APPLIED NUMERICAL METHODS FOR THE MICRO-COMPUTER

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*To the memory of Oscar L. Shoup and David R. Spoon,
who helped me learn about inventiveness, electronics,
and a host of other fun things.*

The small computer is a truly remarkable tool for digital manipulation. Recent growth trends in the area of microcomputer technology predict the widespread use of small computers in many aspects of our lives in the decades ahead. Yet with all its widespread acceptance in the public domain, the small computer has seen relatively little use in applications for numerical methods in science and engineering. A primary reason for this fact is that most numerical methods texts and software packages are written from the perspective of the large, mainframe computer. Yet the use of the small computer as a tool for implementing certain types of numerical methods problems is rapidly becoming preferable to the larger computer for reasons of cost, utility, and convenience.

Of the limited information and software that is available for implementing numerical methods on small computers, very little has been developed by those who have an appreciation for both the power of modern numerical methods and the versatility of the small computer. It is the purpose of this text to exploit the best characteristics of these combined domains. This text is intended to provide not only a source of information about the area of numerical methods, but also to provide a software data base containing quality algorithms tailored for implementation on the small computer.

The text and its accompanying software should provide a package suitable for teaching numerical methods courses at universities and for individual scientific and engineering personnel who want to expand the utility of their computational equipment.

Preface

The spectrum of topics in this text includes the problem areas most frequently encountered in scientific and engineering problem solving. The basic approach of this text is one in which fundamental topics in numerical methods are presented along with the key mathematical relationships necessary for the development of useful algorithms. Then special-purpose software is provided to implement the algorithms on the small computer. Example applications are presented. A summary is presented at the end of each chapter to help the user choose the best algorithm for a given problem-solving task and to alert the user to potential problems in the application of these algorithms on the small computer. End of chapter references are provided to help the user discover more information about specific algorithms. An appendix of problems and exercises is also included. For those who find the BASIC programs used in this text to be useful, a software package will soon be available.

The author would like to thank all those who contributed to the construction of this text and the approach it provides. Special thanks for encouragement are due to Dr. R.H. Page. A special note of appreciation is due to those of my colleagues who provided helpful suggestions to improve the text. The author is especially grateful to Dr. Don Riley, Dr. Farrokh Mistree, Dr. Douglas Green, and Dr. Ken Waldron. A special thank you is in order for Billie Gresham, whose remarkable talents and cheerful disposition made the preparation of the draft manuscript materials a pleasant activity. Finally, I want to express my gratitude to my family for their patience and encouragement during the many hours spent on this project.

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Because of its remarkable capability and size, the microcomputer is rapidly gaining widespread use in scientific and engineering problem solving. *(Photo courtesy of Digital Equipment Corporation.)*

Introduction

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We live in an era of unprecedented scientific and engineering progress. In observing the technological advancements of the past few decades and how they have influenced the way we presently live and work, two exciting trends become apparent. The first of these is that significant achievements have tended to occur with increasing frequency as time progresses. The second trend is that the rate at which we accept these achievements and integrate them into our lives has also increased. As an illustration of these two trends, consider the fact that the invention of the telephone by Alexander Graham Bell in 1876 was slow to be perfected and to be accepted commercially. It was not until 1954 that a majority of households in this country had equipment for long-distance dialing. Yet the equally significant development of the first modern home television set in 1939 took less than 10 years to gain widespread acceptance. In 1974, the U.S. Census Bureau reported that 97 percent of U.S. households contained at least one television set and 45 percent had two or more sets. The rationale for this contrast seems quite clear. Early technological advancements in communications and product-delivery systems actually created a situation in which the acceptance of the later invention was accelerated. Thus, expanded technologies have a synergistic effect in fostering further innovation. The phenomenon is even more pronounced in our present era. Perhaps the best example of this is the microcomputer revolution. The first microprocessor chip was developed in 1971 and the first microcomputer appeared in 1975. Less than a decade later there are nearly a half-million microcomputers in use for tasks ranging from entertainment to business and scientific applications. Recent industrial surveys predict that nearly every home in this nation will have a microcomputer before the end of the decade. The microcomputer may well be as influential to our future life-style as was the development of the telephone or the television in the past. Contemporary writers have predicted a significant role for the microcomputer in the next social revolution following the industrial revolution.

Application areas for the microcomputer continue to emerge as the computational capability and hardware versatility of this device expand. Tasks once thought suitable for only large mainframe computers are now not only physically practical but are, in fact, economically preferably for implementation by a microcomputer. One such emerging application is the field of numerical problem solving in science and engineering (Figure 1-1). It is the purpose of this text to serve as a numerical methods resource to those who wish to apply the computational power of the microcomputer to scientific and engineering problem solving. This text will focus on three important goals:

1. To identify those characteristics of present and future microcomputer systems that suggest when to use and when not to use these important computational devices.

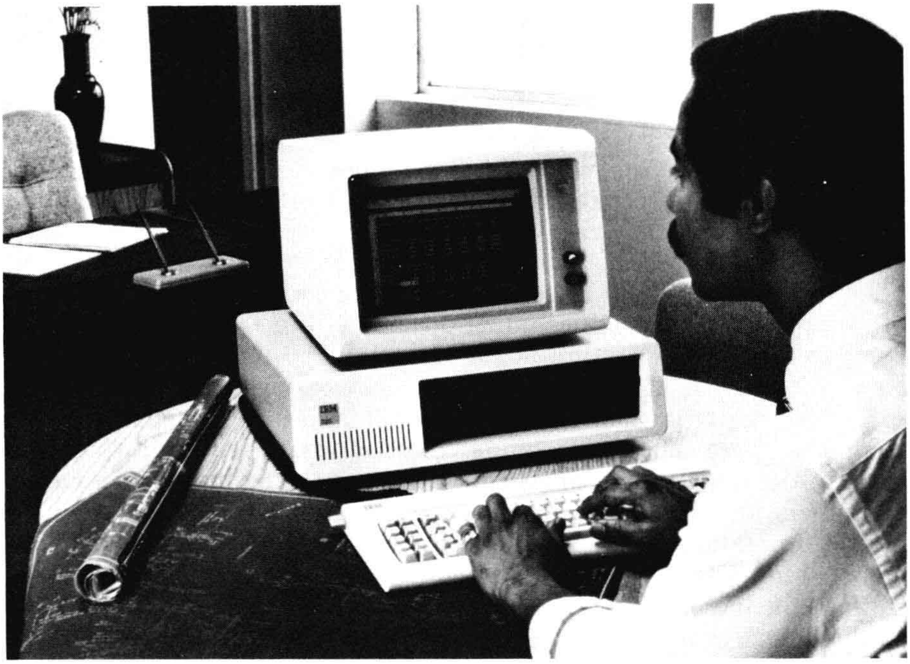

Figure 1-1 The microcomputer is rapidly becoming a significant tool for numerical problem solving in science and engineering. *(Photo courtesy of International Business Machines Corporation.)*

2. To identify those numerical tasks that are frequently encountered in engineering and scientific problem solving.
3. To present practical computational algorithms that represent an efficient merger of numerical method need with microcomputer capability.

1.1 THE DIGITAL COMPUTER

The first electronic digital computer was the ENIAC (from Electronic Numerical Integrator and Computer) built at the University of Pennsylvania between 1943 and 1949. Early computers such as this were large enough physically to fill a room the size of a small house. Yet, with all their size, their computational capability was rather primitive by present standards. The ENIAC computer contained over 18,000 vacuum tubes, and the overall reliability of this device was rather low owing to failures in these electronic tubes. Finding and replacing malfunctioning tubes took countless hours. The tubes also generated considerable heat and consumed large amounts of electric power. Even with its disadvantages, however, the ENIAC performed well enough to demonstrate the utility of the digital computer and to encourage later development of improved devices.

The transistor was invented in 1948 and by 1959 began to be used in digital computers to achieve a significant reduction in size, energy consumption, and costs, with a corresponding increase in both reliability and computational capability. The next quantum jump in size and capability came in the early 1960s when several transistor companies developed ways to place complete electronic circuits on the surface of silicon. These integrated circuits formed the basis for a new generation of computers with qualities much different from the early, large-scale computers used for multipurpose tasks. For the first time, computers could be made small enough and inexpensive enough to be dedicated to specific computational or data-management tasks. These small dedicated computers were called minicomputers because of their size, which was roughly the same as that of a small file cabinet. The first of these minicomputers was the PDP-8 manufactured in 1965 by the Digital Equipment Corporation. During this same time period, the integrated circuit made an important impact on another field of computational electronic equipment, the electronic calculator. In the late 1960s the capabilities of these devices increased dramatically, while the purchase price fell by more than an order of magnitude. The credit for this economic paradox is due mainly to improved manufacturing technology. It is not surprising then that the next breakthrough in computational equipment came on the interface between the calculator and the digital computer. Like so many revolutionary inventions, this innovation started with an attempt to solve a problem in one area and led to a breakthrough in another.

In 1971 the Intel Corporation was trying to design a single integrated circuit that would be a complete calculator on a single integrated-circuit chip. What resulted was a far more versatile device that we now call a microprocessor or a "computer on a chip." The technology that supported this important breakthrough is now called *large-scale integration* and allows the placement of thousands of transistors on a single silicon chip. The microprocessor is a complete computer central processing unit on a silicon chip smaller than a square centimeter (Figure 1-2). The microprocessor is capable of being used for a variety of dedicated applications including timing and controlling industrial processes, controlling traffic lights, guiding and controlling vehicles, and a host of other useful applications. One of the more interesting applications is possible when some memory is added to a microprocessor along with devices for input and output. Such a combination is called a *microcomputer.* It is interesting to note that, even though the size and cost of a microcomputer are small, it is easily capable of outperforming

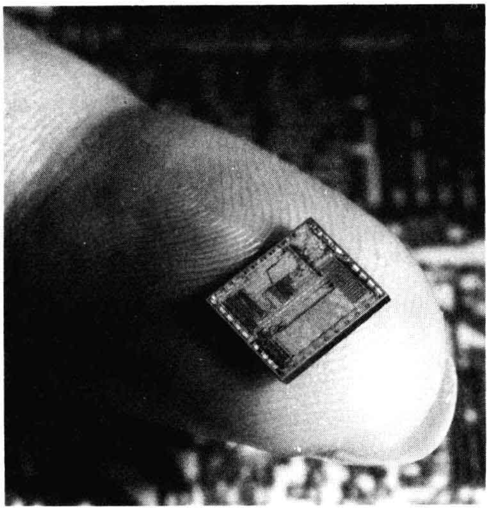

Figure 1-2 The microprocessor "computer on a chip." (*Photo courtesy of Intel Corporation.*)

the large, expensive computers of the vacuum-tube and transistor eras. The classificational boundaries of definition for a large computer, a minicomputer, and a microcomputer are not clearly defined, owing to the continuous technological change that is contributing to their evolution. For convenience, the various types of computers presently in use may be classified in terms of size, speed, cost, and overall utility. Table 1-1 provides a comparison of characteristics for typical equipment in this rapidly evolving field. The implications of these characteristics to scientific and engineering problem solving will be discussed in the following paragraphs.

Size

At one time the physical size of a computer could be used as a measure of its overall computational capability; however, because of rapid changes in computer technology, this is no longer true. The same advancements make it more difficult to classify a computer as either large, medium, or small. Some minicomputers may be larger than some of the smaller of the large mainframe computers. In like manner, a microcomputer together with all its peripheral equipment may appear to be physically larger than some minicomputers. The significance of word size is that it indicates how many significant digits are possible in a given calculation. A typical 8-bit microcomputer is capable of calculations providing nine-significant-digit accuracy. In general, the more binary digits that are used in a computer word, the higher will be the number of significant digits of accuracy. Double precision capability

Table 1-1 Comparison of Characteristics for Various Types of Computers

Characteristic	*Medium/Large Computer*	*Minicomputer*	*Microcomputer*
Size			
Physical size	Room sized	Desk sized	Typewriter sized
Word size	32–64 binary digits	16–32 binary digits	8–16 binary digits
Max. memory size	12,000 K bytes	4000 K bytes	128 K bytes
Speed			
CPU cycle time	< 70 nanoseconds	~100 nanoseconds	> 200 nanoseconds
Memory cycle time	< 250 nanoseconds	~300 nanoseconds	> 400 nanoseconds
Cost	$2 to $4 million	$20 to $40,000	$2 to $4000
Utility			
Operating system	Multiprogramming (large numbers)	Multiprogramming (a few)	Single task
Languages supported	Most commonly used languages	Some	A few
Operating requirements	Special space and trained operator	Limited need for an operator	User operated
Available applications packages	Excellent	Good	Limited at present

frequently used by larger computers to expand accuracy at the expense of storage space is seldom available in microcomputers. Comparisons based on the size of working memory can be misleading. This number should be viewed in terms of word size and in terms of how much of the maximum memory size is actually available to a single user at a given time. As peripheral storage equipment becomes faster in access time, the limitations imposed by an active memory size of 64 K bytes are less severe. In general, it is rare for an engineering or scientific calculation to use all the memory capacity in a large computer. Such applications require the use of the large mainframe computer and are the type of task for which it is most efficiently suited. The microcomputer is best suited for computational applications requiring modest memory storage capacity and modest accuracy.

Speed

In spite of its small size, the microcomputer is surprisingly fast in operation. Based on CPU cycle time, the microcomputer operates at a rate of about one-tenth of the speed of its larger counterpart. Since many engineering and scientific problems performed on a large mainframe computer take less than 6 seconds of CPU time, it would seem reasonable to predict that programs of this same level of complexity would execute in less than 1 minute on a microcomputer. This rule of thumb should, of course, be applied with care since most engineering and scientific programs require more time for I/O operations than for actual CPU time. For these situations the processing speed is limited more by the speed of the peripheral units than by CPU speed. Since most microcomputers are connected to CRT screens or to slow-speed printers, it would seem prudent to restrict their use to problems requiring moderate amounts of overall processing time and moderate amounts of output.

Cost

The cost justification is perhaps one of the strongest arguments for using a microcomputer to do engineering and scientific problem solving. The normalized purchase price based on memory size for a microcomputer is about 5¢ per byte and for a large mainframe computer about 40¢ per byte. This eightfold advantage is possible because the microcomputer is designed to perform only one task at a time. A similar argument based on normalized purchase price for processing speed is even more dramatic than that for storage capacity. The CPU processing time for a large computer is about an order of magnitude faster than that for a microcomputer, whereas the overall purchase price is three orders of magnitude greater. In addition, since the microcomputer does not require specially trained operators nor specially prepared operating environments, its overall operation cost may be far more favorable than that predicted from a comparison of purchase price. Because it is based on relatively new technologies, it is likely that the purchase price of microcomputer systems will continue to decrease in the future. This fact adds to the already strong economic reasons for using the microcomputer for computational tasks within its utility range.

Utility

A major difference between a large and a small computer is in the number of users that can simultaneously interact with the machine. Under the multiprogramming environment of a large computer, several hundred users can simultaneously use this computational resource. On the other hand, for the microcomputer, operating systems are designed so that the machine functions entirely for a single user. For this reason, microcomputers are often referred to as "personal" computers. In the area of computer languages supported, most large-scale computers support a wide variety of commonly used high-level languages. Microcomputers, on the other hand, frequently support only one higher-level language. In most cases this is the BASIC language. Although