STATISTICS FOR PSYCHOLOGY

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PREFACE

An introduction to statistics that concentrates on methods of investigation and problems common to the field of psychology must still focus primarily on an understanding of the theory and techniques of statistics. For that reason it should be advantageous that the author be a statistician. But a statistician cannot be that familiar with the particular-day-to-day uses of statistics in psychology, and for that reason it should be advantageous that the author be a psychologist. By synthesizing these points of view, we feel that we have produced a text that is more valuable than the typical general introductory statistics book, which cannot show the student in psychology how statistical concepts and techniques can be applied specifically to psychological experimentation; but also a text that is more valuable than the typical introductory statistics-for-psychologists book, which may *inform* the student about statistics rather than *teach* him statistics.

We have used as a guide to our approach the basic objectives of the senior author's well-tried-and-tested *An Introduction to Probability and Statistics*, now in its third edition. Like *Introduction*, the present text stresses the role of statistical inference in the scientific method. This theme, carried by the chapter introductions and summaries, relates one body of material to another and produces a connectivity that aids the student in understanding and learning statistics. By concentrating on applications within a single discipline, psychology, the student's familiarity with the language and experimental techniques of his field is reinforced while he is learning statistics.

The mathematical difficulty associated with an introductory statistics course can be increased or decreased by including or excluding material on the theory of probability. One can explain the role that probability plays in inference making by teaching the student how to calculate the probability of certain events for the discrete case, or you can simply assume that approximate values for required probabilities can be obtained, by observing the relative frequencies of the events in many repetitions of an experiment. This text takes the latter approach, thus substantially reducing the mathematical level of the content without sacrificing an explanation of the role that probability plays in making inferences.

The ordering of topics has been directed specifically at the type of experimentation common in psychological research. Specifically, we introduce nonparametric statistics early in the text (Chapter 5) because of the frequent occurrence of ordinal data in psychological experimentation. This alteration to the usual ordering of topics, presenting nonparametric before parametric statistical methods, is of particular value to students in psychology. It provides them with statistical techniques that are easy to comprehend, easy to apply, and applicable for analyzing both ordinal and measurement data. Because the techniques are intuitively easy to comprehend, they provide an excellent vehicle for conveying the notions involved in a statistical test of a hypothesis. Tests of hypotheses, a topic of considerable difficulty to the beginning student, are presented early in the text and are reinforced in later chapters.

Statistics for Psychology presents a wide variety of topics suitable for inclusion in an introductory course. Of particular importance is the introduction of the concept of experimental design through the paired-difference experiment of Chapter 8. This is followed by an elementary but careful discussion of experimental design in Chapter 9. The chapter on analysis of variance (Chapter 10) includes the analyses, both parametric and nonparametric, for the completely randomized and the randomized block designs. It also includes the analysis for a two-way classification and an explanation of the important concept of factor interaction.

Note that the authors' course does not preclude the possibility of altering the order of topical presentations. For example, the instructor need not present all the nonparametric methods of Chapter 5 at that particular point in the course. Some can be deferred until their parametric counterparts are discussed, or they can be omitted. Likewise, the applied chapters at the end of the text can be interchanged with little difficulty.

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PREFACE TO THE STUDENT

The purpose of this book is to familiarize students in the behavioral sciences with a valuable research tool. Typically, behavioral science research involves inference making—using behavioral observations on a limited number of subjects and making decisions or predictions about the behavior of the larger group represented by these subjects. Since statistics is a science concerned with making inferences, an understanding of statistical concepts and applications can be of enormous value to the behavioral scientist. Necessarily, an applied statistics book must present a variety of statistical techniques; different kinds of data and different types of research design call for different methods of treatment. Thus a frequent major pitfall for the beginning student of statistics is that of becoming immersed in specific methodologies and losing sight of the underlying objective—to make inferences. To avoid this pitfall, we offer several suggestions to the student using this text.

- Regard the first chapter as a pivotal one, to be referred back to as you proceed through the book.
- Read carefully the introduction to each chapter, since the introductory sections are intended to provide continuity, relating each new topic to the major objective, inference making.
- After reading each chapter, use the summary section to review the major points covered within the chapter.
- To ensure that these points are firmly established, go back once more over the material that is set off in color or enclosed in boxes.
- It is strongly suggested that you work through all the exercises at the end of each section and as many as possible at the end of each chapter, since these exercises are representative of the types of problems encountered in behavioral-science research.

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STATISTICS AND THE BEHAVIORAL SCIENTIST

INTRODUCTION

In recent years, statistics has come to play an increasingly important role in the behavioral sciences. Indeed, an experimental plan or research report that does not include some statistical method is fast becoming a rarity. This state of affairs has not come about by accident but is due to an increasing recognition that statistics supplies a methodology particularly well suited to the needs of behavioral research. What these needs are and how statistics meets them become more apparent when we consider the essential features of a scientific investigation, pointing out some of the specific problems involved and indicating how statistics can be used in dealing with these problems.

Scientific inquiry is essentially a three-step procedure. First, a relationship is formulated; that is, the problem is defined. Then relevant information is obtained. Finally, the relationship is tested. What distinguishes investigation in the behavioral sciences is the content. The relationships of interest are those between given behaviors or behavioral properties and their modifiers or determinants. For example, an investigator observes that when an unfamiliar object, such as a doll, is placed in its cage, a young chimpanzee shows a reaction very similar to curiosity in humans. He circles the object, sniffs at it, nudges and pokes it. Yet when an identical object is placed in the cage of an older but still active chimp, the response is seemingly one of indifference. On the basis of this comparison the investigator hypothesizes that "curiosity" in chimpanzees decreases with age. He selects a number of chimps of different ages, introduces the unfamiliar object to each, and records the amount of time spent by each in curiosity behavior—operationally defined as circling, sniffing, nudging, poking, and so on. He then uses these recorded values as evidence for (or against) the hypothesized relationship.

In this example, the relationship formulated is that between curiosity—defined in terms of particular behaviors—and age. More specifically, the investigator predicts that curiosity decreases with increasing age. In order to bring his prediction out of the realm of mere speculation, the investigator next obtains relevant information. Here, he obtains measures of the amount of curiosity shown at each age level. The final or testing stage of the investigation consists in showing that the data support the predicted relationship. Here the investigator must show that the observed curiosity measurements bear out the prediction that curiosity decreases as age increases.

This description of scientific investigation as a sequence of three distinct and readily identifiable steps is, of course, oversimplified. In practice, any one phase of an investigation is related to and influenced by subsequent phases. A good researcher does not move blindly from formulating a relationship to obtaining information and then to testing the relationship. Rather, he defines his phenomena and otherwise specifies the relationships of interest with a view toward collecting data that are accurate, representative of the phenomena, and can be meaningfully described and analyzed. He also collects and analyzes his data in such a way that results can be used in testing the relationships under investigation. Nevertheless, a representation of scientific inquiry as a stepwise process serves two useful purposes: It summarizes the essential features, and it provides a convenient framework for discussing some of the more specific problems associated with scientific research.

1.1. THE POPULATION AND THE SAMPLE

Implicit in the preceding definition is a restriction on the generalizability of the relationship being investigated. It is presumed that the relationship applies to some defined *population*. In the example in the Introduction, the prediction of an age–curiosity relationship might, for instance, be limited to male chimpanzees raised in captivity whose ages fall within a specific range. Measurements of curiosity on all animals fitting this description comprise the population of interest. However, it is not ordinarily possible to observe every member of the population. The set of measurements actually obtained is usually a *sample* from the population. The investigator of chimpanzee behavior cannot observe every chimp in the prescribed experimental setting, so he obtains measurements of curiosity on a limited number of chimps. The obtained measurements comprise a sample that the investigator uses to make an inference about the population: that curiosity in captive male chimpanzees does (or does not) decrease with increasing age.

The terms *population* and *sample* can be defined quite simply:

Definition 1.1

A population *is a set of measurements of interest to the experimenter.*

The population associated with the chimpanzee experiment is the set of curiosity measurements made on all captive male chimpanzees whose ages fall within a specific range. In this example, the population is a large set of measurements that exists conceptually.

In contrast, suppose that we wish to determine the fraction of students at a particular university that favors Sam Smith for student-body president.

Each student in favor could be counted as a 1; each opposed or having no opinion, as a 0. Thus the population of measurements is a set of 1s and 0s associated with the entire student body. Unlike the conceptual chimpanzee population, this population exists in reality and the number of measurements in the population is equal to the number of students enrolled in the university.

Now we examine the term *sample*.

Definition 1.2

A sample *is a subset of measurements selected from the population.*

For example, the measurements actually obtained in the chimpanzee curiosity experiment represent a sample selected from the conceptual population of all chimpanzee curiosity measurements. Similarly, a sample of the opinions of 200 students is a subset of measurements selected from the population of all students' opinions.

In both examples, and in general, the statistical problem involves sampling from a population with a very definite objective in mind. We wish to use the partial information contained in a subset of measurements—the sample—to infer the nature of the larger set—the population from which the sample was selected.

1.2. WHAT IS STATISTICS? WHAT IS ITS ROLE IN THE SCIENTIFIC METHOD?

The scientific method has been described as a three-step inference-making procedure in which inferences (decisions or predictions) are made about the population on the basis of sample information. Statistics, a branch of applied mathematics, provides the methodology for making inferences. To illustrate where and how statistical methods enter into the process of scientific investigation, consider our chimpanzee example.

In order to decide whether or not curiosity decreases with age, the investigator must obtain measurements on a number of chimps at various ages. Questions that immediately arise are: How many measurements should be obtained? How should we sample the age range of interest? Questions such as these, concerning *experimental design* or *sampling procedure*, can be answered by using statistical methodology. Statistics provides the strategy and methods for gathering the maximum amount of information for a given expenditure of time and other resources. Once the relevant information is obtained, the investigator requires methods to describe and summarize his data so that results are interpretable and can be communicated. Statistics supplies these methods of *data analysis*—techniques for extracting the essential features of a data set. The culminating stage of the investigative process is that of testing a hypothesized relationship. The investigator must decide whether or not the sample data support the hypothesis that curiosity in captive chimpanzees decreases with increasing age. Statistics offers procedures for making this sort of *inference* from sample data to the population. Moreover, these statistical inference procedures are based on a rationale that permits error evaluation. Not only do these procedures supply the principles and methodology for making inferences, but they afford information about the chance of error in the decision or prediction. If it is decided that curiosity does indeed decrease with age, what is the chance that this decision is incorrect? Statistical-inference procedures provide an answer to this question.

In short, statistics is a science concerned with inference. One selects a sample of measurements from a larger set, a population, in order to make an inference about the population. The sampling procedure involves the collection of a quantity of *information* at a specified cost. Hence we think of statistics as a *theory of information* concerned with inference. The *objective of statistics* is stated as follows:

Objective of Statistics

The objective of statistics *is to make an inference about a population based on information contained in a sample.*

SUMMARY

Investigation in the behavioral sciences is a three-step process in which a behavioral relationship is formulated, relevant information is collected, and the postulated relationship is tested. This process is necessarily one of inference; the information collected is a *sample* from the *population* of interest, and these sample data are used to make inferences (decisions, predictions) about the population relationship. Statistics provides the vehicle for making inferences about the population on the basis of sample data. The behavioral scientist also relies upon statistical procedures for the *experimental-design* and *data-analysis* aspects of information collecting. He uses the principles and methodology of *statistical inference* to test behavioral relationships and to control the chance of error in his decision or prediction.

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