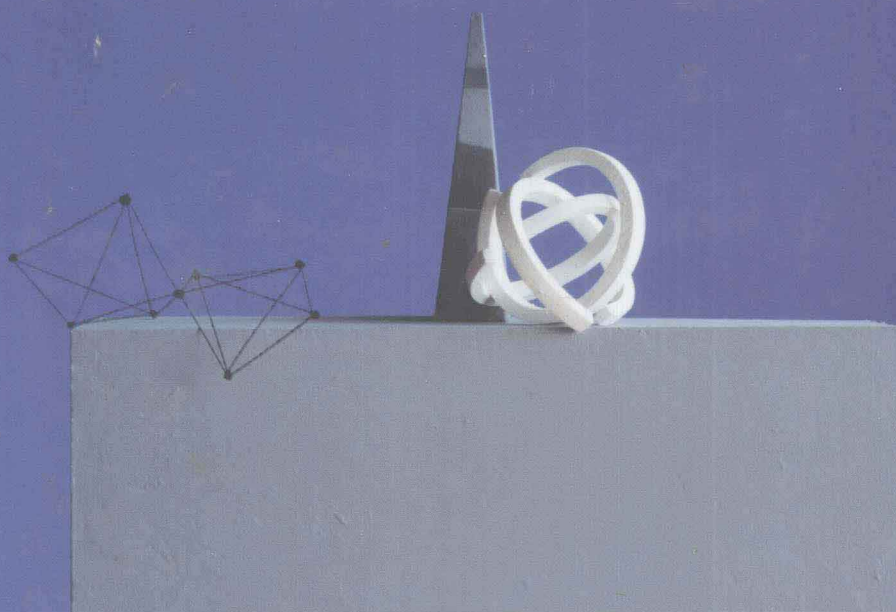


# Applied Mathematics

*for Business, Economics, Life Sciences, and Social Sciences*

Seventh Edition



*Raymond A. Barnett / Michael R. Ziegler / Karl E. Byleen*



# APPLIED MATHEMATICS

FOR BUSINESS, ECONOMICS, LIFE SCIENCES,  
AND SOCIAL SCIENCES

SEVENTH EDITION

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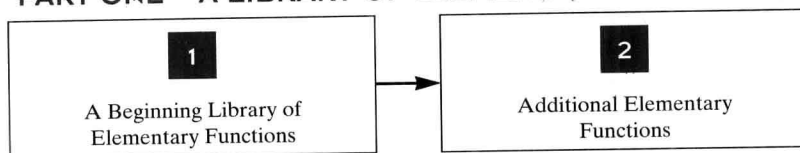
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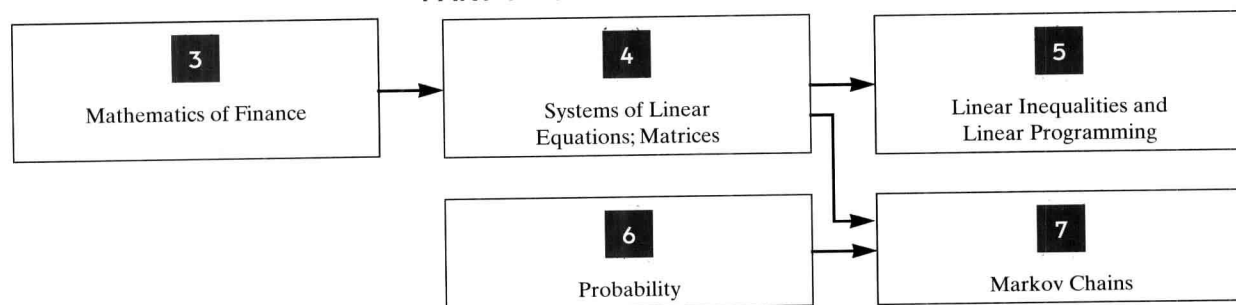
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# CHAPTER DEPENDENCIES

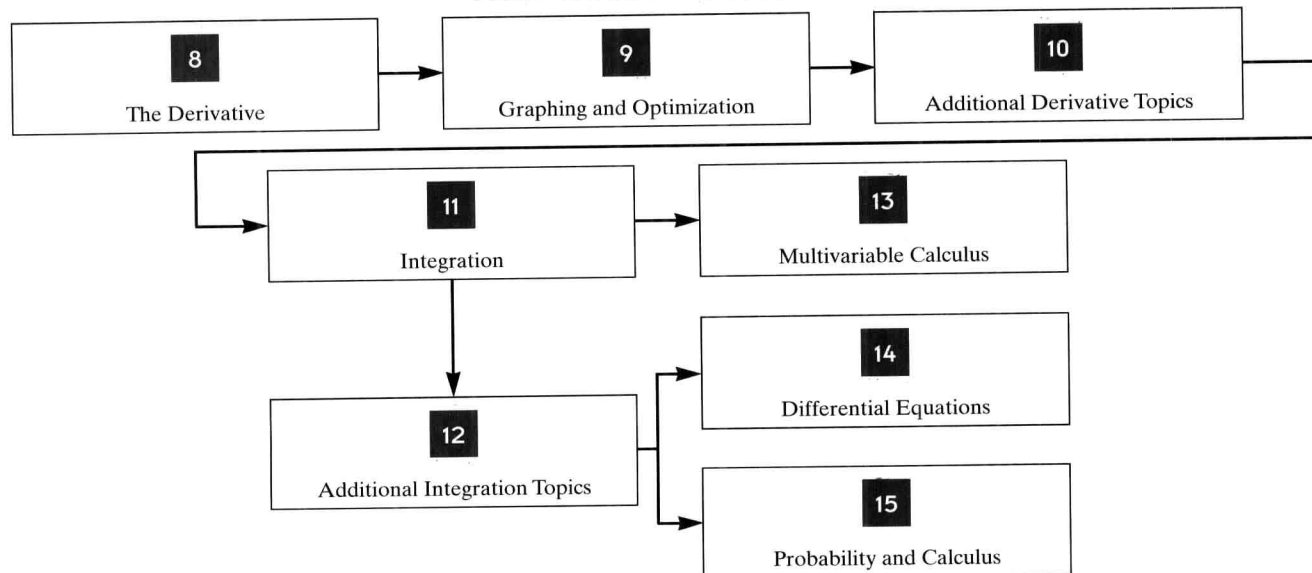
## PART ONE A LIBRARY OF ELEMENTARY FUNCTIONS\*



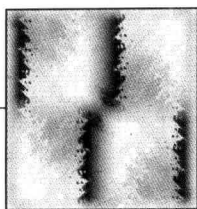
## PART TWO FINITE MATHEMATICS



## PART THREE CALCULUS



\*Selected topics from Part One may be referred to as needed in Parts Two and Three or reviewed systematically before starting either part.



# PREFACE

The seventh edition of *Applied Mathematics for Business, Economics, Life Sciences, and Social Sciences* is designed for a two-term (or condensed one-term) course in finite mathematics and calculus and for students who have had  $1\frac{1}{2}$ –2 years of high school algebra or the equivalent. The choice and independence of topics make the text readily adaptable to a variety of courses (see the Chapter Dependency Chart on page viii). It is one of eight books in the authors' college mathematics series.

Improvements in this edition evolved out of the generous response from a large number of users of the last and previous editions as well as survey results from instructors, mathematics departments, course outlines, and college catalogs. Fundamental to a book's growth and effectiveness is classroom use and feedback. Now in its seventh edition, *Applied Mathematics for Business, Economics, Life Sciences, and Social Sciences* has had the benefit of having a substantial amount of both.

## ■ EMPHASIS AND STYLE

The text is **written for student comprehension**. Great care has been taken to write a book that is mathematically correct and accessible to students. Emphasis is on computational skills, ideas, and problem solving rather than mathematical theory. Most derivations and proofs are omitted except where their inclusion adds significant insight into a particular concept. General concepts and results are usually presented only after particular cases have been discussed.

## ■ EXAMPLES AND MATCHED PROBLEMS

**Over 420 completely worked examples** are used to introduce concepts and to demonstrate problem-solving techniques. Many examples have multiple parts, significantly increasing the total number of worked examples. Each example is followed by a similar **matched problem for the student to work** while reading the material. This actively involves the student in the learning process. The answers to these matched problems are included at the end of each section for easy reference.

## ■ EXPLORATION AND DISCUSSION


Every section contains **Explore–Discuss** problems interspersed at appropriate places to encourage the student to think about a relationship or process before a result is stated, or to investigate additional consequences of a development in the text. **Verbalization** of mathematical concepts, results, and processes is encouraged in these Explore–Discuss problems, as well as in some matched problems, and in some problems in almost every exercise set. The Explore–Discuss material also can be used as in-class or out-of-class **group activities**. In

addition, at the end of every chapter, we have included two special **chapter group activities** that involve several of the concepts discussed in the chapter. Problems in the exercise sets that require verbalization are indicated by color problem numbers.

## ■ EXERCISE SETS



**The book contains over 5,500 problems.** Many problems have multiple parts, significantly increasing the total number of problems. Each exercise set is designed so that an average or below-average student will experience success and a very capable student will be challenged. Exercise sets are mostly divided into A (routine, easy mechanics), B (more difficult mechanics), and C (difficult mechanics and some theory) levels.

## ■ APPLICATIONS

A major objective of this book is to give the student substantial experience in **modeling and solving real-world problems**. Enough applications are included to convince even the most skeptical student that mathematics is really useful (see the Applications Index inside the back cover). Worked examples involving applications are identified by .

**Almost every exercise set contains application problems**, usually divided into business and economics, life science, and social science groupings. An instructor with students from all three disciplines can let them choose applications from their own field of interest; if most students are from one of the three areas, then special emphasis can be placed there. Most of the applications are simplified versions of actual real-world problems taken from professional journals and books. No specialized experience is required to solve any of the applications.

## ■ TECHNOLOGY

The generic term **graphing utility** is used to refer to any of the various graphing calculators or computer software packages that might be available to a student using this book. (See the description of the software accompanying this book later in this Preface.) Although **access to a graphing utility is not assumed**, it is likely that many students will want to make use of one of these devices. To assist these students, **optional graphing utility activities** are included in appropriate places in the book. These include brief discussions in the text, examples or portions of examples solved on a graphing utility, problems for the student to solve, and a **group activity that involves the use of technology** at the end of each chapter. Beginning with the group activity at the end of Chapter 1, and continuing throughout the text, **linear regression** on a graphing utility is used at appropriate points to illustrate **mathematical modeling with real data**. All the optional graphing utility material is clearly identified by either  or  and can be omitted without loss of continuity, if desired.

## ■ GRAPHS

**All graphs are computer-generated** to ensure mathematical accuracy. Graphing utility screens displayed in the text are actual output from a graphing calculator.



## ■ ADDITIONAL PEDAGOGICAL FEATURES

**Annotation** of examples and developments, in small color type, is found throughout the text to help students through critical stages (see Sections 1-1 and 4-2). **Think boxes** (dashed boxes) are used to enclose steps that are usually performed mentally (see Sections 1-1 and 4-1). **Boxes** are used to highlight important definitions, results, and step-by-step processes (see Sections 1-1 and 1-4). **Caution** statements appear throughout the text where student errors often occur (see Sections 4-3 and 4-5). **Functional use of color** improves the clarity of many illustrations, graphs, and developments, and guides students through certain critical steps (see Sections 1-1 and 4-2). **Boldface type** is used to introduce new terms and highlight important comments. **Chapter review** sections include a review of all important terms and symbols and a comprehensive review exercise. **Answers to most review exercises**, keyed to appropriate sections, are included in the back of the book. Answers to all other odd-numbered problems are also in the back of the book. Answers to application problems in linear programming include both the mathematical model and the numeric answer.

## ■ CONTENT

The text begins with the development of a library of elementary functions in Chapters 1 and 2, including their properties and uses. We encourage students to investigate mathematical ideas and processes **graphically** and **numerically**, as well as **algebraically**. This development lays a firm foundation for studying mathematics both in this book and in future endeavors. Depending on the syllabus for the course and the background of the students, some or all of this material can be covered at the beginning of a course, or selected portions can be referred to as needed later in the course.

The material in Part Two (Finite Mathematics) can be thought of as four units: **mathematics of finance** (Chapter 3); linear algebra, including **matrices, linear systems, and linear programming** (Chapters 4 and 5); **probability** (Chapter 6); and applications of linear algebra and probability to **Markov chains** (Chapter 7). The first three units are independent of each other, while the last chapter is dependent on some of the earlier chapters (see the Chapter Dependency Chart on page viii).

Chapter 3 presents a thorough treatment of simple and compound interest and present and future value of ordinary annuities. Appendix A contains a section on arithmetic and geometric sequences that can be covered in conjunction with this chapter, if desired.

Chapter 4 covers linear systems and matrices with an **emphasis on using row operations and Gauss–Jordan elimination** to solve systems and to find matrix inverses. This chapter also contains numerous applications of **mathematical modeling** utilizing systems and matrices. To assist students in formulating solutions, **all the answers in the back of the book to application problems** in Exercises 4-3, 4-5, and the chapter Review Exercise **contain both the mathematical model and its solution**. The row operations discussed in Sections 4-2 and 4-3 are required for the simplex method in Chapter 5. Matrix multiplication, matrix inverses, and systems of equations are required for Markov chains in Chapter 7.

Chapter 5 provides **broad and flexible coverage of linear programming**. The first two sections cover two-variable graphing techniques. Instructors who

wish to emphasize techniques can cover the basic simplex method in Sections 5-3 and 5-4 and then discuss any or all of the following: the dual method (Section 5-5), the big  $M$  method (Section 5-6), or the two-phase simplex method (Group Activity 1). Those who want to emphasize modeling can discuss the formation of the mathematical model for any of the application examples in Sections 5-4, 5-5, and 5-6, and either omit the solution or use software to find the solution (see the description of the software that accompanies this text later in this Preface). To facilitate this approach, **all the answers in the back of the book to application problems** in Exercises 5-4, 5-5, and 5-6, and the chapter Review Exercise **contain both the mathematical model and its solution.**

Chapter 6 covers **counting techniques and basic probability**, including Bayes' formula and random variables.

Chapter 7 ties together concepts developed in earlier chapters and applies them to **Markov chains**. This provides an excellent unifying conclusion to the finite mathematics portion of the text.

The material in Part Three (Calculus) consists of **differential calculus** (Chapters 8–10), **integral calculus** (Chapters 11 and 12), and a variety of **additional calculus topics** (Chapters 13–15). In general, Chapters 8–11 must be covered in sequence; however, certain sections can be omitted or given brief treatments, as pointed out in the discussion that follows (see the Chapter Dependency Chart on page viii).

Chapter 8 introduces the **derivative**, covers the **limit properties** essential to understanding the definition of the derivative, develops the **rules of differentiation** (including the chain rule for power forms), and introduces **applications** of derivatives in business and economics. The interplay between graphical, numerical, and algebraic concepts is emphasized here and throughout the text.

Chapter 9 focuses on **graphing and optimization**. The first three sections cover continuity and first-derivative and second-derivative graph properties, while emphasizing **polynomial graphing**. **Rational function** graphing is covered in Section 9-4. In a course that does not include graphing rational functions, this section can be omitted or given a brief treatment. Optimization is covered in Section 9-5, including examples and problems involving end-point solutions.

The first three sections of Chapter 10 extend the derivative concepts discussed in Chapters 8 and 9 to **exponential and logarithmic functions** (including the general form of the chain rule). This material is required for all the remaining chapters. **Implicit differentiation** is introduced in Section 10-4 and applied to **related rate problems** in Section 10-5. These topics are not referred to elsewhere in the text and can be omitted.

Chapter 11 introduces **integration**. The first two sections cover **antidifferentiation** techniques essential to the remainder of the text. Section 11-3 discusses some applications involving **differential equations** that can be omitted. Sections 11-4 and 11-5 discuss the **definite integral** in terms of **Riemann sums**, including **approximations** with various types of sums and some **simple error estimation**. As before, the interplay between graphical, numerical, and algebraic properties is emphasized. These two sections also are required for the remaining chapters in the text.

Chapter 12 covers **additional integration topics** and is organized to provide maximum flexibility for the instructor. The first section extends the **area** concepts introduced in Chapter 11 to the area between two curves and related applications. Section 12-2 covers three more **applications** of integration, and



Sections 12-3 and 12-4 deal with additional **techniques of integration**. Any or all of the topics in Chapter 12 can be omitted.

The first five sections of Chapter 13 deal with **differential multivariable calculus** and can be covered any time after Section 10-3 has been completed. Section 13-6 requires the **integration** concepts discussed in Chapter 11.

After introducing the basic concepts and terminology used in the study of **differential equations** in Section 14-1, **separable differential equations** (Section 14-2) and **first-order linear differential equations** (Section 14-3) and related applications are covered thoroughly. All the growth laws introduced in Section 11-3 are covered again—this time with more emphasis on student recognition of the relevant growth law.

After discussing **improper integrals** in Section 15-1, the remainder of Chapter 15 discusses properties of **continuous probability density functions**, including the **uniform, exponential, and normal probability distributions**.

Appendix A contains additional topics that can be covered in conjunction with certain sections in the text, if desired.

## ■ SUPPLEMENTS FOR THE STUDENT

1. **A Student Solutions Manual, Explorations in Finite Mathematics, and Visual Calculus** by Garret J. Etgen and David Schneider is available through your book store. The manual includes detailed solutions to all odd-numbered problems and all review exercises. *Explorations in Finite Mathematics* and *Visual Calculus* by David Schneider each contain over twenty routines that provide additional insight into the topics discussed in the text. Although these software packages have much of the computing power of standard mathematical software packages, they are primarily teaching tools that focus on understanding mathematical concepts, rather than on computing. All the routines in these software packages are menu-driven and very easy to use. Included in *Explorations in Finite Mathematics* are routines for Gaussian elimination, matrix inversion, solution of linear programming problems by both the geometric method and the simplex method, Markov chains, probability and statistics, and mathematics of finance. The matrix routines use and display rational numbers, and matrices may be saved and printed. The *Visual Calculus* routines incorporate graphics whenever possible to illustrate topics such as secant lines; tangent lines; velocity; optimization; the relationship between the graphs of  $f$ ,  $f'$ , and  $f''$ ; and the various approaches to approximating definite integrals. The software will run on DOS or Windows platforms.
2. **A Graphing Calculator Manual** by Carolyn L. Meitler contains examples illustrating the use of a graphics calculator to solve problems similar to those discussed in the text. The manual follows the chapter organization of the text, making it easy to find examples in the manual illustrating appropriate calculator solution methods for problems in the text. The manual includes keystrokes for the TI-82, TI-83, TI-85, and TI-86 calculators. However, the examples and techniques can be used with any graphing utility.
3. **The PH Companion Website**, designed to complement and expand upon the text, offers a variety of teaching and learning tools, including links to related websites, practice work for students, and the ability for instructors to monitor and evaluate students' work on the website. For more information, contact your local Prentice Hall representative.  
[www.prenhall.com/barnett](http://www.prenhall.com/barnett)

## ■ SUPPLEMENTS FOR THE INSTRUCTOR

For a summary of all available supplementary materials and detailed information regarding examination copy requests and orders, see page xvii.

1. **PH Custom Test, a menu-driven random test system** for either Windows or Macintosh is available to instructors. The test system has been greatly expanded and now offers **on-line testing**. Carefully constructed algorithms use random-number generators to produce different, yet equivalent, versions of each of these problems. In addition, the system incorporates a unique **editing function** that allows the instructor to create additional problems, or alter any of the existing problems in the test, using a full set of mathematical notation. The test system offers **free-response, multiple-choice, and mixed exams**. An almost unlimited number of quizzes, review exercises, chapter tests, midterms, and final examinations, each different from the other, can be generated quickly and easily. At the same time, the system will produce answer keys, student worksheets, and a gradebook for the instructor, if desired.
2. **A Test Item File**, prepared by Laurel Technical Services, provides a hard copy of the test items available in PH Custom Test.
3. An **Instructor's Resource Manual** provides over 100 transparency masters and all the answers not included in the text. This manual is available to instructors without charge.
4. **A Student Solutions Manual, Explorations in Finite Mathematics, and Visual Calculus** by Garret J. Etgen and David Schneider (see Supplements for the Student) is available to instructors.
5. **A Graphing Calculator Manual** by Carolyn L. Meitler (see Supplements for the Student) is available to instructors. The manual contains all the necessary information for a student with no previous experience with a graphing calculator, eliminating the need for the instructor to prepare materials related to calculator usage. In particular, separate appendixes for the TI-82, TI-83, TI-85, and TI-86 graphing calculators contain detailed instructions, including calculator-specific keystrokes, for performing the various operations required to effectively use each of these calculators to solve problems in the text. Furthermore, the methods illustrated for these calculators are easily adapted to other graphing utilities. The manual is very effective both for a class where all students purchase the same calculator and in a setting where students are using a variety of different calculators—an important consideration as more and more students arrive at college having already purchased a graphing calculator.
6. The **PH Companion Website**, designed to complement and expand upon the text, offers a variety of interactive teaching and learning tools, including links to related websites, practice work for students, and the ability for instructors to monitor and evaluate students' work on the website. For more information, contact your local Prentice Hall representative.  
*www.prenhall.com/barnett*

## ■ ERROR CHECK

Because of the careful checking and proofing by a number of mathematics instructors (acting independently), the authors and publisher believe this book to be substantially error-free. For any errors remaining, the authors would be

grateful if they were sent to: Karl E. Byleen, 9322 W. Garden Court, Hales Corners, WI 53130; or, by e-mail, to: [byleen@execpc.com](mailto:byleen@execpc.com)

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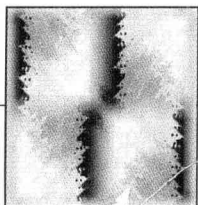
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## ■ ORDERING INFORMATION

When requesting examination copies or placing orders for this text or any of the related supplementary material listed below, please refer to the corresponding ISBN numbers.

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