RISK MANAGEMENT: VALUE AT RISK AND BEYOND

edited by

M.A.H. Dempster

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Introduction

The modern world of global finance had its antecedents in two significant events which occurred approximately thirty years ago: the breakdown of the post-war Bretton Woods system of fixed exchange rates between national currencies and the (re-) introduction of option trading in major financial markets emanating from the creation of the Chicago Board of Trade Options Exchange.

The latter coincided with the Nobel Prize-winning work of Black, Scholes and Merton who produced both a formula for the 'fair' valuation of stock options and an idealised prescription for the option seller to maintain a self-financing hedge against losing the premium charged – the famous delta hedge – which involved trading in the underlying stock only. The essence of their argument involved the concept of perfectly replicating the uncertain cash flows of European options. This argument, which required a continually rebalanced portfolio consisting only of the underlying stock and cash, applied more generally to other financial derivatives products whose introduction followed rapidly and at a rate which is still accelerating today. The new concepts were soon applied to futures and forwards and to the burgeoning market in foreign exchange in terms of derivatives written on currency rates, as FX market makers and participants attempted respectively to profit from, and to employ the hedging capabilities of, the new contracts in order to protect cross border cash flows in domestic terms in a world of uncertain exchange rates.

The market for derivative products in the fixed income sphere of bills, notes and bonds – although the basic theoretical foundations were established early on by Vasicek – has been much slower to develop, not least because fixed income instruments, even those issued by major sovereigns such as the US, Japan or the UK, are subject to multiple risk factors associated with their different multiyear tenors so that they are considerably more complex to value and hedge. Nevertheless, in less than twenty years the global market for swaps – in which two cash flows are exchanged for a specified period between counterparties – has grown from a single deal between IBM and the World Bank to over a trillion US dollar market accounting for about 40% of the global value of the derivatives markets. When the credit risk involved in similar instruments issued by less creditworthy sovereigns or public corporations must be factored in, derivative product valuation and hedging becomes even more complicated. Only recently a rough consensus on at least the alternative approaches to credit migration and default risk valuation has begun to emerge. Further, the derivatives markets are currently attempting to meet head on the risk inherent in all banking intermediation by using the new derivative tools and techniques both to securitize all types of risky cash flows such as mortgages, credit card payments and retail and commercial loan repayments and to create a global market in credit derivatives.

In the meantime, the use of derivative products in risk management is also spreading to such virtual commodities as energy, weather and telecommunications bandwidth. While futures contracts have been in use for agricultural commodities

: two centuries and for oil products and minerals for more than a hundred :he markets for forward, futures and option contracts written on kilowatt f electricity, heating or cooling degree days and gigabits of fibre optic trans- ke their traditional commodity predecessors, introduce a spatial location ; that adds to valuation complexity. Moreover, the nature of the asset price es underlying these new areas often results in a very poor fit to the classi- ısion processes used to model the equity, FX and major sovereign treasury Arising originally from the impacts of credit events on fixed income asset ›n, research continues unabated into valuation models and hedging schemes ıg jumping diffusions, extreme value processes and the unpriced uncertain- ɩo-called incomplete markets.

ough often denied, it was a maxim of nineteenth century commodity and markets that speculative trading led to excessive price fluctuations – today volatility. A new development is that investment banks currently operating najor financial markets have switched from being comfortable fee earners ›ting the equity and bond flotations of major corporations, together with hem advice on mergers and acquisitions, to deriving a considerable portion profits from derivative product sales and trading on own account. Like elopment of modern derivatives trading, the subsequent introduction of ·isk management techniques to cope with the effects of increased volatility cial markets can be traced to two relatively recent events.

first of these was the 1988 recommendation of the Bank of International Set- s in Basle of a flat 8% capital charge meant to be appropriate to all financial ons to cover all types of risks - market (due to price changes), credit (due terparty defaults), liquidity (due to market imbalance), etc. This Capital :y Accord was a more or less direct reaction to credit problems following ity market crash of October 1987 and was subsequently refined in an at- o cover off-balance-sheet derivatives and enacted into law in many of the economies with varying lags. In the absence of a global financial regulator :alled 'soft law' has been remarkably effective in the leading economies. the current BIS proposals to revise the Accord and to explicitly cover the ıerent in banking operations is enjoying heated debate largely in recognition act that the lags in national enforcement are likely to be much shorter this ound.

second, more technical, event occurred on Wall Street about seven years J.P. Morgan in response to an earlier demand by the Chairman for a 4:15 :ach day on the potential trading earnings at risk overnight due to global price movements. The result was the concept of *Value at Risk* (VaR) which ın the title of this volume, together with a formal model for the evaluation such market risks for portfolios and trading desks over short periods of trading days. This concept has been taken up by financial regulators in 6 Basle Accord supplement and has subsequently been extended – more :rsially – to measuring credit risks over much longer horizons. Moreover, :d to the Risk Metrics spin-off which markets data and software systems pon its previously published approaches and has become a major player in dly growing market for so-called enterprise-wide risk management solutions appropriate to the world's financial institutions at all levels. This market trend will no doubt continue under the pressure of the new BIS Capital Adequacy Accord and it is hoped that the present book can play some small role in helping to clarify the complex issues revolving around the future stability of the global financial system.

We now turn to a brief description of the contributions to this volume which are based to a greater or lesser degree on a very successful Workshop on Risk Management held at the Isaac Newton Institute for Mathematical Sciences on 2–3 October 1998, organized by its Director, Professor H.K. Moffat FRS, and attended by both practitioners and academics. The contents of the volume reflect the mix of theory and practice which is required for survival in today's capital markets.

The opening chapter by Picoult, the senior risk analyst at Citicorp, the world's largest and arguably most global bank, sets the practical context for the rest of the book. In a clear and parsimonious style the author discusses in some detail techniques for three of the four most important risks of trading: valuation risk, market risk and counterparty credit risk. (The fourth, operational risk, will be discussed in the last chapter of this volume, where the impact of the Russian Crisis of late summer and early autumn 1998 upon trading profits of an anonymous European bank will be analysed.) Chapter 1 begins by describing the important features of (expected) discounted cash flow models used for the valuation of financial instruments and portfolios. The author points out that valuation error can stem not only from the model error beloved of quantitative analysts, but also from erroneous or misused data and human misunderstanding, and he goes on to clarify the factors required to establish market value. The next two sections of the chapter discuss in detail the methods used to 'measure, monitor and limit' market and counterparty credit risk respectively. The principal approaches to statistical analysis of market risk – *parametric* (Gaussian or mean-variance), *historical* (empirical) and full *Monte Carlo* VaR analysis and *stress testing* – are described precisely. Analysis of credit risk is as indicated above usually more complex, and techniques for the measurement of both pre-settlement and settlement risks are set out next. Finally the main attributes of market and credit risk are compared and contrasted.

In Chapter 2, Srivastava uses parametric VaR analysis based on a binomial tree implementation of the popular Heath–Jarrow–Morton model for forward interest rates to provide a succinct dissection of one of a string of celebrated derivative fiascos of the early 1990s – the fixed-floating five year semi-annual swap between Bankers Trust and Proctor and Gamble (P&G) initiated in November 1993. The author's step-by-step exposition demonstrates that had P&G carried out such a straightforward analysis using modern risk management tools, they would have seen that the VaR of the contract was about seven times its value. In the event this so-called *unexpected loss* amount – \$100M – was actually lost. Using the expected excess loss over the VaR limit – a *coherent* risk measure as introduced in Chapter 6 and applied in subsequent chapters – a factor of about ten times the market value of the contract would have been found.

Kupiec proposes in Chapter 3 a methodology to parametrize extreme or *stress test scenarios*, as used by many banks to evaluate possible market value changes in a large portfolio *in addition* to VaR analysis, in a context which is completely ıt with VaR. The author shows how assuming multivariate normal return ions for all risk factors leads to automatic consideration of value changes ıe non-stressed factors which are commonly ignored in stress testing. He :ates on data for the period of the 1997 Asian crisis that his conditional ı *Stress VaR* (95%) approach to stress testing leads to historically accurate l value changes for a global portfolio with instruments in the US, European .n time zones. The chapter concludes with a detailed discussion of the problems involved in stressing the correlations and volatilities needed in ısian analysis.

last chapter to deal primarily with market risk, Chapter 4, Dempster and n return to the fundamental Black–Scholes concept of accurate trading ·eplication of risk characteristics in the context of dynamic portfolio repli- a large target portfolio by a smaller self-financing replicating portfolio le instruments. Two applications are identified: *portfolio compression* for olio VaR calculation and *dynamic replication* for hedging by shorting the g portfolio or for actual target portfolio simplification. The first (virtual) ›n involves no transaction costs and is shown to be a promising alternative ›ortfolio compression techniques such as multinomial factor approxima- , full daily portfolio revaluation using Monte Carlo simulation. With or ıe use of variance reduction techniques such as low-discrepancy sequences, Monte Carlo simulation to value large portfolios for VaR analysis is for itutions barely possible overnight. The authors demonstrate that the use ;tic programming models and standard solution techniques for portfolio on can produce an expected average absolute tracking error of the easily replicating portfolio which (at about 5% of the initial target portfolio superior to both more static replicating strategies and target portfolio ;ing and within acceptable limits for fast VaR calculations.

·r 5, by Kiesel, Perraudin and Taylor, turns to an integrated consideration and credit risks for VaR calculations. Reporting on part of a larger ve study of credit risk models for US corporate bonds supported by the Ingland, the authors emphasize the very different horizons needed for ıd credit risk VaR calculations – respectively several days and one or more · which the time value of money clearly cannot be ignored. They note that ıte risk should always be included in long horizon credit VaR calculations rates and credit spreads are less than perfectly correlated and they set dy this correlation and its analogue for ratings transition risks. They find at counter intuitively but in agreement with some previous studies – that ıte changes and both credit spreads and ratings transitions are negatively even over one year horizons. Recently it has been suggested that such .y be explained by the empirical fact that expected default rates – and possible credit transitions – account for a surprisingly small proportion d credit spreads, the bulk of which may be due to state tax effects and · nondiversifiable systemic risk in the bond markets analogous to equity

naining four chapters of this volume take the reader well beyond the con- aR analysis. The first, Chapter 6 by Artzner, Delbaen, Eber and Heath, is a classic. The authors *axiomatize* the concept of financial risk measurement in terms of the risk or economic capital required to neutralize potential losses from the current position and relate such *coherent* risk measures to existing VaR and stress testing techniques. They show by example that VaR is *not* a coherent risk measure in that it fails to possess the subadditivity – i.e. portfolio diversification – property. This property assures that the risk capital required to cover two risky positions is never more than the sum of those required to cover each individually. It has the important demonstrated consequence that individual coherent risk measures for classes of risk factors – for example relevant to market and credit risk individually – can be combined into an overall conservative coherent risk measure based on all risk factors present. The abstract approach to risk measurement is applied in the chapter both to improve the stress testing schemes for margining proposed by the Chicago Mercantile Exchange and the US Securities and Exchange Commission and to demonstrate that the *expected excess* over a VaR level added to the VaR yields a coherent measure – an idea with its roots in nearly 150 years of actuarial practice.

Embrechts, McNeil and Straumann provide in Chapter 7 a thorough *primer* on the measurement of static statistical dependencies from both the actuarial and financial risk management viewpoints. They demonstrate, both by theory and illuminating example, that the concept of linear correlation is essentially valid only for the multivariate Gaussian and other closely related spherical distributions. Correlation analysis is based on second moments, breaks down for fat-tailed and highly stressed distributions and is not defined for many *extreme value* distributions. From the risk management perspective, these facts constitute a different criticism of VaR analysis to that studied in the previous chapter: namely correlation matrices calculated from data non-spherically distributed but used in practice for parametric Gaussian VaR calculations can lead to highly misleading underestimates of risk. As well as classical rank correlation and concordance analysis, the use of the *copula* function, appropriate to the study of dependencies amongst the coordinates of *any* multivariate distribution, is proposed and its basic properties set out. Much work remains to be done in this area – particularly with respect to practical computational multivariate techniques – but this chapter provides among many other things a basic grounding in the copula concept.

Following its extensive use by insurance actuaries, possible uses of *extreme value theory* (EVT) in risk management are discussed by Smith in Chapter 8. After a brief exposition of EVT and maximum likelihood estimation of extreme value parameters, these concepts are illustrated on both fire insurance claims and S&P500 equity index data. Next the author introduces the Bayesian approach to the predictive EVT distributions with unknown parameters which are needed for risk management in the presence of extreme loss events. He goes on to describe the limited progress to date in handling multivariate extreme value distributions and then to propose a dynamic changepoint model to attack the volatility clustering of the S&P500 index data. The latter allows the extreme value parameters to change at a fixed number of timepoints, which number is estimated from the data along with the other parameters using hierarchical Bayesian methods. The posterior distributions of all parameters are simultaneously estimated using reversible jump *Markov chain Monte Carlo* (MCMC) sampling. The suggested conclusion of this analysis is

NYSE enjoys periods of (short-tailed) normal returns but exhibits extreme ·haviour in periods of high volatility.

themes of Chapter 8 are continued in the final chapter by Medova and u in the context of extreme operational risks in financial institutions. The set the scene by describing current definitions of operational risk and the proposed Basle Accord revisions to cover it and then argue that all risks considered in an integrated framework in which extreme operational risks nd to events in the unexpected loss tail of the appropriate integrated profit (P&L) distribution *whatever* their underlying source. Next they provide riew of the relationship between the classical limit theory for stable dis-ıs and extreme value theory and then set up a time homogeneous version ·ayesian hierarchical model of the previous chapter, again estimated by sampling, in the context of extreme operational risk measurement and .llocation in terms of the coherent expected excess loss measure. Bayesian es are appropriate to the measurement of operational risk in financial in-s in that, although such data is scarce, using median posterior distribution value parameter estimates – i.e. absolute value loss functions – and the *er threshold* (POT) model, stable accurate estimates are produced for very nple sizes (10–30). The methods are illustrated using five quarters of pro-data on the daily P&L of four trading desks of a European investment ·ough the Russian crisis of 1998. The authors demonstrate that for this ır techniques could have been used to get relatively accurate estimates of :apital required to cover actual losses throughout the period and that this rovision enjoyed a portfolio diversification effect across trading desks in he presence of extreme events.

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CONTENTS

Quantifying the Risks of Trading

Evan Picoult

Abstract

This article defines and describes methods for measuring three of the prominent risks of trading: valuation risk, market risk and counterparty credit risk. A fourth risk, operational risk, will not be discussed. The first section of the article describes the essential components of discounted cash flow models used for valuation, identifies the sources of valuation error and classifies the types of market factors needed to measure market value. The second section of the article describes the nature of and the methods that can be used to measure, monitor and limit market risk. A similar analysis of counterparty credit exposure and counterparty credit risk follows. Finally, the nature of and methods for measuring market risk and counterparty credit exposure will be compared and contrasted.

1 Introduction

The term 'risk' is used in finance in two different but related ways: as the magnitude of (a) the potential loss or (b) the standard deviation of the potential revenue (or income) of a trading or investment portfolio over some period of time.[1]

Our discussion and analysis of market risk and counterparty credit risk will almost exclusively focus on risk as potential loss. That is, we will describe methods for measuring, in a specified context, the potential loss of economic value of a portfolio of financial contracts. The context that needs to be specified includes the time frame over which the losses might occur (e.g. a day, a year), the confidence level at which the potential loss will be measured (e.g. 95%, 99%) and the types of loss that would be attributed to the risk being measured (e.g. losses due to changes in market rates vs. losses due to

[1]The quantitative relationship between risk as potential loss and risk as uncertainty in future revenue is a function of the estimated probability distribution of future revenue. For example, if the estimated probability distribution of potential revenue is normally distributed around an expected value of zero then the potential loss at some confidence level can simply be expressed in terms of the standard deviation of potential revenue. In many cases the expected total revenue from a trading business is not zero (else the firm would not be in the business) and the probability distribution of future revenue may not be symmetric about its expected value.

the default of a counterparty). Part of the context for measuring the potential loss, whether due to market or credit risk, is the distinction between an economic perspective and an accounting perspective.

The distinction arises for market risk because the income from financial contracts may be accounted for in one of two ways: by accrual accounting (e.g. as is typically done for a portfolio of deposits and loans) or by mark to market accounting (e.g. as is typically done for a trading portfolio). The primary difference between the two approaches is in the timing of their recognition of financial gains or losses.[2] Only the mark to market approach is equivalent to the continual measurement of economic value and change in economic value. The relative merits and demerits of measuring the income and risk of a particular business on an accrual basis will not be evaluated here This article is focused on the risks of trading and will analyze and describe market risk from an economic perspective.

A similar issue arises for credit risk. One example of this is the potential difference between the loss in the market (economic) value of a loan caused by the default of the borrower and the timing of the recognition of the loss in the income statement.

A more important example of this issue for credit risk is the treatment of the deterioration of a borrower's credit worthiness. Consider as an example a corporate loan. Assume that in the period after the loan was made the only relevant factor to undergo a material change was a deterioration of the credit worthiness of the borrower. In more detail, assume that one year ago a bank made a three year loan to a corporation for which the corporate borrower was required to make periodic interest payments and to pay back the principal and the remaining interest payment on the maturity date of the loan. Assume further that both the public credit rating and the bank's internal credit rating of the borrower has deteriorated since the loan was initially made. Finally, assume that general market rates have remained unchanged since the loan was made and that the borrower has made all interest payments on time.

[2] As a simple example of the difference consider two portfolios. Portfolio A is a standard deposit and loan portfolio. It consists of a ten year \$100 million loan to firm X at a fixed semi-annual rate of 10.00% and a one year \$100 million deposit from firm Y, at a semi-annual rate of 9.50% interest. Portfolio B is a trading portfolio. It consists of a long position in a ten-year debt security issued by firm X at a fixed semi-annual rate of 10.00% and a short position in a one-year debt security issued by firm Y at a fixed semi-annual rate of 9.50%. If both portfolios were viewed from a marked to market perspective, they would have identical market risks. However, under standard accounting practices, the effect of a change in market rates on the reported revenues of the portfolios will differ. Assume the only change in market rates is a 1% parallel increase in the risk free rate at all tenors. On a marked to market basis the net value of the securities in Portfolio B would fall in value. In contrast, the accrued interest earned by Portfolio A is locked in for the year and is independent of the level of interest rates. If interest rates should continue at their higher level the accrued interest earned by Portfolio A will only be affected after its one year deposit matures and has to be replaced with a deposit at a higher interest rate.

Under standard accrual accounting the loan would be recorded on the balance sheet of the bank at its par value. The bank would only record a loss if the borrower defaulted on a payment. The bank would not in general recognize any loss due to the deterioration of the credit quality of the borrower. At most the bank could establish a general loan loss reserve for the expected credit loss of the portfolio.

In contrast, the market value of the loan would fall if the borrower's credit worthiness deteriorated. To appreciate the reality of this loss, assume the bank took action to actively managing its credit risk to this corporate borrower, after the borrower's credit risk had deteriorated. For example, if the bank were to sell the loan in the secondary loan market, then *pari passu*, it would suffer an economic loss – i.e. the market value of the loan would be less than its par amount because of the increased credit riskiness of the borrower. Or, if the bank tried to hedge its credit exposure to the borrower by buying a credit derivative on its underlying loan it might have to pay an annual fee for that credit insurance that was higher than the net interest income it was earning on the loan. Both of these examples of active portfolio management illustrate that a deterioration in the credit quality of the borrower, all other things held constant, will cause the market value of the loan to decrease, even if the borrower had not defaulted.

From the accrual accounting perspective no credit loss would occur without a default by the borrower. From the economic perspective, the increased riskiness of the borrower would cause the economic value of the loan to decrease.

This article will not focus on loan portfolio credit risk. It will however analyze another form of credit risk, the risk that the counterparty to a forward or derivative trade could default prior to the final settlement of the cash flows of the transaction. This form of credit risk is called *counterparty pre-settlement credit risk* and will be analyzed in detail below.

We will describe methods for measuring four aspects of the risks of trading:

- Methods for measuring and controlling *valuation uncertainty* and valuation error.
- Methods for measuring *market risk*. These methods measure the potential decrease in the economic value of contracts caused by potential future changes in market rates.
- Methods for measuring a *counterparty's pre-settlement credit exposure*. These methods measure the potential future replacement cost of the forward and derivative contracts transacted with a counterparty, should the counterparty default at some time in the future before all the contracts mature. The potential credit exposure will depend on the potential future market value of the contracts transacted with the counterparty, on any risk mitigating agreements (such as netting) that have

been contracted with the counterparty and on the legal enforceability of such agreements.

- Methods for measuring *counterparty credit risk*. These methods measure the probability distribution of loss due to counterparty default and rest, in part, on measurements of the potential future credit exposure to the counterparty, the future default probability of the counterparty and the potential loss in the event of counterparty default.

Our measurements of market risk, counterparty credit exposure and counterparty credit risk all rest on our ability to measure the current and the potential future economic value of financial contracts. At the end of the article we will summarize and contrast each type of risk measurement. Because of the crucial connection between methods for valuing contracts and methods of risk measurement, we shall begin our discussion with a review of revaluation models, valuation errors and market factors.

2 Market valuation and valuation uncertainty

2.1 Discounted cash flow formula

Marking to market is the activity of ascertaining the market value of each financial instrument in a trading portfolio. Market value is ascertained in one of two ways: directly, by observing the market price of identical (or nearly identical) instruments or indirectly, by using a discounted cash flow revaluation model. When a discounted cash flow model is used, it is necessary to periodically calibrate the model against the market to ensure that the model's valuation corresponds to the market's.

Very liquid, cash-like financial instruments such as spot FX, equities and simple debt securities are marked to market by discovering the prices or rates at which identical (or nearly identical) instruments are traded in the market. For example, the market value of a portfolio of US Treasury securities of different maturities and coupon rates would be calculated simply by discovering the unit market price of each security in the portfolio and by multiplying the unit price by the number of units owned (positive for long, negative for short). Nothing beyond simple arithmetic would be needed to calculate the mark to market value of the portfolio.

In contrast, forward and derivative contracts are revalued in terms of discounted cash flow models (reval models). In essence, a *reval model* calculates the net present value of the expected future cash flows of the contract. It does this by representing the economic value of a contract as a function of its terms and conditions, basic market rates, and the current date:

$$PV(t)_k = f(T\&C_k, \{X_j(t)\}, t), \tag{2.1}$$