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THE MULTIPLEX COST AND RATE SYSTEM

BY OTTO B. GOLDMAN

ABSTRACT OF PAPER

The author gives the method of determining the cost of service having a variable demand, with special application to electric service, because the cost does and must always determine the selling price; competition and commissioners can only regulate the profit therein contained. Competition is destructive, involving an immense amount of unnecessary expenditure and lost motion. In order to insure the lowest price to the consumer we must have regulation, and monopoly. The author starts with the premise that an equipment must earn its cost and profits when in use; certainly it cannot do so when idle. The greater the percentage of total time an equipment or part thereof stands idle, the more it must earn when in use, because interest, etc., runs continuously. The subject is a big one and of tremendous import not only to the electric business, but to all business, and it has been the center of discussion for many years. It is now time to get down to absolute proofs of every conclusion, based upon fundamental facts.

THE SUBJECT of rates has received much attention of late, due naturally to its importance, and consequently many "systems" more or less related have been devised. A system is simply an attempt to straight-jacket judgment and make it all good. It must be correct or it is worse than none, and in order to be so, there must appear in the derivations no unbased assumption, such as are so often found. It must be apparent at once that no rate system can really be devised until a cost system has been obtained. For cost governs price, competition—and railroad commissions—only the profit therein contained. The multiplex cost system is a means of determining the cost of a variable service such as power service, gas, transportation, etc. In presenting this, I wish to acknowledge my indebtedness to the ability and patience of Mr. F. D. Weber, for valuable assistance. We are presenting the following only in its application to electric power service.

The demand for electric power service varies greatly during the day, from a very light night load, a tolerable day load, to an excessive peak load. Besides this we have seasonal variations of load. That must mean that part of our equipment is idle or only partly used during the day and year. We have here assumed that an equipment must pay for itself when in use—certainly it cannot do so when idle. As usual, we consider our costs as divided into two parts: (1) fixed charges, being all costs pertaining to an equipment when standing idle, and (2) operating costs, being all additional costs. It is evident that the first are continuous, the latter intermittent, depending on use. But the entire or total fixed charges for the year must be paid for during those hours in the year that the equipment is in use. Thus if F represents the fixed charges per year of a given equipment and N the number of hours in the year that the equipment is used, then F/N is the fixed charge cost per hour. This becomes $F/24 \times 365$ only when the plant is in continuous use.

Daily Variations

In actual power service, we have primarily to consider the load variations during the day. In treating this variation, we consider the day divided into a number of periods. The greater the number of periods the more exact our results may be made, but the more intricate. Where we have two periods, we have the duplex system; where three, the triplex, etc. The latter we will now treat, calling the three periods the night, day and peak periods. Let us call M_1 the maximum demand during the night period, K_1 the mean demand, and N_1 the duration in hours of the night period; similarly M_2, K_2 and N_2 for the day period and M_3, K_3 and N_3 for the peak period, where $N_1 + N_2 + N_3 = 24$. We consider the total number of customers as divided into three classes according to the periods in which they demand service, charging each class with the cost burden that it places on the equipment, except as modified by the relation of one period to the others. In the above the night customers need an equipment of size M_1, the day customers one of size M_2 and the peak customers one of size M_3. Since $M_1 < M_2 < M_3$, the equipment of size M_1 is needed throughout the 24 hours, that of size $M_2 - M_1$ for $N_2 + N_3$ hours and that of size $M_3 - M_2$ for N_3 hours. For the present we will assume that the conditions assumed above are the same for each day of the year. Later we will consider seasonal variations.

Let us call C the cost of equipment per kw. and P the per cent of fixed charges, per year, so that CP becomes the fixed charge cost per kw-year. For the night period size we have $M_1\,CP$ as the total annual fixed charges and since this part of the equipment is in use 24 hours per day the fixed charge cost per hour becomes $M_1CP/365 \times 24$ or $M_1CP/365 \times 24\ K_1$ per kw-hr., so that we have for the fixed charge cost per hour

$$R'_1 = \frac{M_1CP}{365 \times 24 \times K_1} = \frac{M_1CP}{365 \times (N_1 + N_2 + N_3)\, K_1} \qquad (1)$$

During the day period we have for the total fixed charge cost per hour

$$\frac{M_1CP}{365\,(N_1 + N_2 + N_3)} + \frac{(M_2 - M_1)\,CP}{365\,(N_2 + N_3)}$$

Dividing this by the mean load during this period we get

$$R'_2 = \frac{CP}{365\,K_2}\left[\frac{M_1}{(N_1 + N_2 + N_3)} + \frac{(M_2 - M_1)}{(N_2 + N_3)}\right] \qquad (2)$$

which is the fixed charge cost for this period. During the peak period we have for the total cost per hour

$$\frac{M_1CP}{365\,(N_1 + N_2 + N_3)} + \frac{(M_2 - M_1)\,CP}{365\,(N_2 + N_3)} + \frac{(N_3 - M_2)\,CP}{365\,N_3}$$

whence the fixed charge cost per kw-hr. becomes

$$R'_3 = \frac{CP}{365\,K_3}\left[\frac{M_1}{(N_1 + N_2 + N_3)} + \frac{(M_2 - M_1)}{(N_2 + N_3)} + \frac{(M_3 - M_2)}{N_3}\right] \qquad (3)$$

If now we call O_1 the operating cost per hour during the night period, O_2 and O_3 those during the day and peak periods respectively, then our costs (total) per kw-hr. become for the night, day and peak respectively:

$$R_1 = \frac{CP}{365\,K_1}\left[\frac{M_1}{(N_1 + N_2 + N_3)}\right] + \left[\frac{O_1}{K_1}\right] \qquad (4)$$

$$R_2 = \frac{CP}{365\,K_2}\left[\frac{M_1}{(N_1 + N_2 + N_3)} + \frac{(M_2 - M_1)}{N_2 + N_3}\right] + \frac{O_2}{K_2} \qquad (5)$$

$$R_3 = \frac{CP}{365\,K_3}\left[\frac{M_1}{(N_1 + N_2 + N_3)} + \frac{(M_2 - M_1)}{(N_2 + N_3)} + \frac{(M_3 - M_2)}{N_3}\right] + \frac{O_3}{K_3} \qquad (6)$$

This is, as stated above, on the basis of continuous use at the rates stated throughout the year.

Seasonal Variations

Taking into consideration the variations of load during the year we will usually find a comparatively light summer load, a medium spring and fall load and a very heavy winter load. That means that not only must part of our plant be idle part of each day, but that part of our equipment must lie idle often for months at a time, due to the above-mentioned conditions. Again, as above, in the daily load variations, the aggregate of those customers causing any special burdens on the equipment must pay for the burden they create by their demand. To determine this, we proceed as follows. Consider the year as divided into, say, three periods, the minimum of D_1 days duration, the mean of D_2 days duration, and the maximum of D_3 days duration. As above, let us call M_1, M_2 and M_3 the maximum demands for the night, day and peak periods respectively of the first or minimum period of the year, similarly K_1, K_2, and K_3 the mean demands in kw., and N_1, N_2, and N_3 the lengths in hours of the night, day and peak periods, where $D_1 + D_2 + D_3 = 365$. Then for this minimum period, the equations (**4**), (**5**) and (**6**) again hold.

For the mean period, let us call

M_1', M_2', and M_3', the maximum demands
K_1', K_2', and K_3', the mean demands
N_1, N_2, and N_3, the length in hours, and
O_1', O_2', and O_3' the operating cost per hour of the night, day and peak periods respectively. Then during the night period we have for the cost per hour

$$\frac{M_1 CP}{(D_1 + D_2 + D_3)\,(N_1 + N_2 + N_3)} + \frac{M_1' - M_1}{(D_2 + D_3)\,(N_1 + N_2 + N_3)} + O_1'$$

whence the cost per kw-hr. is

$$R_1' = \frac{CP}{(N_1 + N_2 + N_3) K_1'} \left[\frac{M_1}{D_1 + D_2 + D_3} + \frac{M_1' - M_1}{D_2 + D_3} \right] + \frac{O_1'}{K_1'} \tag{7}$$

During the day period, we have for the cost per hour

$$\frac{M_1 CP}{(D_1 + D_2 + D_3)(N_1 + N_2 + N_3)} + \frac{(M_1' - M_1) CP}{(D_2 + D_3)(N_1 + N_2 + N_3)}$$

$$+ \frac{(M_2 - M_1) CP}{(D_1 + D_2 + D_3)(N_2 + N_3)} + \frac{M_2' - M_2 CP}{(D_2 + D_3)(N_2 + N_2)} + O_2'$$

whence the cost per kw-hr. becomes

$$R_2' = \frac{CP}{K_2'} \left\{ \frac{1}{(N_1 + N_2 + N_3)} \left[\frac{M_1}{D_1 + D_2 + D_3} + \frac{M_1' - M_1}{D_2 + D_3} \right] \right.$$

$$\left. + \frac{1}{(N_2 + N_3)} \left[\frac{M_2 - M_1}{D_1 + D_2 + D_3} + \frac{M_2' - M_2}{D_2 + D_3} \right] \right\} + \frac{O_2'}{K_2} \tag{8}$$

During the peak period, we have for the cost per hour

$$\frac{M_1 CP}{(D_1 + D_2 + D_3)(N_1 + N_2 + N_3)} + \frac{(M_2 - M_1) CP}{(D_1 + D_2 + D_3)(N_2 + N_3)}$$

$$+ \frac{(M_3 - M_2) CP}{(D_1 + D_2 + D_3) N_3} + \frac{(M_1' - M_1) CP}{(D_2 + D_3)(N_1 + N_2 + N_3)}$$

$$+ \frac{(M_2' - M_2) CP}{(D_2 + D_3)(N_2 + N_3)} + \frac{(M_3' - M_3) CP}{(D_2 + D_3) N_3} + O_3'$$

Dividing this by the mean load, we get

$$R_3' = \frac{CP}{K_3'} \left\{ \frac{1}{(N_1 + N_2 + N_3)} \left[\frac{M_1}{(D_1 + D_2 + D_3)} + \frac{(M_1' - M_1)}{(D_2 + D_3)} \right] + \frac{1}{(N_2 + N_3)} \left[\frac{M_2 - M_1}{D_1 + D_2 + D_3} + \frac{M_2' - M_2}{D_2 + D_3} \right] + \frac{1}{N_3} \left[\frac{M_3 - M_2}{D_1 + D_2 + D_3} + \frac{M_3' - M_3}{D_2 + D_3} \right] \right\} + \frac{O_3'}{K_3'} \qquad (9)$$

For the maximum period let us call

M_1'', M_2'', and M_3'', the maximum demands
K_1'', K_2'', and K_3'', the mean demands
N_1, N_2, and N_3, the length in hours, and
O_1'', O_2'' and O_3'', the operating costs per hour of the night, day and peak periods respectively of this maximum period. Then during the night period of this season, we have for the cost per hour

$$\frac{M_1 CP}{(N_1 + N_2 + N_3)(D_1 + D_2 + D_3)} + \frac{M_1' - M_1}{(N_1 + N_2 + N_3)(D_2 + D_3)} + \frac{M_1'' - M_1'}{(N_1 + N_2 + N_3) D_3} + O_1''$$

Whence the cost per kw-hr. is

$$R_1'' = \frac{CP}{(N_1 + N_2 + N_3) K_1''} \left[\frac{M_1}{(D_1 + D_2 + D_3)} + \frac{(M_1' - M_1)}{(D_2 + D_3)} + \frac{M_1'' - M_1'}{D_3} \right] + \frac{O_1''}{K_1''} \qquad (10)$$

Similarly we get, for the cost per kw-hr. during the day period,

$$R_2'' = \frac{CP}{K_2''} \left\{ \frac{1}{(N_1 + N_2 + N_3)} \left[\frac{M_1}{D_1 + D_2 + D_3} + \frac{M_1' - M_1}{D_2 + D_3} \right.\right.$$

$$\left. + \frac{M_1'' - M_1'}{D_3} \right] + \frac{1}{(N_2 + N_3)} \left[\frac{(M_2 - M_1)}{(D_1 + D_2 + D_3)} \right.$$

$$\left.\left. + \frac{M_2' - M_2}{(D_2 + D_3)} + \frac{M_3'' - M_2'}{D_3} \right] \right\} + \frac{O_2''}{K_2''}$$

and for the peak period

$$R_3'' = \frac{CP}{K_3''} \left\{ \frac{1}{(N_1 + N_2 + N_3} \left[\frac{M_1}{(D_1 + D_2 + D_3)} + \frac{(M_1' - M_1)}{(D_2 + D_3)} \right.\right.$$

$$\left. + \frac{M_1'' - M_1'}{D_3} \right] + \frac{1}{(N_2 + N_3)} \left[\frac{(M_2 - M_1}{(D_1 + D_2 + D_3)} \right.$$

$$\left. + \frac{(M_2' - M_2)}{(D_2 + D_3)} + \frac{(M_2'' - M_2')}{D_2} \right] + \frac{1}{N_3} \left[\frac{M_3 - M_2}{(D_1 + D_2 + D_3)} \right.$$

$$\left.\left. + \frac{(M_3' - M_3)}{(D_2 + D_3)} + \frac{M_3'' - M_3'}{D_3} \right] \right\} + \frac{O_3''}{K_3''}$$

For brevity's sake we are leaving out, for the present, the case of more than three periods in the year and the rather important condition where in each of the seasonal periods, the corresponding periods of the day are different, *i.e.*, where we have for the length of the night, day and peak periods, N_1, N_2 and N_3 during the minimum period, N_1,' N_2' and N_3' during the mean period, and N_1'', N_2'' and N_3'' during the maximum period. We will instead proceed at once to show the application of a concrete example under the first case. This application is merely illustrative.

Application of Case I

In computing the cost of power in the following, we have segregated each factor contributing thereto; *i.e.*, generation, transmission, transformation, and distribution. This must be done