THE DYNAMICS AND THERMODYNAMICS OF
COMPRESSIBLE FLUID FLOW

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and Thermodynamics of
COMPRESSIBLE FLUID FLOW

By

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IN TWO VOLUMES

VOLUME I

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PREFACE

During the past two decades a rapid growth of interest in the motion of compressible fluids has accompanied developments in high-speed flight, jet engines, rockets, ballistics, combustion, gas turbines, ram jets and other novel propulsive mechanisms, heat transfer at high speeds, and blast-wave phenomena. My purpose in writing this book is to make available to students, engineers, and applied physicists a work on compressible fluid motion which would be suitable as an introductory text in the subject as well as a reference work for some of its more advanced phases. The choice of subject matter has not been dictated by any particular field of engineering, but rather includes topics of interest to aeronautical engineers, mechanical engineers, chemical engineers, applied mechanicians, and applied physicists.

In selecting material from the vast literature of the field the basic objective has been to make the book of practical value for engineering purposes. To achieve this aim, I have followed the philosophy that the most practical approach to the subject of compressible fluid mechanics is one which combines theoretical analysis, clear physical reasoning, and empirical results, each leaning on the other for mutual support and advancement, and the whole being greater than the sum of the parts.

The analytical developments of this book comprise two types of treatments: those leading to design methods and those leading to exemplary methods. The design methods are direct and rapid, and easily applied to a variety of problems. Therefore, they are suited for use in the engineering office. The discussions of these design methods are detailed and illustrative examples are often given. The exemplary methods, on the other hand, comprise those theoretical analyses which are time consuming, which generally require mathematical invention, and which are not easily applied to a variety of problems. Such methods are primarily of value for yielding detailed answers to a small number of typical problems. Although they are not in themselves suitable for the engineering office, the examples which they permit to be worked out often provide important information about the behavior of fluids in typical situations. Thus they serve as guides to the designer in solving the many complex problems where even the so-called design methods are not sufficient. The treatment of exemplary methods in this book usually consists of a brief outline of the method, together with a presentation of those results obtained by the method which illuminate significant questions concerning fluid motion and which help to form the vital "feel" so desired by designers.

In keeping with the spirit of the several foregoing remarks, all the important results of the book have been reduced to the form of convenient charts and tables. Unless otherwise specified, the charts and tables are for a perfect gas with a ratio of specific heats (k) of 1.4.

In those parts of the book dealing with fundamentals, emphasis is placed on the introduction of new concepts in an unambiguous manner, on securing a clear physical understanding before the undertaking of an analysis, on the rigorous application of physical laws, and on showing fruitful avenues of approach in analytical thinking. The remaining part of the work proceeds at a more rapid pace befitting the technical maturity of advanced students and professionals.

The work is organized in eight parts. Part I sets forth the basic concepts and principles of fluid dynamics and thermodynamics from which the remainder of the book proceeds and also introduces some fundamental concepts peculiar to compressible flows. In Part II is a discussion of problems accessible by the most simple picture of fluid motion —the one-dimensional analysis. Part III constitutes a summary of the basic ideas and concepts necessary for the succeeding chapters on two- and three-dimensional flow. Parts IV, V, and VI then present in order comprehensive surveys of subsonic flows, of supersonic flows (including hypersonic flow), and of mixed subsonic-supersonic flows. In Part VII is an exposition of unsteady one-dimensional flows. Part VIII is an examination of the viscous and heat conduction effects in laminar and turbulent boundary layers, and of the interaction between shock waves and boundary layers. For those readers not already familiar with it, the mathematical theory of characteristic curves is briefly developed in Appendix A. Appendix B is a collection of tables which facilitate the numerical solution of problems.

The "References and Selected Bibliography" at the end of each chapter will, it is hoped, be a helpful guide for further study of the voluminous subject. Apart from specific references cited in each chapter, the lists include general references appropriate to the subject matter of each chapter. The choice of references has been based primarily on clarity, on completeness, and on the desirability of an English text, rather than on historical priority.

My first acknowledgment is to Professor Joseph H. Keenan, to whom I owe my first interest in the subject, and who, as teacher, friend, and colleague, has been a source of inspiration and encouragement.

In an intangible yet real way I am indebted to my students, who have made teaching a satisfying experience, and to my friends and colleagues

at the Massachusetts Institute of Technology who contributed the climate of constructive criticism so conducive to creative effort.

Many individuals and organizations have been cooperative in supplying me with helpful material and I hope that I have not failed to acknowledge any of these at the appropriate place in the text. The National Advisory Committee for Aeronautics and the M.I.T. Gas Turbine Laboratory have been especially helpful along these lines.

I was fortunate in being able to place responsibility for the important work of the drawings in the competent hands of Mr. Percy H. Lund, who, with Miss Prudence Santoro, has been most cooperative in this regard.

For help with the final revision and checking of the manuscript I wish to give thanks to Dr. Bruce D. Gavril and Dr. Ralph A. Burton.

Finally, but by no means least, I must express a word of appreciation to Sylvia, and to young Peter, Mardi, and Bunny, who, one and all, made it possible for me to escape from the office into the somewhat less trying atmosphere of the home, and there to carry this work forward to its completion.

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PART I

BACKGROUND

Chapter 1

FOUNDATIONS OF FLUID DYNAMICS

1.1. Introductory Remarks

An engineering science like fluid dynamics rests on foundations comprising both theory and experiment. With fluid dynamics, progress has been especially dependent upon an intimate cross-fertilization between the analytical and empirical branches; the experimental results being most fruitfully interpreted in terms of theoretical reasoning, and the analyses in turn suggesting critical and illuminating experiments which further amplify and strengthen the theory.

The analytical branch of a science is constructed from concepts, definitions, and the statements of physical laws. The latter are in terms of the concepts and definitions and are in conformity with experimental observations. All analyses concerning the motion of compressible fluids must necessarily begin, either directly or indirectly, with the statements of the four basic physical laws governing such motions. These laws, which are independent of the nature of the particular fluid, are

(i) the law of conservation of mass
(ii) Newton's second law of motion
(iii) the first law of thermodynamics
(iv) the second law of thermodynamics

In addition to these fundamental principles, it is usually necessary to bring into an analysis certain subsidiary laws relating to the particular fluid or class of fluids in question. Examples are the equation of state of a perfect gas, the proportionality law between shear stress and rate of shear deformation in a Newtonian fluid, the Fourier law of heat conduction, etc.

In this book emphasis is placed on the manner in which important conclusions spring from analyses growing out of the four basic laws mentioned above. For this reason the first two chapters are devoted to a review of these principles and the associated concepts and definitions. In this way the general point of view and phraseology of the book will be established.

Definition of a Fluid. The rigorous classification of substances in various ways is usually thwarted because certain substances behave so anomalously as to defy being neatly placed in a pigeonhole. For our present purpose, we wish principally to distinguish between the numerous common substances we call fluids and such other substances as solids and plastics.

We shall define a *fluid* as a substance which *deforms continuously under the action of shearing forces.* When shearing forces are applied to a solid, the latter undergoes a certain deformation which does not change so long as the applied forces are maintained constant. A fluid, however, whether viscous or nonviscous, exhibits relative motion between its elementary parts so long as shearing forces are applied. Thus we say that "a fluid cannot withstand shearing stresses." 流体与固体的区别

An important corollary which follows from the definition of a fluid is the observation that if there is no relative motion within the fluid, i.e., if fluid particles are not deformed, then there can be no shear stresses acting on such particles.

LIQUIDS VS. GASES. The usual methods of attempting to distinguish rigorously between a liquid and a gas, both of which are fluids, are futile and indeed not of any practical use. That this is so may be seen by considering that a mass of "water" at 1 atm and 100°C in a glass cylinder closed by a piston may, by suitable heat transfers and motions of the piston, be transformed to a mass of "steam" at 1 atm and 100°C, without a meniscus once being observed! For most practical purposes the words "liquid" and "gas" are of value insofar as the former denotes a fluid which generally exhibits only small percentage changes in density.

The subject matter of this book relates for the most part to highly compressible fluids, and so we shall generally speak of gases.

The Concept of a Continuum. Matter, while seemingly continuous, is composed of myriads of molecules in constant motion and collision. The most fundamental approach in analyzing the motion of matter in the aggregate is, therefore, to set down the laws of motion for each individual molecule and to trace the history of each molecule, or of statistical groups of molecules, subsequent to some initial state of affairs. This approach, which usually goes under the name of kinetic theory or statistical mechanics, has obvious merits, but, on the other hand, is often too cumbersome for practical calculations.

In most engineering problems our primary interest lies not in the motions of molecules, but rather in the gross behavior of the fluid thought of as a continuous material. Although the postulate of a continuous fluid is nothing but a convenient fiction, it is fortunately a valid approach to many practical problems where only macroscopic or phenomenological information is of interest. The treatment of fluids as continua may be said to be valid whenever the smallest volume of fluid of interest contains so many molecules as to make statistical averages meaningful.

The great simplification afforded by the concept of a continuum is that instead of dealing with instantaneous states of innumerable molecules, we deal instead with certain macroscopic properties describing the gross behavior of the substance. In the motion of compressible fluids the relevant properties are density, pressure, shear stress, velocity, coefficient of viscosity, temperature, internal energy, entropy, and coefficient of thermal conductivity. These are defined in Arts. 1.3 and 2.4.

This book concerns the motion of compressible fluids which may be treated as continua. To avoid the impression that the methods and results of this book are universally valid, it seems well at this point to mention that the macroscopic approach fails whenever the mean free path of the molecules is of comparable size with the smallest significant dimension of the problem. Thus, whenever we deal with highly rarefied gases (as in rocket flight at extreme altitudes, high vacuum technology, or electronic tubes), the continuum approach of classical fluid mechanics and thermodynamics must be abandoned in favor of the microscopic approach of kinetic theory.

NOMENCLATURE

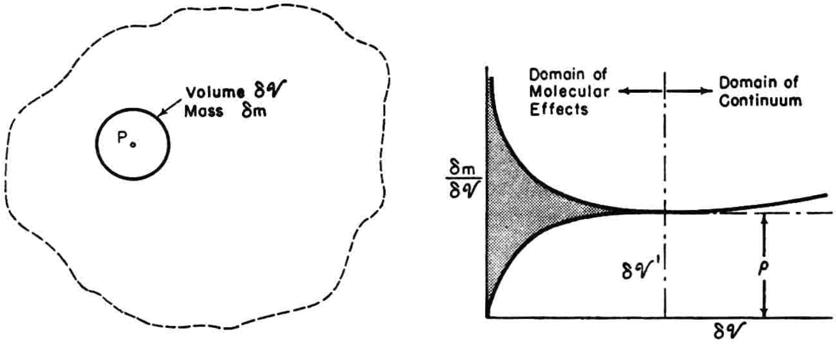
a	acceleration
A	area
$\mathbf{A}$	area vector
F	force
$\mathbf{F}$	force vector
g	magnitude of body force per unit mass
g_0	constant of proportionality in Newton's second law
m	mass
$\mathbf{M}$	moment of a force, or torque
p	normal force per unit area, or pressure
r	magnitude of radius vector
$\mathbf{r}$	radius vector
t	time
u	component of velocity in x-direction
v	component of velocity in y-direction
$\mathcal{V}$	volume
V	speed
$\mathbf{V}$	velocity
w	mass rate of flow
x, y, z	Cartesian coordinates
γ	angle
μ	coefficient of viscosity
ρ	mass density at a point
τ	tangential force per unit area, or shear stress

1.2. Properties of the Continuum

We discuss here those continuum properties relevant to the laws of motion.

Density at a Point. Consider the mass of fluid δm in a volume $\delta\mathcal{V}$ surrounding the point P in a continuous fluid (Fig. 1.1a). The ratio $\delta m/\delta\mathcal{V}$ is called the average mass density of the fluid within the volume

$\delta\mathcal{V}$. Now suppose that at first $\delta\mathcal{V}$ is rather large, and that it is subsequently shrunk about the point P. Then a plot of $\delta m/\delta\mathcal{V}$ versus $\delta\mathcal{V}$ would be typified by Fig. 1.1b. At first the average density tends to approach an asymptote as the volume encloses fluid more and more homogeneous in nature. However, when $\delta\mathcal{V}$ becomes so small as to contain relatively few molecules, the average density fluctuates substantially with time as molecules pass into and out of the volume, and so it is impossible to speak of a definite value for $\delta m/\delta\mathcal{V}$. We may then imagine the smallest volume which can be regarded as continuous to be $\delta\mathcal{V}'$, and then define the *density at a point* as



(a) (b)

FIG. 1.1. Definition of density at a point.

$$\rho \equiv \lim_{\delta\mathcal{V}\to\delta\mathcal{V}'} \frac{\delta m}{\delta\mathcal{V}} \tag{1.1}$$

This definition illuminates the idea of a continuum and shows the true nature of a continuum property "at a point" as a fictitious but highly useful concept.

Fluid Velocity at a Point. The fluid velocity at a point is quite independent of the instantaneous velocity of the molecule nearest that point. Rather we consider the motion of the center of gravity of the volume $\delta\mathcal{V}'$ (Fig. 1.1b) instantaneously surrounding that point, and define the *fluid velocity at the point P* as the instantaneous velocity of this center of gravity. Thus the fluid velocity at a point is the instantaneous velocity of the fluid particle which at that moment is passing through the point. By *fluid particle* we mean here a small mass of fluid of fixed identity and of size comparable with $\delta\mathcal{V}'$.

Whereas density at a point is a scalar quantity, fluid velocity at a point is a vector. After the introduction of a coordinate system, it is therefore possible to resolve the vector velocity into three scalar components.