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REVISTA DE FISICA DEL PLASMA Y FUSION TERMONUCLEAR

1962 SUPPLEMENT — PART 2

PLASMA PHYSICS AND CONTROLLED NUCLEAR FUSION RESEARCH

CONFERENCE PROCEEDINGS,
SALZBURG, 4—9 SEPTEMBER 1961

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NUCLEAR FUSION
FUSION NUCLEAIRE
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FUSION NUCLEAR

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PART 2

PROCEEDINGS OF THE CONFERENCE ON PLASMA PHYSICS AND CONTROLLED NUCLEAR FUSION RESEARCH, 4—9 SEPTEMBER 1961, SALZBURG, AUSTRIA

LES ACTES DE LA CONFERENCE SUR LA PHYSIQUE DES PLASMAS ET LA RECHERCHE CONCERNANT LA FUSION NUCLEAIRE CONTROLEE, 4—9 SEPTEMBRE 1961, SALZBOURG, AUTRICHE

ТРУДЫ КОНФЕРЕНЦИИ ПО ИССЛЕДОВАНИЯМ В ОБЛАСТИ ФИЗИКИ ПЛАЗМЫ И УПРАВЛЯЕМОГО ЯДЕРНОГО СИНТЕЗА, 4—9 СЕНТЯБРЯ 1961 г., ЗАЛЬЦБУРГ, АВСТРИЯ

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FOREWORD — AVANT-PROPOS — ПРЕДИСЛОВИЕ — PREFACIO

In 1958, at the Second United Nations Conference on the Peaceful Uses of Atomic Energy, the results of research on controlled nuclear fusion obtained in a few technically advanced countries were first disclosed to the world at large. Since then, it has become more and more evident that a better understanding of fundamental phenomena is needed before the goal of energy extraction from nuclear fusion may be reached. Consequently, the intensive research undertaken in recent years has been primarily basic research in plasma physics.

The fact that such research is most complex and costly has enhanced the desirability of co-operation and exchange of information and experience between all those engaged in this field of nuclear science and technology. It has become obvious that the International Atomic Energy Agency can play an important part in promoting such co-operation on a world-wide scale.

After consultation with a number of leading scientists, the Agency convened an international conference on Plasma Physics and Controlled Nuclear Fusion Research. The extent of the interest shown by Member States did not merely confirm that such a conference was actually needed at the present time, but greatly exceeded expectations. The quality and volume of the papers submitted, the number of participants and of countries represented, all bore witness to this interest.

Today, plasma physics and controlled thermonuclear fusion research is a more-or-less academic study. All that can be said at this stage is that it should eventually lead to a practical energy source. The day may come when the energy from nuclear fusion will be needed and when the well-being of mankind may depend on the ability to draw on this almost limitless reservoir.

The publication of the conference proceedings is intended to promote international co-operation and accelerate progress in this most important field of scientific endeavor.

INTRODUCTION – ВВЕДЕНИЕ – INTRODUCCIÓN

The Conference on Plasma Physics and Controlled Nuclear Fusion Research was held in Salzburg, Austria, on 4—9 September 1961. More than 500 scientists, representing 29 nations and 6 international organizations, participated in the Conference. The Proceedings are published in three parts as a 1962 supplement to this journal.

Because of the many interconnections between the various problems of plasma physics, it was decided to have no parallel plenary sessions. Accordingly, nine sessions were held during the six days of the Conference. During these sessions, 111 papers were presented. The "free" afternoons and evenings were devoted to at least fourteen informal discussions of topics of special interest to the participants. The present Proceedings do not include the records of these informal discussions (the discussions would have ceased to be "informal" if recorded), although it seems certain that new ideas generated in these discussions will lead to publication of papers elsewhere.

"Part 1" contains the texts (in original language only) of all papers delivered in Sessions I, II and IV of the Conference, the records (in English) of the discussions of these papers, as well as the texts (in English and Russian) of the two concluding speeches by Prof. Artsimovich and Dr. Rosenbluth summarizing the Conference. Translations of the abstracts of each paper (Sessions I, II, IV) are given at the end of this part of the Proceedings. In addition there is an author index.

The remainder of the Proceedings is published in "Part 2" (Sessions III, V, VI, VIII) and in "Part 3" (Sessions VII, IX). The abstracts of those papers accepted but not presented to the Conference, a list of participants, subject and author indexes for the entire Proceedings are included in the third part.

In preparing the Proceedings the Editor gratefully acknowledges the substantial help of B. Buras, P. A. Davenport, C. Etievant, W. F. Gauster, W. A. Newcomb and E. V. Piskarev.

SESSION III — SÉANCE III
ЗАСЕДАНИЕ III — SESIÓN III

5 SEPTEMBER 1961 — 5 SEPTEMBRE 1961

5 СЕНТЯБРЯ 1961 Г. — 5 DE SEPTIEMBRE DE 1961

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[Papers presented: CN-10/43, 73, 87, 88, 132, 152, 164, 199, 220]

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ON THE STRUCTURE OF HYDROMAGNETIC SHOCK WAVES WITH TRANSVERSE FIELD AND VISCOUS DISSIPATION*

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The problem of hydromagnetic shock waves has recently received increasing interest in connection with experiments carried out to obtain extremely high temperatures for fusion processes. In such experiments a tendency toward appreciable densities (obtained, e.g., by magnetic compression) becomes noticeable.

Theories of hydromagnetic shock structure available at present either neglect the inertial term in the differential equation for the magnetic field if dissipation processes are considered (Marshall, Sen), or if the rigorous equations for the magnetic field are used, a collision-free plasma is considered (Grad, Morawetz).

This paper deals with a study of the effect of viscous dissipation in the rigorous hydromagnetic shock structure equations for the purpose of deriving a theory applicable to relatively high densities. First, the shock structure for a pure hydromagnetic case (neglecting viscosity but considering ohmic resistance as a damping mechanism) is considered in some detail. A stability investigation is carried out and it is shown that there exist two regimes for the shock transition, one characterized by an oscillatory transition, the other by an aperiodic damped transition. Considering viscosity, the range for which this dissipation mechanism becomes appreciable is determined. It can furthermore be shown that whereas solutions for Mach numbers $M < 1$ behind the shock do not exist for the collision-free hydromagnetic case, the inclusion of viscous dissipation in the Navier-Stokes approximation indicates the possibility for such solutions.

1. Hydromagnetic shock structure with ohmic damping

1.1 General remarks and basic equations

The structure of a hydromagnetic shock wave as discussed by C. S. Morawetz [1] is not based on a truly collision-free theory, as the damping mechanism considered, namely ohmic dissipation, is produced by collisions. For magnetic energy density large compared to thermal energy density (small β values), however, this damping becomes small and in this limit the theory may be regarded as a collision-free one. For appreciable damping, it is very likely that, e.g., viscous dissipation can no longer be neglected, so that for simplicity all those solutions shall be termed "collision-free" where viscous effects can be disregarded, even though a small value of the ohmic resistance is retained for damping purposes.

In our investigations we start with such a collision-free theory for hydromagnetic shock structure. However, unlike C. S. Morawetz, we shall use a definite and concrete equation for the ohmic resistance. The calculations shall furthermore be restricted to the temperature and density range currently of particular interest in hydromagnetic compression experiments, namely from some 10^4 °K to 10^5 °K and particle densities of 10^{16} and 10^{17} cm^{-3}.

As basic equations for the shock structure, the conventional one-fluid equations [1—4] are employed, the equation of continuity and the equation of motion:

$$\varrho u = m, \tag{1}$$

$$\varrho u^2 + p + \frac{B^2}{8\pi\mu} = P, \tag{2}$$

the energy equation,

$$\frac{u^2}{2} + \frac{\varkappa}{\varkappa - 1} \cdot \frac{p}{\varrho} + \frac{c}{4\pi\mu m} E B = Q, \tag{3}$$

and a generalized form of Ohm's law derived from a two-fluid description of the y-momentum equations,

$$\frac{\mathrm{d}^2 B}{\mathrm{d}t^2} + \gamma_e \gamma_i \eta_R \varrho \frac{\mathrm{d}B}{\mathrm{d}t} + \frac{4\pi\mu}{c^2} \gamma_e \gamma_i (\varrho u) \cdot (cE - uB) = 0, \tag{4}$$

together with the Maxwell equation (in gaussian units)

$$j = -\frac{c}{4\pi\mu} \cdot \frac{\mathrm{d}B}{\mathrm{d}x}. \tag{5}$$

With $u = \mathrm{d}x/\mathrm{d}t$, Eq. (4) can also be written in the form

$$\frac{\mathrm{d}^2 B}{\mathrm{d}x^2} + \frac{1}{u}\frac{\mathrm{d}u}{\mathrm{d}x}\frac{\mathrm{d}B}{\mathrm{d}x} + \gamma_e\gamma_i \frac{\varrho}{u} \eta_R \frac{\mathrm{d}B}{\mathrm{d}x} + \frac{4\pi\mu}{c^2} \gamma_e \gamma_i \frac{\varrho}{u} (cE - uB) = 0. \tag{6}$$

In the above equations, u is the x-component of the velocity, ϱ the mass density, p the hydrostatic pressure, B the z-component of the magnetic field, E the y-component of the electric field ($E = $ const.

* Conference paper CN-10/43 presented by H. J. Kaeppeler. Discussion of this paper is given on page 491. Translations of the abstract are at the end of this volume of the Conference Proceedings.
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due to curl $\mathbf{E}=0$), j the y-component of the current density, η_R the specific ohmic resistance, $\varkappa$ the ratio of the specific heats,

$$\varkappa = c_p/c_v\,, \tag{7}$$

and γ an abbreviation,

$$\gamma_e = e/m_e\,, \qquad \gamma_i = e/m_i\,, \tag{8}$$

where e is the elementary charge, m_e the electron mass and m_i the ionic mass.

Assumptions in the above equations are ionic charge $eZ=e$ ($Z=1$) and quasi-neutrality $n_e=n_i$, where n is the particle density.

1.2 Singularities of the differential equation

We consider those states for which the differential equation has singular points, i.e. for which all derivatives vanish (stationary flow). The stationary solution requires that

$$\frac{\mathrm{d}^2 B}{\mathrm{d}x^2} = \frac{\mathrm{d}B}{\mathrm{d}x} = 0;$$

hence,

$$E = \frac{u}{c}\,B\,.$$

On the other hand, we find from the conservation laws Eqs. (1) through (3) after elimination of ϱ and p,

$$u^2 - \frac{2\varkappa}{\varkappa+1}\,\frac{1}{m}\left(P - \frac{B^2}{8\pi\mu}\right)\cdot u - 2\,\frac{\varkappa-1}{\varkappa+1}\left(\frac{cE}{4\pi\mu m}\cdot B - Q\right) = 0\,. \tag{9}$$

Inserting $B=cE/u$ into this equation, there follow those values of u for which the flow is steady. There results a third-order equation in u,

$$u^3 - \frac{2\varkappa}{\varkappa+1}\,\frac{P}{m}\cdot u^2 + 2\,\frac{\varkappa-1}{\varkappa+1}\,Q\cdot u + \frac{2-\varkappa}{\varkappa+1}\,\frac{c^2E^2}{4\pi\mu m} = 0\,. \tag{10}$$

Expressing m, P and Q by ϱ_0, u_0, p_0 and B_0, there follows that

$$u = u_0$$

is a solution of Eq. (10). Splitting off the factor $(u-u_0)$, there remains a quadratic equation for u,

$$u^2 + \left(u_0 - \frac{2\varkappa}{\varkappa+1}\,\frac{P}{m}\right)u + \left(2\,\frac{\varkappa-1}{\varkappa+1}\,Q + u_0^2 - \frac{2\varkappa}{\varkappa+1}\,\frac{P}{\varrho_0}\right) = 0\,. \tag{11}$$

Introducing the new quantities

$$\hat u = \frac{u}{u_0}\,,\quad M_0^2 = \frac{u_0^2}{\varkappa\,p_0/\varrho_0}\,,\quad \beta_0 = \frac{p_0}{B_0^2/8\pi\mu}\,, \tag{12}$$

the equation for $\hat u$ becomes

$$\hat u^2 - \frac{1}{\varkappa+1}\left[\varkappa - 1 + \frac{2}{M_0^2}\left(1+\frac{1}{\beta_0}\right)\right]\cdot\hat u - \frac{2}{\varkappa}\,\frac{2-\varkappa}{\varkappa+1}\,\frac{1}{\beta_0 M_0^2} = 0\,,$$

or

$$\hat u_{1,(2)} = \frac{1}{2(\varkappa+1)}\left[\varkappa - 1 + \frac{2}{M_0^2}\left(1+\frac{1}{\beta_0}\right)\right] \overset{+}{(-)} \left\{\left[\frac{1}{2(\varkappa+1)}\right]^2\left[\varkappa - 1 + \frac{2}{M_0^2}\left(1+\frac{1}{\beta_0}\right)\right]^2 + \frac{2}{\varkappa}\,\frac{2-\varkappa}{\varkappa+1}\,\frac{1}{\beta_0 M_0^2}\right\}^{1/2}. \tag{13}$$

Only the positive sign has a physical meaning, the negative sign results in negative u. If $\hat u=1$ designates the singular point in front of the shock, then Eq. (13) yields the singular point behind the shock.

Two special cases for the above Eq. (13) shall be given.

(a) $\varkappa=2$ (two degrees of freedom):

$$\hat u_1 = \frac{1}{3}\left[1 + \frac{2}{M_0^2}\left(1+\frac{1}{\beta_0}\right)\right].$$

(b) $\varkappa=5/3$ (three degrees of freedom):

$$\hat u_1 = \frac{1}{8}\left[1 + \frac{3}{M_0^2}\left(1+\frac{1}{\beta_0}\right)\right] + \left\{\left(\frac{1}{8}\right)^2\left[1 + \frac{3}{M_0^2}\left(1+\frac{1}{\beta_0}\right)\right]^2 + \frac{3}{20}\,\frac{1}{\beta_0 M_0^2}\right\}^{\frac{1}{2}}.$$

In both cases, the final velocity is greater than in the gas-dynamic case.

Another way of ascertaining these two singular points is the derivation of the hydromagnetic analogue of the Rankine-Hugoniot relations.

1.3 Solution of the equations in the neighborhood of the singularities

The behavior of the system of equations in the vicinity of the singular points becomes evident from the higher derivatives. Differentiating the differential equation of second order for B, Eq. (6), with respect to x, there follows

$$u\,\frac{\mathrm{d}^3 B}{\mathrm{d}x^3} + \varrho\,\gamma_e\,\gamma_i\,\eta_R\,\frac{\mathrm{d}^2 B}{\mathrm{d}x^2} - \frac{4\pi\mu}{c^2}\,\varrho\,\gamma_e\,\gamma_i\,\frac{\mathrm{d}}{\mathrm{d}B}(uB)\cdot\frac{\mathrm{d}B}{\mathrm{d}x} = 0\,,$$

where small members of second order were neglected. Inserting the Maxwell Equation (5) results in

$$\frac{\mathrm{d}^2 j}{\mathrm{d}x^2} + \frac{\varrho}{u}\,\gamma_e\gamma_i\eta_R\,\frac{\mathrm{d}j}{\mathrm{d}x} - \frac{4\pi\mu}{c^2}\,\frac{\varrho}{u}\,\gamma_e\,\gamma_i\,\frac{\mathrm{d}}{\mathrm{d}B}(uB)\cdot j = 0\,. \tag{14}$$

The term $\mathrm{d}(uB)/\mathrm{d}B$ is immediately derived from Eq. (9). This yields

$$\frac{\mathrm{d}}{\mathrm{d}B}(uB) = u\left(1 - \frac{2}{\varkappa\beta}\,\frac{1}{M^2-1}\right), \tag{15}$$

where M is the gas-dynamic Mach number ($M=u/a$, a=sonic velocity). Substituting (15) into (14), there follows the differential equation for j in the neighborhood of the singularities,

$$\frac{\mathrm{d}^2 j}{\mathrm{d}x^2} + \frac{\varrho}{u}\,\gamma_e\gamma_i\eta_R\,\frac{\mathrm{d}j}{\mathrm{d}x} - \frac{4\pi\mu}{c^2}\,\gamma_e\,\gamma_i\,\varrho\left(1 - \frac{2}{\varkappa\beta}\,\frac{1}{M^2-1}\right)\cdot j = 0\,. \tag{16}$$

In this neighborhood, the dynamical quantities ϱ, u, etc. may be considered as constants. We thus may write the solution,

$$j = \alpha \cdot e^{\omega x}, \tag{17}$$

where α is a small perturbation. From Eqs. (17) and (16), ω becomes

$$\omega_{1,2} = -\frac{\varrho}{2u} \cdot \gamma_e \gamma_i \eta_R \pm \sqrt{\left(\frac{\varrho}{2u} \cdot \gamma_e \gamma_i \eta_R\right)^2 + \frac{4\pi\mu}{c^2} \gamma_e \gamma_i \varrho \cdot \left(1 - \frac{2}{\varkappa\beta} \frac{1}{M^2 - 1}\right)}. \tag{18}$$

This equation is now used for a discussion of the behavior of the system near the singularities in front of and behind the shock wave.

Singularity in front of the shock wave. In this case we have the hydromagnetic Mach number M^*, expressed by the gas-dynamic Mach number M and the ratio of gas pressure to magnetic pressure β,

$$(M_0^*)^2 = \frac{M_0^2}{1 + 2/\varkappa\beta_0} > 1,$$

and thus

$$1 - \frac{2}{\varkappa\beta_0} \frac{1}{M_0^2 - 1} > 0.$$

The two roots of Eq. (18) yield

ω_1 = positive, real: exponential increase,
ω_2 = negative, real: exponential decrease.

In the low temperature, high density region of our range of interest ($T < 10^5$ °K, $n \geqslant 10^{17}$ cm^{-3}), the first term in the root of Eq. (18) predominates and the "time constant" ω_1 can be represented by

$$\omega_1 = \frac{4\pi\mu}{c^2} \cdot \frac{u_0}{\eta_R} \cdot \left(1 - \frac{2}{\varkappa\beta_0} \cdot \frac{1}{M_0^2 - 1}\right).$$

Example: For $T_0 = 4 \times 10^4$ °K, $n_0 = 10^{17}$ cm^{-3}, $\beta_0 = 1$, $M_0^* = 2$ follows $\omega_1 = 10{,}66$ cm^{-1}.

Singularity behind the shock wave. At the singularity behind the shock wave we have

$$(M_1^*)^2 < 1; \quad 1 - \frac{2}{\varkappa\beta_1} \frac{1}{M_1^2 - 1} < 0.$$

Here, three cases are to be distinguished.

Case (i):

$M_1^2 > 1$;

$$\left(\frac{\varrho_1}{2u_1} \gamma_e \gamma_i \eta_R\right)^2 > \frac{4\pi\mu}{c^2} \cdot \gamma_e \gamma_i \varrho_1 \left(\frac{2}{\varkappa\beta_1} \frac{1}{M_1^2 - 1} - 1\right).$$

The two solutions of Eq. (18) are

ω_1 = negative, real: exponential damping,
ω_2 = negative, real: exponential damping.

The result in this case is an aperiodic damped transition from the singularity in front of the shock to that behind the shock.

Case (ii):

$M_1^2 > 1$;

$$\left(\frac{\varrho_1}{2u_1} \gamma_e \gamma_i \eta_R\right)^2 < \frac{4\pi\mu}{c^2} \cdot \gamma_e \gamma_i \varrho_1 \left(\frac{2}{\varkappa\beta_1} \frac{1}{M_1^2 - 1} - 1\right).$$

Here, ω becomes

$$\omega_{1,2} = -\frac{\varrho_1}{2u_1} \cdot \gamma_e \gamma_i \eta_R \pm i \cdot \sqrt{\frac{4\pi\mu}{c^2} \cdot \gamma_e \gamma_i \varrho_1 \cdot \left(\frac{2}{\varkappa\beta_1} \frac{1}{M_1^2 - 1} - 1\right) - \left(\frac{\varrho_1}{2u_1} \gamma_e \gamma_i \eta_R\right)^2}.$$

The result is a damped oscillatory transition with the damping constant

$$\alpha = \frac{\varrho_1}{2u_1} \cdot \gamma_e \gamma_i \eta_R$$

and the wavelength

$$\lambda = \frac{2\pi}{\sqrt{\frac{4\pi\mu}{c^2} \cdot \gamma_e \gamma_i \varrho_1 \cdot \left(\frac{2}{\varkappa\beta_1} \frac{1}{M_1^2 - 1} - 1\right) - \left(\frac{\varrho_1}{2u_1} \gamma_e \gamma_i \eta_R\right)^2}}.$$

The transition from damped oscillations to a damped aperiodic shock profile is given by the relation

$$\eta_R^2 = 16\pi\mu \cdot \left(\frac{u_1}{c}\right)^2 \cdot \frac{1}{\varrho_1 \gamma_e \gamma_i} \cdot \left(\frac{2}{\varkappa\beta_1} \cdot \frac{1}{M_1^2 - 1} - 1\right). \tag{19}$$

If η_R^2 becomes larger than the right-hand side of Eq. (19), then there exists aperiodic damping. As η_R is a function of temperature T_1, the regime for aperiodic damping can be expressed by temperature, density, Mach number and β-factor.

Case (iii):

$$M_1^2 < 1.$$

For this case the following roots result:

ω_1 = positive, real: exponential increase of a perturbation,
ω_2 = negative, real: exponential decrease.

This means that the solution is unstable near the singularity behind the shock. This will be discussed further in the following section of this paper.

The investigations of behavior near the singularities may be summarized by dividing the entire range of Mach numbers into three regimes:

$0 < M^2 < 1$,	exponential increase of perturbation;
$1 < M^2 < 1 + 2/\varkappa\beta$,	exponential decrease of perturbation (a damped oscillation for the smaller values of M^2, and aperiodic damping for the larger values);
$1 + 2/\varkappa\beta < M^2$,	exponential increase of perturbation.

1.4 Numerical calculations and discussion of damping coefficient, wavelength and entire shock width

The formulae derived above hold only in the vicinity of the singularities. However, they already give a satisfactory picture of the behavior of the

TABLE I. Data of the calculated hydromagnetic shocks

Case	I	II	III	IV
temperature T_0 [°K]	4×10^4	10^5	10^5	10^5
particle density n_0 [cm^{-3}]	10^{17}	10^{17}	10^{16}	10^{16}
† hydromagnetic Mach number M_0*	1,4	1,4	1,4	1,4
† pressure ratio β_0	10^{-2}	10^{-2}	6×10^{-3}	10^{-3}
† Mach number M_0	15,4	15,4	19,85	48,5
† pressure ratio β_1	$3{,}13\times10^{-2}$	$3{,}13\times10^{-2}$	$2{,}9\times10^{-2}$	$2{,}34\times10^{-2}$
† Mach number M_1	4,72	4,72	4,92	5,44
ϱ_1/ϱ_0	1,503	1,503	1,503	1,503
p_1/p_0	7,06	7,06	10,85	53
T_1/T_0	4,7	4,7	7,2	35

† $M_0{}^* = u_0 / \left(\frac{\varkappa p_0}{\varrho_0} + \frac{B_0{}^2}{4\pi\mu\varrho_0}\right)^{1/2}$, $\beta_0 = \frac{p_0}{B_0{}^2/8\pi\mu}$, $M_0 = \frac{u_0}{(\varkappa p_0/\varrho_0)^{1/2}}$, $\beta_1 = \frac{p_1}{B_1{}^2/8\pi\mu}$, $M_1 = \frac{u_1}{(\varkappa p_1/\varrho_1)^{1/2}}$.

solution. In order to ascertain the extent of their applicability, the shock equations, Eqs. (1) through (6), were solved numerically for certain selected cases. These are presented in Table I and Figs. 1—4. The selection was motivated by interest for actual experimental application.

Case I has almost complete aperiodic damping. The maximum value of the magnetic field B occurs well towards the end of the transition and is approximately 0,05 percent greater than the final value.

Case II also is quite close to aperiodic damping. It seems that the small temperature increase does not change the situation essentially.

In Case III we reduce both β and the density and obtain a pronounced oscillatory transition with good damping. This case will be discussed in detail below.

Case IV shows oscillatory transition with very small damping. The damping per wavelength is approximately 0,73 percent. Comparing this with Case III, the difference between III and IV consists in a lowering of β only, from 6×10^{-3} to 10^{-3}. It seems that the value of β has a decisive influence on the nature of the oscillatory transition.

The following is a more detailed discussion of Case III with respect to damping coefficient, wavelength and entire shock-width.

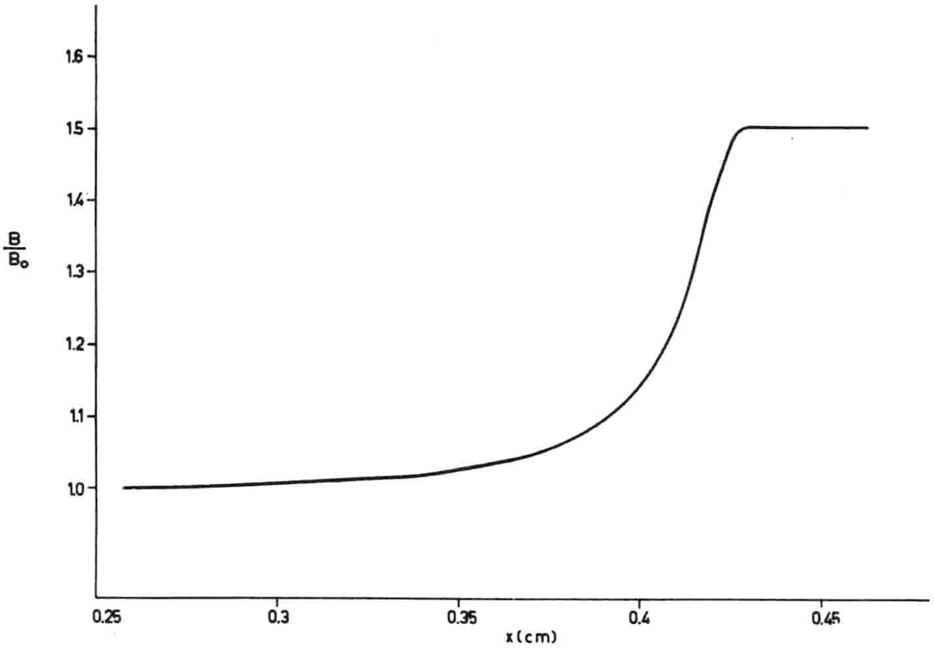

Fig. 1 Shock structure for $M^*_0 = 1{,}4$; $\beta_0 = 10^{-2}$; $n_0 = 10^{17}$ cm^{-3}; $T_0 = 4\cdot10^4$ °K.

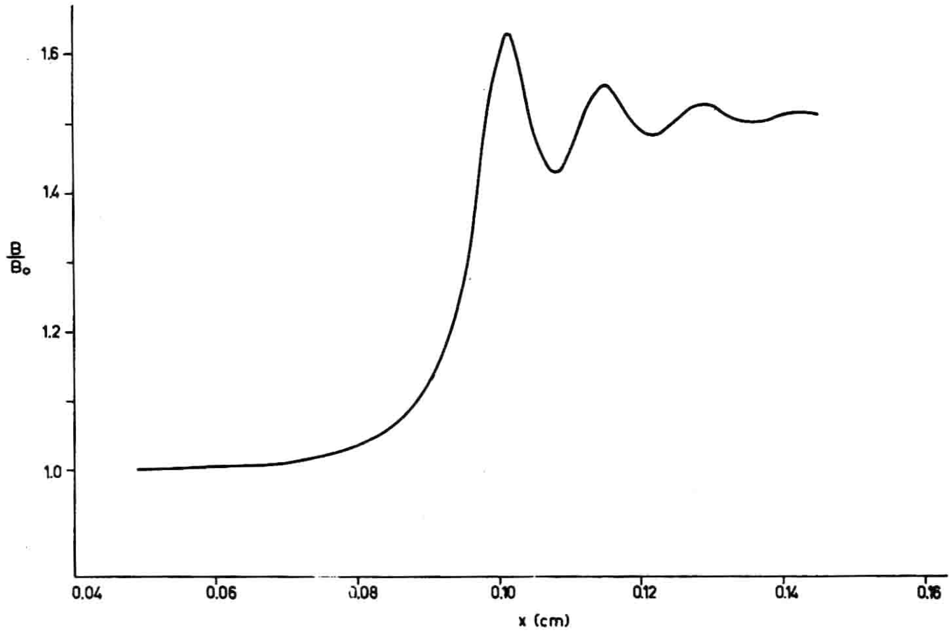

Fig. 2 Shock structure for $M^*_0 = 1{,}4$; $\beta_0 = 10^{-2}$; $n_0 = 10^{17}\ \mathrm{cm}^{-3}$; $T_0 = 10^5\ ^\circ\mathrm{K}$.

Wavelength. If the transition is sufficiently far from being aperiodic, the first term under the root of Eq. (18) may be neglected compared with the second term. This yields for the wavelength,

$$\lambda = \sqrt{\frac{\pi c^2}{\gamma_e \gamma_i \varrho \cdot (C-1)}},$$

where C is an abbreviation

$$C = \frac{2}{\varkappa \beta_1} \cdot \frac{1}{M_1^2 - 1}.$$

The value of C is practically constant in the cases calculated,

$$C = 1.8.$$

The wavelength λ is, from the above equation, essentially proportional to $1/\sqrt{\varrho}$. As ϱ varies quite slowly and $\sqrt{\varrho}$ still more slowly, then λ shows very little variation during the shock transition. Using ϱ_1 (behind the shock) for the calculation of λ, the wavelength is somewhat too small. The calculation yields for Case III

$$\lambda = 4{,}33 \times 10^{-2}\ \mathrm{cm}.$$

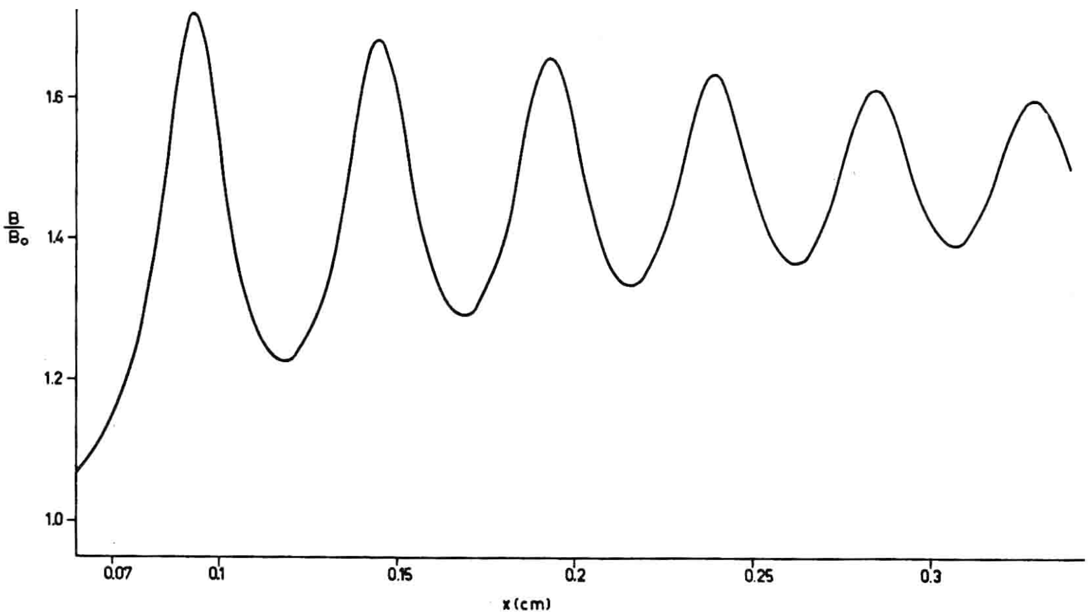

Fig. 3 Shock structure for $M^*_0 = 1{,}4$; $\beta_0 = 6 \cdot 10^{-3}$; $n_0 = 10^{16}\ \mathrm{cm}^{-3}$; $T_0 = 10^5\ ^\circ\mathrm{K}$.

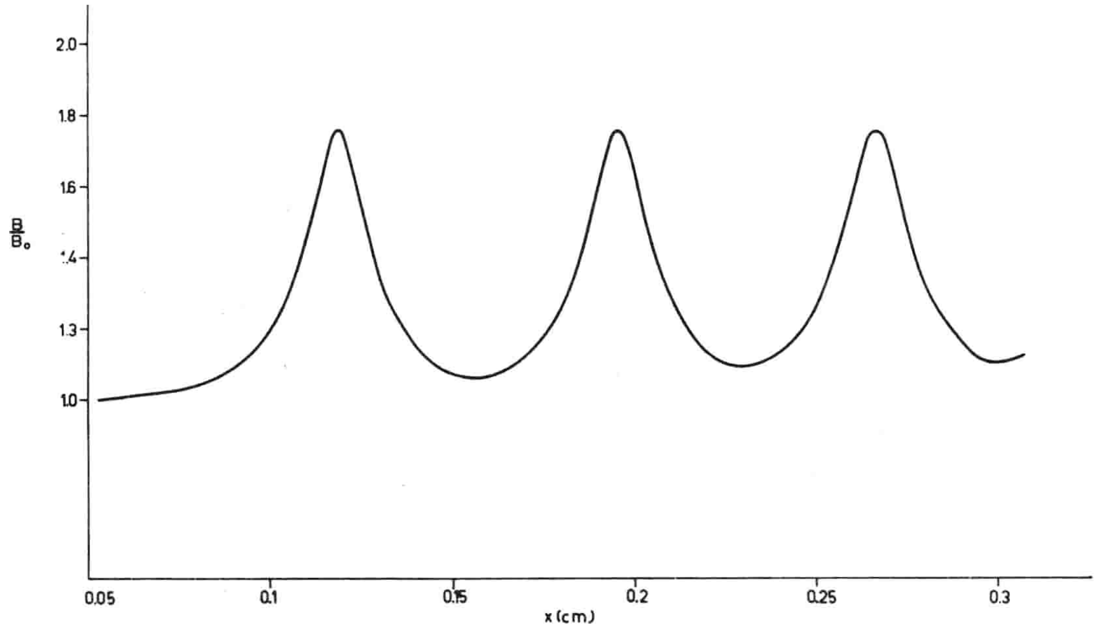

Fig. 4 Shock structure for $M^*_0 = 1{,}4$; $\beta_0 = 10^{-3}$; $n_0 = 10^{16}\ \mathrm{cm}^{-3}$; $T_0 = 10^5\ ^\circ\mathrm{K}$.

The numerical solution of the shock equations yields (cf. Fig. 3)

$$5{,}2 \times 10^{-2} \ldots 4{,}2 \times 10^{-2}\ \mathrm{cm}.$$

Damping per wavelength. The damping per wavelength, for relatively small damping, may be expressed by

$$e^{-\alpha\lambda} \approx 1 - \alpha\lambda, \quad \text{for } \alpha\lambda \ll 1.$$

The relative decrease per wavelength thus becomes

$$\alpha \cdot \lambda = \frac{c}{2} \sqrt{\pi \gamma_e \gamma_i} \sqrt{\frac{\varrho u}{C-1} \frac{\eta_R}{u^{3/2}}}$$

$$= 3{,}28 \times 10^{26} \sqrt{\frac{\varrho u}{C-1} \frac{\eta_R}{u^{3/2}}}.$$

The damping per wavelength is essentially proportional to $\eta_R/u^{3/2}$. For Case III there follows from the above equation,

$$\alpha\lambda = 0{,}158 = 15{,}8\,\%.$$

Evaluation of the numerical solution of the shock equations results in

$$\alpha = 16\,\%$$

as the decrease of the current j relative to its value at the beginning of the interval.

Entire shock-width. Defining the shock-width as the distance d along which the amplitude of the oscillation has decreased to 1/10th of its initial value, there follows from

$$e^{-\alpha d} = 0{,}1$$

for the entire shock-width,

$$d = \frac{\ln 10}{\alpha} = \frac{\ln 10}{\varrho/2u \cdot \gamma_e \gamma_i \eta_R} = 3{,}0 \times 10^{-32} \cdot \frac{u}{\varrho \cdot \eta_R}.$$

Using the values behind the shock, there follows from the above formula

$$d = 0{,}62\ \mathrm{cm}.$$

From the numerical solution we find $d = 0{,}58$ cm. The distance from the beginning of the shock until the first maximum of the current is 0,08 cm. In agreement with Davis, Lüst and Schlüter [5], we find that the entire shock-width is proportional to the flow velocity; we furthermore find, however, that it is inversely proportional to the specific ohmic resistance.

2. The influence of viscous dissipation on hydromagnetic shock structure

2.1 General remarks and basic equations

In the studies presented in the preceding Section of this paper, a certain inconsistency arises from the fact that, while collisions were considered in the damping mechanism (ohmic resistance), all other dissipation mechanisms arising from collisions were neglected. The purpose of the following considerations is to determine in what ranges this is permissible and to what extent viscous dissipation influences the transition through a hydromagnetic shock wave.

For this purpose, the basic equations for a hydromagnetic shock wave with transverse field in a frame of reference moving with the shock shall be given. **B** again is in z-direction, **j** and **E** in y-direction while the flow velocity **u** is in x-direction. Considered are the elements p_{xx} and p_{xy} of the trace-less pressure tensor $p_{ij} = P_{ij} - p\delta_{ij}$. We start from a two-fluid description, where the subscript e denotes the quantities for electrons, the subscript i those for ions. In the derivation we proceed from the conventional differential equations describing a plasma (cf. e.g. [6]).

Demanding quasi-neutrality, $n_e = n_i$, and assuming $T_e = T_i$, the component equations are added to form pseudo-one-fluid equations. There result

(a) the equation of continuity

$$\varrho\, u = m\,, \tag{20}$$

(b) the equation of motion

$$\varrho\, u^2 + p + p_{xx} + \frac{B^2}{8\pi\mu} = P, \tag{21}$$

(c) the energy equation

$$\frac{\varkappa}{\varkappa-1}\frac{p}{\varrho} + \frac{u^2}{2} + \frac{p_{xx}}{\varrho} + \frac{c\,E}{4\pi\mu m}B + \frac{1}{2\gamma_e\gamma_i}\frac{j^2}{\varrho^2} - \left(\frac{1}{\gamma_e} + \frac{1}{\gamma_i}\right)\frac{j}{m\,\varrho}\,p_{e,xy} = Q, \tag{22}$$

(d) the general Ohm's law,

$$\frac{d^2B}{dx^2} + \frac{1}{u}\frac{du}{dx}\frac{dB}{dx} + \gamma_e\gamma_i\eta_R\frac{\varrho}{u}\frac{dB}{dx} + \frac{4\pi\mu}{c^2}\gamma_e\gamma_i\frac{\varrho}{u}(c\,E - u\,B) + \frac{4\pi\mu}{c}(\gamma_e+\gamma_i)\frac{1}{u}\frac{dp_{e,xy}}{dx} = 0\,. \tag{23}$$

The expressions for the components of the pressure tensor are taken from CHAPMAN and COWLING [7], with the approximation for strong magnetic fields,

$$p_{xx} = -\frac{4}{3}\eta\frac{du}{dx}\left[\frac{1 + (4/9)\,\omega^2\tau^2}{1 + (16/9)\,\omega^2\tau^2}\right], \tag{24}$$

$$p_{xy} = \frac{4}{3}\eta\frac{du}{dx}\left[\frac{\omega\,\tau}{1 + (16/9)\,\omega^2\tau^2}\right], \tag{25}$$

where

$$\omega = \frac{e\,B}{m\,c}\,, \quad \tau = \frac{3\eta}{2p}\,, \tag{26}$$

and η is the coefficient of viscosity. In the case of a strong magnetic field $\omega^2\tau^2 \gg 1$.

2.2 Influence of viscosity on shock transition

In order to discuss the influence of viscosity on the shock transition, we derive an equation for du/dx by differentiating Eqs. (20) through (22) and eliminating $d\varrho/dx$ and dp/dx. The result (neglecting the last two terms in the energy equation) is

$$\frac{du}{dx} = \frac{1}{\varrho\,u\,[1 - (1/M^2) - (\varkappa-1)\,(p_{xx}/\varrho\,u^2)]}\left\{\frac{j}{c}\left[\varkappa\,B - (\varkappa-1)\frac{\varrho}{\varrho_0}B_0\right] - \frac{dp_{xx}}{dx}\right\}. \tag{27}$$

In the hydromagnetic case without viscosity this equation is

$$\frac{du}{dx} = \frac{1}{\varrho\,u[1 - (1/M^2)]}\cdot\frac{j}{c}\left[\varkappa\,B - (\varkappa-1)\frac{\varrho}{\varrho_0}B_0\right]. \tag{28}$$

The influence of viscosity is thus constituted by the term $(\varkappa-1)\,(p_{xx}/\varrho u^2)$ in the denominator and the second member in brackets in Eq. (27).

It is readily seen that du/dx according to Eq. (28) approaches infinity for the Mach number M approaching unity. This means that the shock becomes a discontinuity and shock-structure solutions are not possible for $M<1$. This is in agreement with the stability considerations given in the preceding Section of this paper.

In the case of Eq. (27), however, the term $(\varkappa-1)\,p_{xx}/\varrho u^2$ prevents this singularity at $M=1$. Hence, the inclusion of viscosity indicates the possibility of solutions for the shock transition with $M<1$. A stability investigation, however, still has to be carried out.

The second term in brackets in Eq. (27) should essentially influence only the shape of the shock transition. The question as to the magnitude of its influence shall be discussed in the following.

2.3 Comparison of magnitudes of ohmic and viscous dissipation

As an estimate of the influence of viscous dissipation, we may at first assume this influence to be relatively small so that the member p_{xx} can then be treated as a perturbation. This would enable use of the solution with $p_{xx}=0$ for determining du/dx in the calculation of p_{xx} according to Eq. (24). The conditions for permitting such a procedure are, according to Eqs. (27) and (28)

$$\left|\frac{dp_{xx}}{dx}\right| \ll \left|\frac{j}{c}\left\{\varkappa\,B - (\varkappa-1)\frac{\varrho}{\varrho_0}B_0\right\}\right|,$$

$$\left|(\varkappa-1)\frac{p_{xx}}{\varrho\,u^2}\right| \ll \left|1 - \frac{1}{M^2}\right|.$$

A further condition, required in using the Newton approximation for the stress tensor, is

$$\frac{p_{xx}}{p} \ll 1\,.$$

This condition would imply that $p_{xx} \ll B^2/8\pi\mu$ for values of $\beta<1$. For all cases where the above conditions are fulfilled, neglect of viscous dissipation would definitely be permissible for a relatively simple calculation of hydromagnetic shocks. If it can be shown, however, that for a large range these conditions are violated, then neglect of viscous dissipation would result in an erroneous description of hydromagnetic shock behavior.

Several characteristic points of our four cases were calculated and the quantities are presented in Table II. From these data, it is immediately seen that, for the cases selected, neglect of viscous dissipation is definitely not permissible. In Case I, the influence would become noticeable for the sharp bend towards the end of the shock (not given in Table II).

It is of course difficult to give an accurate picture of the influence of viscosity from such a rough estimate. However, a few definite characteristics can be ascertained.

(1) Viscosity influence always becomes dominating for large values of d^2u/dx^2 and du/dx. This influence increases with increasing temperature.

TABLE II. Comparison of ohmic and viscous dissipation

Case	M_0	T_0 [°K]	β_0	x [cm]	$\frac{\lvert p_{xx}\rvert}{p}$	$\frac{\lvert \mathrm{d}p_{xx}/\mathrm{d}x\rvert}{\left\lvert \frac{j}{c}\left(\varkappa B-(\varkappa-1)\frac{\varrho B_0}{\varrho_0}\right)\right\rvert}$	$1-\frac{1}{M^2}$	$\left\lvert -(\varkappa-1)\frac{p_{xx}}{\varrho u^2}\right\rvert$
I	15,4	4×10^4	10^{-2}	0,4	$2{,}4 \times 10^{-2}$	$2{,}1 \times 10^{-3}$	0,992	$7{,}8 \times 10^{-5}$
II	15,4	10^5	10^{-2}	0,09	$4{,}8 \times 10^{-1}$	$3{,}35 \times 10^{-2}$	0,993	$1{,}41 \times 10^{-3}$
III	19,85	10^5	6×10^{-3}	0,11375	$7{,}3 \times 10^{-1}$	$2{,}16 \times 10^0$	0,988	$3{,}49 \times 10^{-3}$
				0,11697	$9{,}8 \times 10^2$	$1{,}11 \times 10^2$	0,963	$1{,}45 \times 10^1$
IV	48,5	10^5	10^{-3}	0,13197	$1{,}23 \times 10^3$	$4{,}19 \times 10^1$	0,986	$7{,}25 \times 10^0$
				0,15447	$2{,}38 \times 10^0$	$4{,}32 \times 10^{-1}$	0,896	$9{,}9 \times 10^{-4}$

The values of $\mathrm{d}^2u/\mathrm{d}x^2$ and $\mathrm{d}u/\mathrm{d}x$ tend to increase for comparable points with increasing Mach number M and decreasing β. The effect of viscosity, then, will likely be a smoothing of the oscillation, strongly damping sharp peaks (cf. Case IV).

(2) The total shock-width (cf. e.g. Sec. I. 4.) will probably not be altered essentially as it is mostly determined by the collision mechanism in general. As the collision integral for the pressure tensor approaches zero with increasing temperature more rapidly than the collision integral for the ohmic dissipation (specific resistance η_R), the entire shock-width probably will be essentially determined by η_R.

Stability calculations have shown that for the system of Eqs. (20) through (25), the singularity behind the shock shows unstable behavior. It may well be that in a more rigorous expression for the stress tensor, this instability is retained. However, only if the inclusion of all possible dissipation mechanisms does not remove this instability, will the question as to the physical validity of the equations or the question of a probable general instability of hydromagnetic flows become pertinent.

3. Conclusions

In the case of hydromagnetic shock structure considering ohmic resistance as the only dissipation mechanism, damped aperiodic and damped oscillatory transitions are possible. A criterion giving the limit between the two ranges was derived.

The above case is furthermore characterized by the fact that no shock-structure solutions are possible for Mach numbers smaller than unity in the shock transition. For $M=1$, there results a true discontinuity and it was shown that perturbations increase exponentially for $M<1$.

Numerical estimates showed that viscous dissipation cannot be neglected against ohmic resistance for most cases of practical interest.

If the Mach number in front of the shock becomes sufficiently great and β sufficiently small, large values of $\mathrm{d}u/\mathrm{d}x$ and $\mathrm{d}^2u/\mathrm{d}x^2$ are obtained and viscous dissipation must be considered. This effect then results in a smoothing of the oscillations.

It is intended to carry out detailed calculations of hydromagnetic shock structure with the equations including viscosity (given in this paper) or with a similar system of equations including a more rigorous expression for the stress tensor.

Acknowledgement

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THÉORIE DES ONDES ADIABATIQUES ASSOCIÉES AUX TERMES NON DIAGONAUX DU TENSEUR DE PRESSION DANS LES PLASMAS*

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La théorie hydrodynamique de la propagation des ondes planes monochromatiques dans un plasma homogène et indéfini, en présence d'un champ magnétique uniforme mais quelconque, est en général développée à l'aide des équations macroscopiques, en admettant que les transformations produites par l'onde satisfont à des conditions d'adiabaticité: $\nabla p_\alpha = \gamma_\alpha k T_\alpha \nabla n_\alpha$ ($\alpha = e,i$); mais cela signifie que la pression reste isotrope pendant le passage de l'onde: on sait que cette hypothèse n'est certainement pas vérifiée pour un plasma plongé dans un champ magnétique quelconque.

Les auteurs présentent une théorie où la variation de pression accompagnant le passage de l'onde est représentée par un tenseur, sans aucune hypothèse restrictive. Le système des équations macroscopiques est fermé maintenant par la condition $\nabla \cdot \mathbf{Q} = 0$, qui exprime que le flux de chaleur est nul et, en conséquence, doit être considérée comme une "condition adiabatique" plus générale.

Cette nouvelle théorie conduit à des résultats dont les caractères généraux s'accordent très bien avec ceux obtenus dans la théorie indiquée au premier alinéa, notamment en ce qui concerne les conditions d'existence des modes purement transversaux ou purement longitudinaux; mais, en plus des modes décrits dans la première théorie, les termes non diagonaux du tenseur de pression font apparaître de nouveaux modes. Leur signification physique reste à discuter, mais une telle théorie permet au moins de délimiter, en principe, le domaine de validité de la théorie à pression scalaire. On a étudié plus particulièrement les modes purement longitudinaux, ainsi que les modes purement transversaux en propagation longitudinale.

1. Introduction

Dans un ouvrage récent [1], une théorie hydrodynamique des divers types d'ondes planes monochromatiques pouvant se propager dans un plasma a été présentée; cette théorie a l'avantage de fournir une description générale et systématique des divers types d'ondes.

Dans toute théorie hydrodynamique, la difficulté essentielle consiste à rendre fermé le système des équations macroscopiques (cf. [2] p. 88): en [1], on a fermé le système en admettant que les ondes se propageant sont adiabatiques, et en écrivant pour les électrons et les ions du plasma cette condition sous la forme:

$$\nabla p = \gamma k T \nabla n \qquad (1)$$

où p est la pression et n la densité des particules considérées. Cette écriture suppose donc, à priori, que la pression conserve son caractère scalaire au cours des transformations produites par l'onde.

D'autre part, on est amené dans de nombreux cas à négliger les collisions entre particules; cette hypothèse est raisonnable dans de nombreux cas expérimentaux; elle est d'autre part nécessaire, si l'on veut pousser assez loin les calculs et la description des ondes. Or, il semble que l'existence d'une pression scalaire et l'absence de collisions sont dans une large mesure contradictoires; de plus, la présence d'un champ magnétique implique des anisotropies de pression; il semble donc préférable d'abandonner la condition (1) et de ne faire aucune hypothèse restrictive sur le tenseur de pression $\mathbf{\Psi}$. Il faudra donc, en plus des équations de conservation et de transport de la quantité de mouvement déjà écrites en [1], écrire l'équation de transport de la pression cinétique $\mathbf{\Psi}$ pour chaque espèce de particule:

$$\frac{\partial n}{\partial t} + \nabla \cdot n\mathbf{v} = 0 \qquad (2)$$

$$n\,m\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)\mathbf{v} = n\,q\,(\mathbf{E} + \mathbf{v}\times\mathbf{B}) - \nabla\cdot\mathbf{\Psi} + \mathbf{P} \qquad (3)$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla + \nabla\cdot\mathbf{v}\right)\mathbf{\Psi} + \nabla v\cdot\mathbf{\Psi} + (\nabla v\cdot\mathbf{\Psi})^{T} + \nabla\cdot\mathsf{Q} = \mathsf{H} + \mathsf{R} \qquad (4)$$

où l'on fera $\mathbf{P} = \mathsf{R} = 0$ (collisions négligées). Ce système sera fermé par la relation:

$$\nabla\cdot\mathsf{Q} = 0 \qquad (5)$$

où Q est le tenseur flux de «chaleur» (cf. [2] p. 67 et 96) et qui semble donc exprimer une condition exacte d'adiabaticité.

Le présent article est consacré à cette théorie "exacte" des ondes adiabatiques dans un plasma; on verra que cette théorie garde les caractères généraux de la théorie «inexacte» faite dans [1], notamment en ce qui concerne les conditions d'existence de modes purement transversaux ou purement longitudinaux; mais les termes non diagonaux du tenseur de pression font apparaître de nouveaux modes.

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