-lb aluminum tee section 12 in. long stands on end and carries an 8,000-lb load on top. Find the unit stress in a cross section. *Ans.* 3,420 psi.

5-8. A block in the form of a frustum of a pyramid is 2 in. square at the top, 3 in. square at the bottom, and 8 in. high. Find the unit stress 2 in. from the bottom and 4 in. from the bottom when a load of 7,200 lb is placed on the top. *Ans.* 952.1 psi; 1,152 psi.

5-9. In a short block 2 in. square, the unit stress increases uniformly from 100 psi in the left face to 700 psi in the right face. Find the total load. *Ans.* P = 1,600 lb.

5-10. In Prob. 5-9, find the location of the resultant force. Represent the stress in the front face by a trapezoid 100 units high on the left and 700 units high on the right. Find the center of gravity of the trapezoidal wedge which represents the force by combining the moment and area of two triangles, or the moment and area of a triangle and a rectangle. *Ans.* 1.25 in. from the left face; 1 in. from the front face.

5-11. A short block of triangular section has two faces each 13 in. wide, and one face 10 in. wide. The block is subjected to compression parallel to its length which causes the unit stress to increase uniformly from 100 psi at the intersection of the 13-in. faces to 700 psi in the 10-in. face. Find the total load by integration. Show that this load equals the area of the section multiplied by the unit stress at the center of gravity of the cross section. *Ans.* P = 30,000 lb.

5-12. By integration of moments, find the line of action of the resultant force of Prob. 5-11. *Ans.* 8.8 in. from the intersection of the 13-in. faces.

6. Working Stress; Allowable Unit Stress. Working stresses are the unit stresses to which the materials of a machine or structure are subjected. The *allowable unit stress* for a given material is the maximum unit stress which, in the judgment of some competent and official authority, should be applied to this material. For instance, the specifications of the American Institute of Steel Construction give 20,000 pounds per square inch as the unit tensile stress for structural steel. For the compressive stress in relatively short blocks of select-grade white oak in situations which are always dry, the American Society for Testing Materials specifies 1,000 pounds per square inch parallel to the grain. The Joint Committee of Concrete and Reinforced Concrete[1] gives 25 per cent of the compressive strength at 28 days as the allowable compressive stress of concrete.

[1] This committee is made up of representatives from the American Society

Table 1 gives a few allowable stresses in tension and compression.

TABLE 1. ALLOWABLE UNIT STRESS
(This table should be memorized.)

Material	Tension, psi	Compression, psi	
Structural steel	20,000	20,000	
Cast steel	16,000	16,000	
Wrought iron	12,000	12,000	
Cast iron	3,000	15,000	
Nickel steel	25,000	25,000	
Bolts on nominal area at root of thread	20,000		
Butt welds, section through throat	20,000		
Aluminum alloy 17S-T and 24S-T	15,000	15,000	
Portland cement concrete	.....	600	
		With grain	Across grain
Common-grade timber in dry location:			
Douglas fir, coast region	......	880	325
Southern yellow pine	......	880	325
White or red oak	.....	800	500

A steel bar 1 foot long and 1 square inch in cross section weighs 3.4 pounds. For estimating purposes 1 cubic inch of steel weighs 0.283 pounds and 1 cubic inch of aluminum weighs 0.1 pound, although alloys will vary considerably from these figures.

Problems

(Use the data of Table 1 unless otherwise specified.)

6-1. Find the total allowable load, in compression parallel to the grain, which may be applied to a 4- by 6-in. short block of southern yellow pine.

Ans. 21,120 lb.

6-2. What must be the dimensions of a cubical block of white oak which supports a load of 50,000 lb? (Two solutions.)

6-3. An I bar of structural steel, 1 in. thick, exerts a pull of 60,000 lb. What is its minimum width?

6-4. A piece of 6-in. wrought-iron water pipe is 2 ft long and 6⅝ in. in outside diameter. What is the allowable load on the pipe standing on end?

Ans. 66,970 lb.

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of Civil Engineers, the American Society for Testing Materials, the American Railway Engineering Association, the American Concrete Institute, and the Portland Cement Association.

6-5. A yellow-pine beam 8 in. wide rests on the end of the pipe of Prob. 6-4. A steel plate, 10 in. square, transmits the load from the beam to the pipe. Find the allowable load. *Ans.* 26,000 lb.

6-6. A 1-in. steel bolt supports a load by means of a square standard nut. What is the allowable load? (Use AISC handbook.) *Ans.* 11,020 lb.

6-7. In Prob. 6-6 what is the bearing stress on the nut when the bolt carries its allowable load? *Ans.* 7,720 lb.

6-8. The bolt of Prob. 6-6 runs vertically through an oak beam. Find the diameter of the washer required.

6-9. In Fig. 7 the pin-connected steel truss is hinged at J and held by a horizontal force at A. Find the diameter of the round rod AB. *Ans.* 1.67 in.

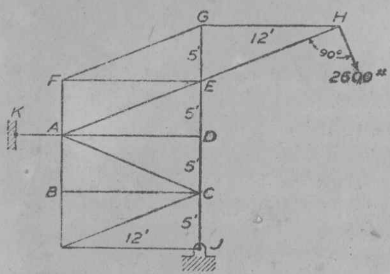

FIG. 7.

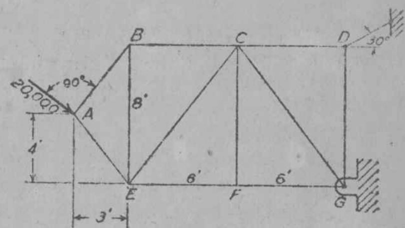

FIG. 8.

6-10. The pin-connected aluminum truss shown in Fig. 8 is hinged to the wall at G and held by a force at D. Find the diameter of a round rod to be used at CD. *Ans.* 1.28 in.

6-11. What is the allowable load in tension on a steel rod which is 5 ft 6 in. long and weighs 70 lb?

7. Deformation; Unit Deformation. The changes in dimensions which occur when forces are applied to a body are called *deformations*. In Fig. 1, the increase in length a, which takes place when the first load is applied, is the deformation caused by that load; the increase b is the deformation caused by the second load; and $a + b$ is the deformation caused by the two loads. The deformation produced by a *tensile* force or *pull* is an *elongation*. The deformation produced by a *compressive* force or *push* is a *compression*. Compression is negative elongation. A deformation which remains after the force is removed is called a *set*.

Unit deformation in a body is the deformation per unit length. In a bar of uniform cross section, the unit deformation is calculated by dividing the total deformation of a given portion of the bar by the original length of the portion. In Fig. 1, the length a divided by the original length of the band is the unit deformation caused by the first load. Unit deformation is frequently called *relative deformation*.

In algebraic equations many authors represent unit deformation by the letter ϵ (epsilon).

Deformation is frequently called *strain*. The word *strain* was formerly used as a synonym for *stress* and is still sometimes heard in that sense. The general practice of technical literature, however, is now to use *strain* to mean *deformation*. When employed in this book, it will always have that meaning. Total deformation in a length L sometimes is represented by e. Unit deformation is then

$$\epsilon = \frac{e}{L} \qquad \text{Formula II}$$

Problems

7-1. When a steel bar is subjected to a tensile stress, a portion, originally 8 in. long, is stretched 0.0052 in. Find the unit elongation. *Ans.* 0.00065

7-2. An oak post under compression is shortened 0.1476 in. in a length of 15 ft. Find the unit deformation. *Ans.* 0.00082

7-3. A ⅞-in. steel rod 20 in. long is subjected to a pull of 15,176 lb. A portion of the rod, originally 8 in. long, is stretched 0.0054 in. when the force is applied. Find the unit stress and the unit deformation.

7-4. The coefficient of expansion of steel is 0.000012 for 1°C. Find the unit deformation and the total deformation in a steel rod 15 ft long when the temperature changes from 50 to 20°C. Solve when the temperature changes from 14°F to 14°C. *Ans.* 0.000288; 0.05184 in.

8. Elastic Limit. When a force is applied to a solid body and then removed, the body returns to its original size and shape, provided the unit stress developed by the force has not exceeded a certain limit. If the stress has gone beyond this limit, the body does not return entirely to its original dimensions but retains some permanent deformation or *set*. The *unit stress* at this limit is called the *elastic limit* of the material. A soft-steel rod may be stretched 0.0054 inch in a gage length of 8 inches by a pull of 20,000 pounds per square inch. When this load is removed, the rod shortens to its original length. A pull of 30,000 pounds per square inch may stretch this rod 0.0081 inch, and the rod may return to its original length when the load is removed. A load of 32,000 pounds per square inch may stretch the rod 0.0200 inch. When this load is removed, the rod may have an elongation of 0.0110 inch. The rod shortens about 0.0090 inch, while the remaining elongation of 0.0110 inch persists as a permanent set. Evidently, the elastic limit is between 30,000 and 32,000 pounds per square inch.

It is difficult to determine the elastic limit with exactness. A test piece may appear to have no residual deformation when measured with the usual apparatus and still show some set when more delicate instruments are employed. **Time is a factor.** If a load is applied for a considerable period, it causes somewhat greater deformation and considerably greater set than it would cause if the time of application were shorter. Some materials, such as steel, after having been subjected to comparatively large unit stress, frequently show a set of more or less temporary character. When the load is first removed, there is a residual deformation, which may partly or wholly vanish after some little interval.

9. Modulus of Elasticity. For all stresses below the elastic limit, the ratio of the unit stress to the unit deformation is *nearly* constant. The quotient obtained by dividing any given change of unit stress by the accompanying change in unit deformation is called the *modulus of elasticity* or *Young's modulus*.[1] Modulus of elasticity is represented in physical equations by the letter E. In algebraic language, the definition of the modulus of elasticity is

$$E = \frac{s}{\epsilon} \qquad \text{Formula III}$$

in which E is the modulus of elasticity, s represents a change in the unit stress, and ϵ is the change in unit deformation which accompanies this change of unit stress.

Problems

9-1. A 2- by 1.5-in. bar is tested in tension. When the load changed from 3,000 to 48,000 lb, the dial reading for a gage length of 8 in. changed from 0.00080 to 0.00492 in. Find the change in unit stress, the change in unit deformation, and the modulus of elasticity. *Ans.* E = 29,130,000 psi.

9-2. A steel rod 0.600 in. in diameter is stretched 0.00536 in. in a gage length of 8 in. when the load changed from 1,415 to 7,057 lb. Using the area to three significant figures, find the modulus of elasticity. *Ans.* E = 29,850,000 psi.

9-3. A timber piece 2 in. square is shortened 0.014 in. in a length of 20 in. Find the force required if the modulus of elasticity is 2,000,000 psi. How does the unit stress compare with the allowable compressive stress for southern pine?

9-4. A 10-in. 30-lb standard channel 10 ft long is subjected to a compressive load of 88,000 lb parallel to its length. How much is the channel shortened if E = 29,300,000? *Ans.* 0.0410 in.

9-5. A 4- by 3- by 5⁄16-in. 2.34-lb aluminum tee 12 in. long shortened 0.000876 in. in an 8-in. gage length when the total load on the end increased from 484 to 6,292 lb. Find the modulus of elasticity. *Ans.* 10,960,000 psi.

9-6. In the pin-connected steel truss of Fig. 7, the member AE is a ½-in. square bar. If the modulus of elasticity is 30,000,000 psi, find the total change in length which occurs when the 2,600-lb load is applied.

[1] Dr. Thomas Young (1773–1829), who was not an engineer, brought the idea to the attention of the English engineers.