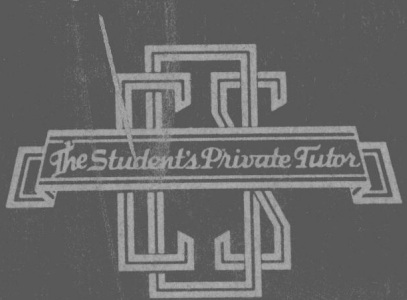
COLLEGE OUTLINE SERIES

25

AN OUTLINE OF STATISTICAL METHODS

FOURTH EDITION

ARKIN and COLTON



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COLLEGE OUTLINE SERIES

AN OUTLINE OF STATISTICAL METHODS

AS APPLIED TO
ECONOMICS, BUSINESS, EDUCATION,
SOCIAL AND PHYSICAL SCIENCES, ETC.

By

HERBERT ARKIN
The College of the City of New York
and
RAYMOND R. COLTON
The College of the City of New York

WITH A PREFACE BY
JUSTIN H. MOORE, Professor of Law
School of Business and Civic Administration
The College of the City of New York

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Preface

In Europe it has long been the practise to publish small books known as *manuals,* giving a condensed and succinct treatment of the subject as an aid to the student in summarizing the more elaborate and detailed contents of a large textbook. Halfway between an article in an encyclopedia and the often discursive texts studied in the classroom, these manuals fulfil a real need; they winnow fundamental principles from a mass of material, give with incisive clarity the contours of the ground already covered and leave with the reader a definite framework which later he may fill in by further practical contact with the subject in real life. In America there has been a noticeable gap in educational literature which now happily will be filled in part by the excellent series of small books published by Barnes & Noble, Inc.

The present volume on statistics does not aim to be a comprehensive treatise on the subject. On the contrary it gives the distilled essence of material which might well require one or more large volumes for a full discussion. For that very reason it ought to be a most useful tool in the hands alike of students and people actually engaged in statistical work, wherever the particular field of activity may happen to lie. The formulas and examples given in it will be ample for the needs of most workers, whether they be concerned with financial, industrial, commercial, social, or educational statistics. Of necessity there is a minimum of verbiage; no formula is included that has not practical applications; the mathematical aspects are not stressed; the philosophy of the subject and many recondite byways are not explored. Thus the reader is spared the need of hunting back and forth to find the special help which he needs on his concrete problems in daily life. To all statistical workers this little volume will be as indispensable as an adding machine.

JUSTIN H. MOORE, Professor of Law
School of Business and Civic Administration
The College of the City of New York

PUBLISHER'S NOTE

In this edition we have included eighteen standard textbooks in the tabulated bibliography table. The following list gives the author, title, and publisher of all the books referred to in the tables on the next two pages.

Camp, B. H., *Mathematical Part of Elementary Statistics,* Heath, 1931.

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Thurstone, L. L., *Fundamentals of Statistics,* Macmillan, 1925.

Yule, G. U., *An Introduction to the Theory of Statistics,* Griffen (London) Rev. 1937.

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THE GREEK ALPHABET

A	α	alpha	N	ν	nu
B	β	beta	Ξ	ξ	xi
Γ	γ	gamma	O	o	omicron
Δ	δ	delta	Π	π	pi
E	ϵ	epsilon	P	ρ	rho
Z	ζ	zeta	Σ	σ	sigma
H	η	eta	T	τ	tau
Θ	θ	theta	Υ	υ	upsilon
I	ι	iota	Φ	ϕ	phi
K	κ	kappa	X	χ	chi
Λ	λ	lambda	Ψ	ψ	psi
M	μ	mu	Ω	ω	omega

Table of Contents

CHAPTER I

STATISTICAL SERIES

Definition of Statistical Method

Statistical Method is a technique used to obtain, analyze and present numerical data.

Elements of Statistical Technique

The elements of statistical technique include the:

1. Collection and assembling of data.
2. Classification and condensation of data.
3. Presentation of data in:
    a. textular form.
    b. tabular form.
    c. graphic form.
4. Analysis of data.

Characteristics and Limitations of Statistical Methods

1. Statistical method is the only means for handling large masses of numerical data.

2. Statistical technique applies only to data which are reducible to quantitative form.

3. Statistical technique is *objective.* The results, however, cannot but be affected by the necessarily *subjective* interpretation

4. Statistical technique is the same for the social as for the physical sciences; i.e., economics, education, sociology and psychology as contrasted with biology, chemistry and astronomy. Method and technique apply alike to these divergent fields.

Statistical Series

In order to analyze numerical data, it is first necessary to arrange them systematically. The data may be arranged in a number of different ways. Technically an arrangement is called a **distribution** or **series.** An example of each type of distribution is shown below:

When data are grouped according to:	*The resulting series is called a:*
1. Magnitude	Frequency distribution
2. Time of occurrence	Time series
3. Geographic location	Spatial distribution

In addition there is a number of special types of distributions in which the data may be arranged by *kind* or by *degree*.

The Frequency Distribution

Definition

The frequency distribution is an arrangement of numerical data according to size or magnitude.

Construction

A frequency distribution is constructed in the following manner:

1. Using the **range** of the data (the interval between the highest and the lowest figure) as a guide, the data are divided into a number of convenient sized groups. The groups are called **class intervals** (compare table 1, column 1).[1]

The size of the class interval is dependent upon the number of values to be included in the distribution. The range of the values (difference between the highest and lowest values) is determined and is divided by the number of class intervals desired. The resulting size is rounded off. Few class intervals are used when a limited number of values are included and a large number when the distribution is to be compiled from many values. The most efficient number of class intervals usually lies between ten and twenty groups.

Other requirements for the determination of the class interval are:

a. The class intervals should not overlap; 0 – 4.9, 5 – 9.9 etc., should be used in preference to 0 – 5, 5 – 10, etc.

b. When the values tabulated coincide with the integers or with selected values, these values should generally constitute the midpoints of the groups.

c. When possible the class intervals should be of a uniform size.

2. The groups are then placed in a column with the lowest class interval at the top and the rest of the class intervals following according to size.
3. The data are then scored. Each figure is checked once next to the class interval into which it falls (see tally, table 1).[2]

Graphic Presentation of Frequency Distribution

If two lines are drawn perpendicular to one another and are divided according to a scale of values, given data may be represented by reference to the scale. The horizontal line is known as the X axis and the vertical line as the Y axis. If the values for

[1] As a preliminary step the raw data may be arranged according to size. The series is then called an array.

[2] In scoring data an efficient procedure is to connect the first and fourth score by the fifth. Using this procedure totals are obtained merely by adding the resulting units multiplying by five and adding the odd scores (see Table 1). A tally of this type saves time in counting frequencies and also eliminates possible inaccuracies.

Table 1 — Score Sheet

City Tax Rate of "True" Valuation in 261 Cities in the United States, 1927

Rate Per Thousand Dollars (Class Intervals)	Tally	Total Number of Cities (frequency)
4 – 7.99	𝍸	5
8 – 11.99	𝍸 𝍸 𝍸	15
12 – 15.99	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 I	46
16 – 19.99	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 III	68
20 – 23.99	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 III	58
24 – 27.99	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 II	32
28 – 31.99	𝍸 𝍸 𝍸 𝍸 II	22
32 – 35.99	𝍸 𝍸	10
36 – 39.99	II	2
40 – 43.99	II	2
44 – 47.99		0
48 – 51.99	I	1
		261

Source: United States Department of Commerce, *Financial Statistics of Cities*, 1927, Table 23.

a point are given the point may then be located on the graph. For example, in figure 1 the point $X = 2$, $Y = 3$ may now be located at the point marked *a*.

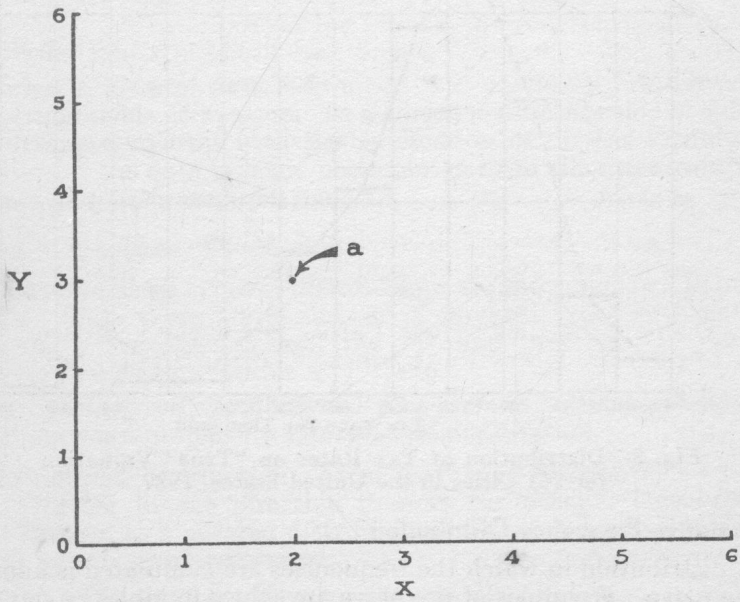

Fig. 1—Location of Plotted point $X = 2$, $Y = 3$.

If the two axes are now marked in the units of the given data the frequency distribution may be presented graphically (pictorially).

1. The class interval grouping which will be termed the **independent variable** is placed on the X (horizontal) axis and the frequency or **dependent variable** is placed on the Y (vertical) **axis.**[1]
2. The number of cases (frequency) is plotted at the midpoint of each respective class interval at the appropriate horizontal level as indicated by the scale on the Y axis.[2]
3. When connected the plotted points form a **frequency polygon.**
4. Rectangles may be constructed by using as the width the size of the class interval and as the height the frequency in each class interval. The rectangles form an **histogram** (also known as **a rectangular frequency polygon**).

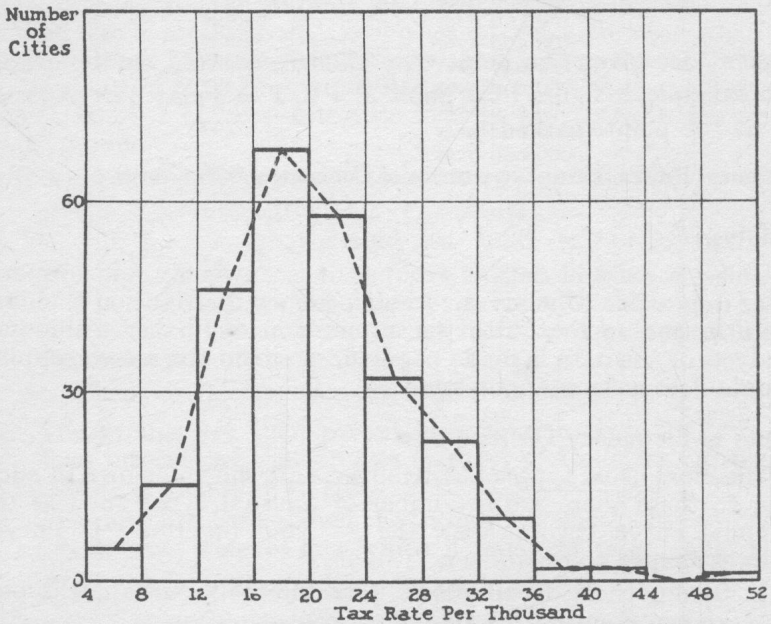

Fig. 2—Distribution of Tax Rates on "True" Valuation for 261 Cities in the United States, 1927.

Cumulative Frequency Distribution

A distribution in which the frequencies are cumulated is known as an ogive. Examples of the ogive are shown in tables 2a and 2b.

[1] The distance along the X axis is called the abscissa while that along the Y axis is called the ordinate.

[2] A cross sectional background or ruling may be used to aid in this work.