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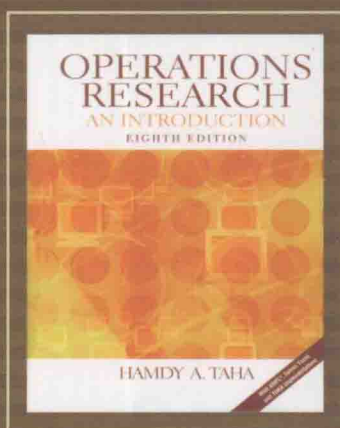
Operations Research
An Introduction

运筹学导论

初级篇

(英文版 · 第8版)

[美] Hamdy A. Taha 著



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内 容 提 要

本书是运筹学方面的经典著作之一, 为全球众多高校采用. 初级篇共 12 章, 内容包括线性规划建模、单纯形方法和灵敏度分析、对偶性和后最优分析、运输模型及其变型、网络模型、目标规划、整数线性规划、确定性动态规划、确定性库存模型、决策分析和博弈、排队系统等, 并附有 AMPL 建模语言简介.

本书可作为经管类专业、数学专业和计算机专业本科生的教材, 也可供相关研究人员参考.

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前 言

本书第8版对内容作了很多的修订,在教材的编排上突出反映运筹学中的应用问题和计算方法.

- 第2章通过城市规划、货币套利交易、投资、生产计划、混合配比、排序以及下料等实际问题的应用,主要介绍了线性规划的建模.新增加的习题也涉及从水质管理和交通控制到军事领域等多个运筹问题.

- 第3章以一种简单和直接的方式介绍了一般性的线性规划灵敏度分析,包括对偶价格和简约费用(reduced cost),作为单纯形表计算部分的直接扩充.

- 本版的第4章主要讲述基于对偶性进行线性规划最优化后的分析.

- 针对旅行商问题(Traveling Salesperson Problem),介绍了一个基于Excel的组合式最近邻点反向启发式算法.

- 新增的第17章扩充了马尔可夫链的处理方法.

- 在全新的第24章里,详细介绍了15个实际应用案例.对这些案例的分析通常涉及多种OR技术(例如启发式算法和线性规划,或者线性整数规划和排队论),用来进行建模、数据采集以及问题的求解计算等.这些应用问题在相关的各章都有引用,使读者能够充分了解实际生活中如何运用运筹学技术.

- 新的附录E收录了按照章节排列的约50个小型实用问题的例子.

- 本书还包含了1000多个节后习题.

- 每章开始都有本章导读,以帮助读者了解教材内容和有效利用附带的软件程序.

- 把教材与软件相结合可以让读者对需要深入介绍的概念进行实际检验:

1. 全书都用到了Excel程序,包括动态规划、旅行商问题、库存问题、层次分析法、贝叶斯概率、“电子化”统计表、排队问题、模拟、马尔可夫链和非线性规划等.一些程序中的交互式用户输入功能有助于对相应方法的更好理解.

2. 对Excel Solver程序的使用扩展到了全书,特别用在线性规划、网络规划、整数规划和非线性规划问题.

3. AMPL[®]是一种强大的商业化建模语言,本书将AMPL结合在大量的例题中,这些例子涉及线性、网络、整数和非线性规划问题.附录A给出了AMPL的语句规则以及本书例题中所引用的语言素材.

4. 本书中,TORA仍然充当教学软件的重要角色.

- 所有与计算机相关的材料都相对独立,有的作为单独的章节,有的按照标题AMPL/Excel/Solver/TORA程序作为一小节,以尽量不影响本书的主要内容的介绍.

为了限制本书的页数,我们把一些小节、一部分整章以及两个附录都放在了附

带的CD里，作者把运筹学导论课程中不太经常用到的内容截选下来，也放在CD里¹。

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C H A P T E R 1

What Is Operations Research?

Chapter Guide. The first formal activities of Operations Research (OR) were initiated in England during World War II, when a team of British scientists set out to make scientifically based decisions regarding the best utilization of war materiel. After the war, the ideas advanced in military operations were adapted to improve efficiency and productivity in the civilian sector.

This chapter will familiarize you with the basic terminology of operations research, including mathematical modeling, feasible solutions, optimization, and iterative computations. You will learn that defining the problem correctly is the most important (and most difficult) phase of practicing OR. The chapter also emphasizes that, while mathematical modeling is a cornerstone of OR, intangible (unquantifiable) factors (such as human behavior) must be accounted for in the final decision. As you proceed through the book, you will be presented with a variety of applications through solved examples and chapter problems. In particular, Chapter 24 (on the CD) is entirely devoted to the presentation of fully developed case analyses. Chapter materials are cross-referenced with the cases to provide an appreciation of the use of OR in practice.

1.1 OPERATIONS RESEARCH MODELS

Imagine that you have a 5-week business commitment between Fayetteville (FYV) and Denver (DEN). You fly out of Fayetteville on Mondays and return on Wednesdays. A regular round-trip ticket costs \$400, but a 20% discount is granted if the dates of the ticket span a weekend. A one-way ticket in either direction costs 75% of the regular price. How should you buy the tickets for the 5-week period?

We can look at the situation as a decision-making problem whose solution requires answering three questions:

1. What are the decision **alternatives**?
2. Under what **restrictions** is the decision made?
3. What is an appropriate **objective criterion** for evaluating the alternatives?

Three alternatives are considered:

1. Buy five regular FYV-DEN-FYV for departure on Monday and return on Wednesday of the same week.
2. Buy one FYV-DEN, four DEN-FYV-DEN that span weekends, and one DEN-FYV.
3. Buy one FYV-DEN-FYV to cover Monday of the first week and Wednesday of the last week and four DEN-FYV-DEN to cover the remaining legs. All tickets in this alternative span at least one weekend.

The restriction on these options is that you should be able to leave FYV on Monday and return on Wednesday of the same week.

An obvious objective criterion for evaluating the proposed alternative is the price of the tickets. The alternative that yields the smallest cost is the best. Specifically, we have

$$\text{Alternative 1 cost} = 5 \times 400 = \$2000$$

$$\text{Alternative 2 cost} = .75 \times 400 + 4 \times (.8 \times 400) + .75 \times 400 = \$1880$$

$$\text{Alternative 3 cost} = 5 \times (.8 \times 400) = \mathbf{\$1600}$$

Thus, you should choose alternative 3.

Though the preceding example illustrates the three main components of an OR model—alternatives, objective criterion, and constraints—situations differ in the details of how each component is developed and constructed. To illustrate this point, consider forming a maximum-area rectangle out of a piece of wire of length L inches. What should be the width and height of the rectangle?

In contrast with the tickets example, the number of alternatives in the present example is not finite; namely, the width and height of the rectangle can assume an infinite number of values. To formalize this observation, the alternatives of the problem are identified by defining the width and height as continuous (algebraic) variables.

Let

$$w = \text{width of the rectangle in inches}$$

$$h = \text{height of the rectangle in inches}$$

Based on these definitions, the restrictions of the situation can be expressed verbally as

1. Width of rectangle + Height of rectangle = Half the length of the wire
2. Width and height cannot be negative

These restrictions are translated algebraically as

$$1. 2(w + h) = L$$

$$2. w \geq 0, h \geq 0$$