Renate A. Schmidt (Ed.)

# Relations and Kleene Algebra in Computer Science

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9th International Conference on Relational Methods in Computer Science and 4th International Workshop on Applications of Kleene Algebra, RelMiCS/AKA 2006 Manchester, UK, August/September 2006, Proceedings

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Manchester, UK, August/September, 2006
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# Preface

This volume contains the joint proceedings of the 9th International Conference on Relational Methods in Computer Science (RelMiCS-9) and the 4th International Workshop on Applications of Kleene Algebra (AKA 2006). The joint event was hosted by the School of Computer Science at the University of Manchester, UK, from August 29 to September 2, 2006. RelMiCS/AKA is the main forum for the relational calculus as a conceptual and methodological tool and for topics related to Kleene algebras. Within this general theme, the conference series is devoted to the theory of relation algebras, Kleene algebras and related formalisms as well as to their diverse applications in software engineering, databases and artificial intelligence. This year, special focus was on formal methods, logics of programs and links with neighboring disciplines. This diversity is reflected by the contributions to this volume.

The Programme Committee selected 25 technical contributions out of 44 initial submissions from 14 countries. Each paper was refereed by at least three reviewers on its originality, technical soundness, quality of presentation and relevance to the conference. The programme included three invited lectures by distinguished experts in the area: "Weak Kleene Algebra and Computation Trees" by Ernie Cohen (Microsoft, USA), "Finite Symmetric Integral Relation Algebras with no 3-Cycles" by Roger Maddux (Iowa State University, USA), and "Computations and Relational Bundles" by Jeff Sanders (Oxford, UK). In addition, for the first time, a PhD programme was co-organized by Georg Struth. It included the invited tutorials "Foundations of Relation Algebra and Kleene Algebra" by Peter Jipsen (Chapman University, USA), and "Relational Methods for Program Refinement" by John Derrick (Sheffield University, UK).

As in previous years, the RelMiCS Conference and the AKA Workshop were co-organized because of their considerable overlap. Previous RelMiCS meetings were held in 1994 at Dagstuhl, Germany, in 1995 at Parati, Brazil, in 1997 at Hammamet, Tunisia, in 1998 at Warsaw, Poland, in 1999 at Québec, Canada, in 2001 at Oisterwijk, The Netherlands, in 2003 at Malente, Germany and in 2005 at St.Catharines, Canada. The AKA Workshop has been held jointly with RelMiCS since 2003, after an initial Dagstuhl Seminar in 2001.

I would like to thank the many people without whom the meeting would not have been possible. First, I would like to thank all authors who submitted papers, all participants of the conference as well as the invited keynote speakers and the invited tutorial speakers for their contributions. I am very grateful to the members of the Programme Committee and the external referees for carefully reviewing and selecting the papers. I thank my colleagues on the Steering Committee for their advice and the support for the changes introduced for this year's event. Special thanks go to the members of the local organization team in the School of Computer Science at the University of Manchester for all their

help: the staff in the ACSO office, especially Bryony Quick and Iain Hart, the staff of the finance office, and the technical staff, as well as Zhen Li and David Robinson. Moreover, I am extremely grateful to Georg Struth for his tremendous amount of effort—as Programme Chair he helped with every aspect of the planning and organization of RelMiCS/AKA 2006 and the PhD Programme. Finally, it is my pleasure to acknowledge the generous support by: the UK Engineering and Physical Sciences Research Council (grant EP/D079926/1), the London Mathematical Society, the British Logic Colloquium, the University of Manchester (President's Fund), and the School of Computer Science, University of Manchester.

Manchester, June 2006

Renate Schmidt
General Chair
RelMiCS/AKA 2006

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# Weak Kleene Algebra and Computation Trees

Ernie Cohen

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**Abstract.** The Kleene algebra axioms are too strong for some program models of interest (e.g. models that mix demonic choice with angelic or probabilistic choice). This has led to proposals that weaken the right distributivity axiom to monotonicity, and possibly weaken or eliminate the right induction and left annihilation axioms (e.g. lazy Kleene algebra, probabilistic Kleene algebra, monodic tree Kleene algebra, etc.). We'll address some of the basic metatheoretic properties of these theories using rational trees modulo simulation equivalence.

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# Finite Symmetric Integral Relation Algebras with No 3-Cycles

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**Abstract.** The class of finite symmetric integral relation algebras with no 3-cycles is a particularly interesting and easily analyzable class of finite relation algebras. For example, it contains algebras that are not representable, algebras that are representable only on finite sets, algebras that are representable only on infinite sets, algebras that are representable on both finite and infinite sets, and there is an algorithm for determining which case holds.

Some questions raised in a preprint by Jipsen [1] are addressed in this paper. Most of the results in this paper date from 1983, but some were found and published independently by Comer [2], Jipsen [1], and Tuza [3]. The first four sections contain background material.

## 1 Relation Algebras and Their Relatives

A **relation algebra** is an algebraic structure

$$\mathfrak{A} = \langle A, +, {}^{-}, ;, {}^{\smile}, 1' \rangle, \tag{1}$$

where $A$ is a nonempty set, $+$ and $;$ are binary operations on $A$, ${}^{-}$ and ${}^{\smile}$ are unary operations on $A$, and $1' \in A$ is a distinguished element, which satisfies these equational axioms:

| | | |
|---|---|---|
| $R_1$ | $x + y = y + x,$ | $+$-commutativity |
| $R_2$ | $x + (y + z) = (x + y) + z,$ | $+$-associativity |
| $R_3$ | $\overline{\overline{x} + \overline{y}} + \overline{\overline{x} + y} = x,$ | Huntington's axiom |
| $R_4$ | $x;(y;z) = (x;y);z,$ | $;$-associativity |
| $R_5$ | $(x + y);z = x;z + y;z,$ | right $;$-distributivity |
| $R_6$ | $x;1' = x,$ | right identity law |

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| | | |
|---|---|---|
| $R_7$ | $\breve{\breve{x}} = x,$ | $\breve{}$-involution |
| $R_8$ | $(x+y)\breve{} = \breve{x} + \breve{y},$ | $\breve{}$-distributivity |
| $R_9$ | $(x;y)\breve{} = \breve{y};\breve{x},$ | $\breve{}$-involutive distributivity |
| $R_{10}$ | $\breve{x};\overline{x;y} + \overline{y} = \overline{y}.$ | Tarski/De Morgan axiom |

RA is the class of relation algebras, and NA is the class of **nonassociative relation algebras**, algebras of the form (1) which satisfy all the RA axioms except ;-associativity. For every $\mathfrak{A} \in$ NA, $\langle A, +, \bar{} \rangle$ is a Boolean algebra by axioms $R_1$–$R_3$ (this fact is due to E. V. Huntington [4,5,6]). Because of this, $\langle A, +, \bar{} \rangle$ is called the **Boolean part** of $\mathfrak{A}$, and standard concepts from the theory of Boolean algebras may be applied to nonassociative relation algebras by referring to the Boolean part. BA is the class of Boolean algebras. For any algebra $\mathfrak{A} \in$ NA, the **identity element** of $\mathfrak{A}$ is 1', and other operations and elements, which correspond to intersection, difference, empty relation, diversity relation, and universal relation, are defined by

$$x \cdot y := \overline{\overline{x} + \overline{y}}, \tag{2}$$

$$x - y := \overline{\overline{x} + y}, \tag{3}$$

$$0 := \overline{1' + \overline{1'}}, \tag{4}$$

$$0' := \overline{1'}, \tag{5}$$

$$1 := 1' + \overline{1'}. \tag{6}$$

The **zero element** is 0, the **diversity element** is 0', and the **unit element** is 1. Every algebra $\mathfrak{A} \in$ NA satisfies the **cycle law**:

$$\begin{aligned} &\breve{x};z \cdot y = 0 \quad \text{iff} \quad x;y \cdot z = 0 \quad \text{iff} \quad z;\breve{y} \cdot x = 0 \\ \text{iff} \quad &y;\breve{z} \cdot \breve{x} = 0 \quad \text{iff} \quad \breve{y};\breve{x} \cdot \breve{z} = 0 \quad \text{iff} \quad \breve{z};x \cdot \breve{y} = 0 \end{aligned} \tag{7}$$

and many other laws, a few of which we gather here:

$$1'\breve{} = 1', \quad 0'\breve{} = 0', \tag{8}$$

$$1';x = x, \tag{9}$$

$$0;x = x;0 = 0, \tag{10}$$

$$1;1 = 1. \tag{11}$$

An algebra $\mathfrak{A} \in$ NA is **symmetric** if it satisfies the equation

$$\breve{x} = x. \tag{12}$$

If $\mathfrak{A} \in$ NA is symmetric then $\mathfrak{A}$ is also **commutative**, *i.e.*, it satisfies the equation

$$x;y = y;x. \tag{13}$$

WA is the class of **weakly associative relation algebras**, those algebras in NA which satisfy the **weak associative law**

$$((x \cdot 1');1);1 = (x \cdot 1');(1;1). \tag{14}$$

Finally, SA is the class of **semiassociative relation algebras**, those algebras in NA which satisfy the **semiassociative law**

$$(x;1);1 = x;(1;1). \tag{15}$$

Clearly $\mathsf{NA} \subseteq \mathsf{WA} \subseteq \mathsf{SA} \subseteq \mathsf{RA}$. Since every NA satisfies $1;1 = 1$, the weak associative law and semiassociative law can be simplified by replacing $1;1$ with 1.

The set of atoms of an algebra $\mathfrak{A} \in \mathsf{NA}$ is $At\mathfrak{A}$. An atom $x \in At\mathfrak{A}$ is an **identity atom** if $x \leq 1'$ and a **diversity atom** if $x \leq 0'$. An algebra $\mathfrak{A} \in \mathsf{NA}$ is **integral** if $0 \neq 1$ ($\mathfrak{A}$ is nontrivial) and $x;y = 0$ implies $x = 0$ or $y = 0$. If $\mathfrak{A} \in \mathsf{NA}$ and $1' \in At\mathfrak{A}$ then $\mathfrak{A}$ is integral. The converse holds if $\mathfrak{A} \in \mathsf{SA}$. However, it fails for some $\mathfrak{A} \in \mathsf{WA}$. Around 1940 J. C. C. McKinsey invented a nontrivial WA with zero-divisors in which $1'$ is an atom. This algebra shows that $\mathsf{WA} \supset \mathsf{SA}$. In fact, all the inclusions are proper (see [7, Cor. 2.6, Th. 3.7] or [8, Th. 450]), so we have

$$\mathsf{NA} \supset \mathsf{WA} \supset \mathsf{SA} \supset \mathsf{RA}.$$

## 2 Representable Relation Algebras

For every set $U$, let $Sb(U)$ be the set of subsets of $U$, and let

$$\mathfrak{Bl}(U) := \langle Sb(U), \cup, \bar{} \rangle,$$

where $\cup$ is the binary operation on $Sb(U)$ of forming the union of any two subsets of $U$, and $\bar{}$ is the unary operation of complementation with respect to $U$ (so $\overline{X} = U \sim X$, where $\sim$ is the operation of forming the set-theoretic difference of two sets). $\mathfrak{Bl}(U)$ is the **Boolean algebra of subsets of** $U$. For every equivalence relation $E$, let

$$\mathfrak{Sb}(E) := \langle Sb(E), \cup, \bar{}, |, {}^{-1}, \mathsf{Id} \cap E \rangle, \tag{16}$$

where $|$ is relative multiplication of binary relations, defined for binary relations $R$ and $S$ by

$$R|S := \{\langle a, c\rangle : \exists_b(\langle a, b\rangle \in R,\ \langle b, c\rangle \in S)\}, \tag{17}$$

${}^{-1}$ is conversion of binary relations, defined for any binary relation $R$ by

$$R^{-1} := \{\langle b, a\rangle : \langle a, b\rangle \in R\}, \tag{18}$$

and $\mathsf{Id} \cap E$ is the identity relation on the field $Fd(E) = \{x : \exists_y(xEy)\}$ of $E$ ($\mathsf{Id}$ is the class of pairs of sets of the form $\langle x, x\rangle$). $\mathfrak{Sb}(E)$ is the **relation algebra of subrelations of** $E$, and we refer to $\mathfrak{Sb}(E)$ as an **equivalence relation algebra**. It is necessary to assume that $E$ is an equivalence relation in order to ensure that $Sb(E)$ is closed under relative multiplication and conversion. $Sb(U^2)$, the powerset of $U^2$, is the **set of binary relations on** $U$. For every set $U$, $\mathfrak{Re}(U)$ is the **square relation algebra on** $U$, defined by

$$\mathfrak{Re}(U) := \mathfrak{Sb}(U^2). \tag{19}$$

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