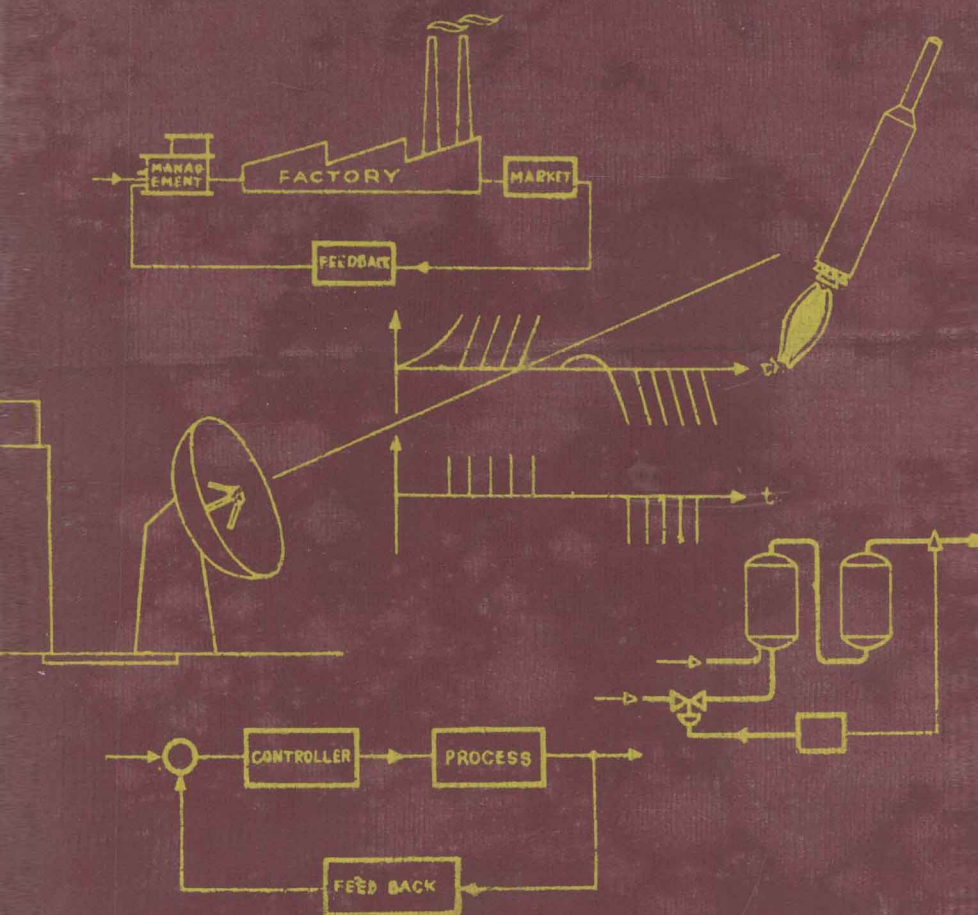


Control System Theory

DR. SUSHIL DAS GUPTA



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CONTROL SYSTEM THEORY

[A TEXTBOOK FOR ENGINEERING STUDENTS]

By

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Calcutta.

1975

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This book is dedicated to

MY TEACHERS

at

Jadavpur University, India.

State University of Iowa, U.S.A.

University of Wisconsin, U.S.A.

Preface

This book on control system theory is written mainly for the undergraduate students in engineering science. Most of the materials presented here is essentially an outgrowth of what is presented to the electrical engineering undergraduates of Jadavpur University. The material presented here should be good for a two year course with three lectures a week through each year. Should it be necessary to give a less detailed course, the teacher concerned may easily choose the topics for class discussion according to his own requirements.

The development of the subject of automatic control systems theory in this book, has been preceded by some elementary discussions of systems and, in particular, linear systems. It is felt that system theory is a basic pre-requisite for, not only students specializing in control systems but also other systems in general. Indeed, this should be a general compulsory course for the students in virtually all the engineering disciplines. It is with this notion, that the author has drawn examples from a diverse field to illustrate the concepts of system. As this is a first course we have intentionally avoided looking at the system from the more mathematical point of view of mapping of functions, etc. However, after going through this book, it should be a simple step forward for the interested and more mathematically oriented reader to extend his ideas in that direction. The required topics of the linear Algebra to help in such venture has been included in appendix II. The materials in matrix theory however, has been presented in a relatively elementary level for more general usage.

To day, particularly with the introduction of discrete devices, solid-state and others, the discrete data systems play a very important role in our every day life, so much so, that it can not be left out of an engineering first course in control system. This has been included in chapter 6. To the best of the author's knowledge, there had been no general classification of discrete data systems in any textbook published so far. This has been attempted in this chapter.

Most of the results in system and in particular control system theory and many of the present works are now based on the concepts of state variables. Added with this is the resultant simplification in analog and digital modelling of systems. These advantages has helped this approach to make its own place in the control systems theory. This therefore has been treated in some detail in chapter 8.

The chapter 12 on the non-linear systems is essentially a continuation of the notions on state-variable theory. Thus the treatment here is only for a limited class of non-linear systems, which however encompasses a reasonably large application area. The central theme in this presentation has been to equip the reader with some "horse sense" with which he could attack a non-linear problem and attempt to explain the non-linear phenomenon in such control systems. To obtain a deeper understanding of the subject, the interested reader may go through the references suggested at the end of chapters 2 and 12 respectively.

We have assumed a reasonable knowledge of Laplace transform theory and hence a note of brevity will be evident in the appendix devoted to it.

Because of the large number of industrial examples provided in this book and because the discussions are given in a detailed fashion with many worked out examples, it is felt that this book will also be useful to practising engineers, who did not have an opportunity of receiving a formal training in the subject. There have been incorporated in appendix III, discussions on some component devices, and circuit arrangement commonly used in servo mechanisms.

The book has come out after five years from when its first writing started. Author's ideas about the subject have changed in the process, in many respects and considerably. Naturally, there had to be a compromise in presenting a distributed effort in time into a lumped form. However it is one of those perennial difficulties and unavoidable problems of evolution.

It is now the duty of the author to make acknowledgements. The blessing of being a teacher is that he earns a double share of inspirations. He is inspired by his teachers and also by his students. The present author is no exception in this matter. The materials presented has been a development of the ideas gathered from his teachers, Prof. H.C. Guha of Jadavpur University, Profs. L.A. Ware and E.B. Kurtz at the State University of Iowa and Profs. J.J. Skiles, T.J. Higgins, and V.C. Rideout at the University of Wisconsin.

Professor J.F. Coales of Cambridge University was greatly responsible in enabling the author to materially increased the content of this book. It was through his untiring effort that the author visited Cambridge University for a year, during which time a major portion of the book was written and some rewritten. In this connection, the authorities of Jadavpur University, Cambridge University, U.G.C. of India, in particular, Dr. D.S. Kothari and Dr. D. Shankar Narayan, Commonwealth Scholarship Association of U.K., President Dr. A.B. Pippard and his associates in Clare Hall at Cambridge University also helped immensely.

Discussions with Dr. A.T. Fuller, Dr. P.C. Young, of Cambridge University, Dr. Jan Willems of M.I.T. and Professor W.L. Root of the University of Michigan also helped.

Among the many students who contributed in the author's effort, some need special mention Mr. S. Mukhopadhyay, Mr. K. Dutta, Mr. P. Das Gupta, Mr. S. Basak solved several problems for this book. Colleagues, and students as well of the author, Mr. T. Ghoshal. Dr. A.K. Roy Mr. Shyamal Goswami and Mr. D. Bhattacharya helped him in preparing portions of chapters, 9, 11, Appendix III and chapter 13 respectively. Mr. Sid Maitra at Stanford University also helped.

One time classmate of the author and then his student and associate, Dr. K.K. Bandyopadhyay has meticulously gone through the proof of the whole book and gave quite a few suggestions for improvement in the process. In spite of his great care some error has appeared, which probably is not unnatural for a book of this size. The responsibility of such errors is of the author. He would welcome any suggestion in this or other matters by critical readers.

Finally, the author wishes to acknowledge the great patience shown by his family while the author had been writing this book, particularly while in Cambridge.

Calcutta

S. DAS GUPTA

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Introduction

The complexity of the problems of man's existence is growing at an ever-increasing rate and it may be expected to continue to do so in future. As such, constantly, human effort and capacity is replaced by machines and control systems, wherever possible, so that these may be employed to other problems, where it is indispensable.

It is indeed not surprising that such replacements perform the job more efficiently than, their human counterpart. These machines or processes are not susceptible to so much fatigue or as many blunders and errors as human beings are. Moreover, they may operate at relatively high speed and precision and can be designed with very little difficulty. This fact that, the machine can compete and miserably beat its human counterpart in some specific jobs, is indeed often embarrassing to the common man. This however should not be so alarming, because it is not yet possible to duplicate a human brain completely. Consequently, there are and will remain many facets of life where human intervention and effort would be indispensable. Such systems and automation will therefore only increase the capability of human being and his horizon of activity.

For the past few decades, the trend of the modern civilization have been in the direction of greater control. Its importance has grown tremendously in almost every field of technical endeavour. Thermostats regulate the temperature in air-conditioners, refrigerators, ovens and furnaces. Numerous control arrangements find their ways into industrial and military applications, for the control of quantities, such as position, speed, tension, temperature, humidity, pressure flow etc. Some specific examples are—tension controllers of sheet rolls in paper mills, thickness controllers of sheet metals in rolling mills, pressure controllers in boilers, concentration controllers, reaction controllers etc., in chemical processes, radar, antenna sweeping control, gun director, missile control and control in space etc.

Complex living has demanded automation of large-scale problems; thus for example, **transportation problems concerning dispatching, switching and signalling in rail roads, reservation system for air passengers, air traffic and landing systems etc. have been partly or fully automated.**

With the limited supply of natural resources and keen competition in allied business, one of the burning problems of the day is that of optimization of cost and material. Thus the electric power generation scheduling is automated to reduce cost. Plant managers seek ways and means to schedule his plant operation to maximize production and profit. Rocket engineer designs his rocket control arrangement to place the satellite in a desired orbit or send a space-craft on a mission with a specific pay load requiring minimum fuel.

Recent efforts in the area of wather prediction, hand-written character recognition, medical diagnosis and other types of pattern recognition deal with the most sophisticated type of control systems, known as learning systems.

1.1. Concept of a System

Before the concept of control system is introduced, it is desirable to discuss the essential features of a system. This is what we shall do in what follows. A more formal treatment will be given later.

A system is a co-ordinated unit of individual elements performing a specific function. It produces an output corresponding to a given input according to some rule. Thus, electrical, mechanical, hydraulic, pneumatic, chemical, analog, digital etc. or any other elements, devices or processes and/or interacting combinations thereof may be regarded as systems.

When, at least one element or member of a system is capable of storing energy or some similar capacity, such a system is known to be dynamic. In such systems one or more aspects or states give a lagging response, following an input. This means that the states pass through a transient condition before their arrive at their respective steady state values.

Some specific examples may help to illustrate the point.

EXAMPLES

A Thermometer. Consider the action of temperature upon a thermometer. We know that the length of the mercury column is directly dependent upon the temperature around the mercury bulb. We may identify the thermometer as a system whose input $u = \theta$ is the temperature and the output $x = l$ is the length of the mercury thread

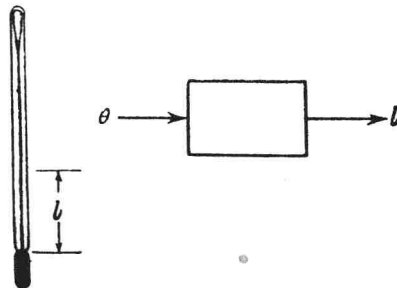


Fig. 1.1-1. A thermometer.