ELECTRONIC CIRCUIT THEORY

Devices, Models, and Circuits

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FOREWORD

This book is one of several resulting from a recent revision of the Electrical Engineering Course at The Massachusetts Institute of Technology. The books have the general format of texts and are being used as such. However, they might well be described as reports on a research program aimed at the evolution of an undergraduate core curriculum in Electrical Engineering that will form a basis for a continuing career in a field that is ever-changing.

The development of an educational program in Electrical Engineering to keep pace with the changes in technology is not a new endeavor at The Massachusetts Institute of Technology. In the early 1930's, the Faculty of the Department undertook a major review and reassessment of its program. By 1940, a series of new courses had been evolved, and resulted in the publication of four related books.

The new technology that appeared during World War II brought great change to the field of Electrical Engineering. In recognition of this fact, the Faculty of the Department undertook another reassessment of its program. By about 1952, a pattern for a curriculum had been evolved and its implementation was initiated with a high degree of enthusiasm and vigor.

The new curriculum subordinates option structures built around areas of industrial practice in favor of a common core that provides a broad base for the engineering applications of the sciences. This core structure includes a newly developed laboratory program which stresses the role of experimentation and its relation to theoretical model-making in the solution of engineering problems. Faced with the time limitation of a four-year program for the Bachelor's degree, the entire core curriculum gives priority to basic principles and methods of analysis rather than to the presentation of current technology.

J. A. Stratton

PREFACE

The importance of electronic devices, circuits, and systems in modern technology is apparent to most electrical engineering students; many have acquired practical experience with circuits before they begin formal study of the subject. Electronic circuit theory can be introduced to the student in a variety of ways. In view of the many devices available and the numerous applications of electronic circuits, it is important in any plan of presentation to seek unifying principles. Such principles permit the student to extend his knowledge in a rapidly advancing field.

We have organized our approach to electronics around circuit models and methods of circuit analysis in order to reduce the number of separate ideas and concepts. The many functions performed by electronic systems can be understood in terms of a few fundamental circuits if similarities are sought.

This book deals with electronic devices, models, basic circuits and circuit functions. Many of the interesting properties of electronic devices are a consequence of nonlinearity accompanied by regional linearity. As a result, piecewise-linear circuit models can be used to convert a nonlinear circuit problem to a number of related linear problems. Thus, the mathematics of linear circuit theory can be applied to a broad class of physical circuits and systems operating in a nonlinear manner.

The model concept emphasizes the need for making approximations as part of the process of analyzing a physical problem. The student is thus encouraged to exercise judgment in order to arrive at the simplest circuit models that will give an adequate result. Extremely simple models can be used to explain the general mode of operation of electronic circuits. Simple resistive models together with one major energy-storage element suffice to explain the behavior of most basic circuits. Refinements in the resistive model and one or two additional energy-storage elements provide adequate accuracy for nearly all design purposes. Sinusoidal or rectangular-wave circuit response illustrates basic operations such as waveform generation, wave shaping, amplification and modulation. A companion volume presents methods of linear and quasi-linear analysis pertinent to more complicated electronic circuits, signals and systems.

The emphasis on general methods illustrated by specific examples is in keeping with the present trend in engineering education. The emphasis on fundamentals is the inevitable consequence of rapid development, particularly in such fields as electronics, communications, and computation, which have literally exploded in the past decade. In today's technology, a specialized education becomes obsolete too soon after graduation. We do not mean to say that real problems and applications should be avoided. However, too much specialization, either in fact or in attitude, deprives the student of the background and the confidence that will enable him to enter new fields. Moreover, technical problems often span several disciplines so that breadth of understanding becomes more important in the long run than detailed knowledge.

Some of the material presented in this book evolved from a graduate subject on pulse circuits, and some was developed during the revision of the introductory undergraduate subject on electronic circuits (part of the core curriculum for all electrical engineering students at M.I.T.). With minor variations, portions of the material in this book have been used for five years in this core subject.

The development of this presentation of electronic-circuit theory was influenced by the early work of Godfrey T. Coate. Contributions have also been made by other colleagues on the teaching staff; in particular, many of the problems were prepared by section instructors. Ideas have come from staff members of the Research Laboratory of Electronics or have resulted from the stimulation of the research environment. The inspiring leadership of Professor Ernst A. Guillemin in circuit theory research and teaching has had both tangible and intangible effects on the project. Many worth-while suggestions have been made by our students.

During the final stages of the book, we had the invaluable aid of Professor Campbell L. Searle, whose critical technical editing contributed greatly to the improvement of the manuscript. In addition, he gave his time and effort unsparingly to galley reading in order to help us meet publication deadlines. As in any other book, a number of errors inevitably remain. The number would have been greater had it not been for the perceptive checking and page proofing done by Professor Richard D. Thornton, who also took major responsibility for organizing the index.

We are most grateful to Professor Gordon S. Brown for creating a departmental environment in which academic experiments are the rule rather than the exception. His constant encouragement has provided a real stimulus throughout the subject revision, note writing, manuscript, and production stages of this project.

Our acknowledgements would be incomplete without an expression of thanks to the secretaries who typed rough draft, notes, and manuscript. They are Bertha Hornby, Rosemarie Connell, Dorothea Scanlon, Marjorie D'Amato, and Louise Juliano.

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CHAPTER ONE

Introduction

1.1 Electronic Circuit Theory

Electronic devices such as diodes, triodes, and transistors operate as switches or valves to control or modulate the flow of electric current. Electronic switches and valves are useful because of their sensitivity and speed of operation, which exceed the sensitivity and speed of mechanical or electromechanical devices.

Electronic circuit theory is the mathematical study of circuits containing electronic devices. The ultimate purpose of electronic circuit theory is to provide a basis for the design of electronic systems. Such systems are combinations of electric and electronic devices assembled and connected to perform some desired operation on an electrical signal. These systems may also be associated with nonelectrical devices as in servomechanisms or industrial process control.

Like any other body of theory, electronic circuit theory deals with *models*. A model is a simple, idealized abstraction which approximates the behavior of a physical system. *A model is always a compromise between simplicity and reality.* The three phases of electronic circuit theory are, therefore, as follows:

1. Development of suitable *circuit models for electronic devices*. Model making is facilitated by a knowledge of the theoretical "models" for the

underlying *physical processes* which determine the externally observed electrical behavior of a given device.

2. Development of *analysis methods* applicable to electronic circuits. A natural and useful approach is the extension of elementary *electric circuit theory* so that familiar techniques can be brought to bear upon *electronic circuits.*
3. Accumulation of a store of ideas from working with the models and analyzing the circuits. A knowledge of device and circuit capabilities and limitations provides specific criteria and general technical intuition for the *design of electronic systems.*

This book emphasizes the first two phases of electronic circuit theory, namely, model making and methods of circuit analysis. Design information accumulates, as we proceed, from illustrative examples, problems, and interpretations of basic results.

1.2 Ideal Circuit Elements

In general, electronic circuits are not linear and do not obey reciprocity. This means that we cannot make satisfactory circuit models from the familiar R, L, and C building blocks of linear circuit theory. However, the difficulty can be overcome with only two additional circuit elements. These are the *ideal diode* and the *controlled source.*

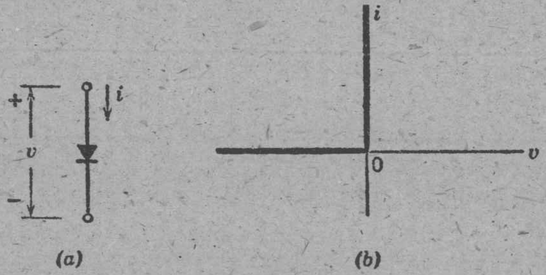


Fig. 1.1. The ideal diode. (*a*) Circuit symbol; (*b*) Current versus voltage curve.

As indicated in Fig. 1.1, the ideal diode is a *self-operated switch* that *opens* when the terminal voltage is negative and *closes* when the current is positive. The ideal diode, sometimes called an ideal rectifier, is a *nonlinear* circuit element. It is an idealized approximation of the class of electronic devices known as diodes or rectifiers.

A controlled source is shown in Fig. 1.2. The control variable and the source variable may each be either a current or a voltage, but only the current-controlled current source is shown here. The controlled source is a linear three-terminal circuit element. It is described by the linear two-terminal-pair relations between its input variables e_1 and i_1 and its output variables e_2 and i_2. The *controlled source* is a *basic unilateral circuit element.* "Unilateral" means that a signal at the input has an effect on the output but a signal applied at the output has no influence on the input. Thus in Fig. 1.2, the input current i_1 controls the position of the line relating v_2 and i_2. Varying i_1 from $+2$ to -2 sweeps the v_2 vs. i_2 line across the plane from left to right. Conversely, v_1 and i_1 are independent of v_2 and i_2. Such performance is impossible to achieve with resistance, capacitance, or inductance (including mutual inductance). Coupling between two terminal-pairs due to any of these elements is "bilateral" or "reciprocal" (two-way coupling).

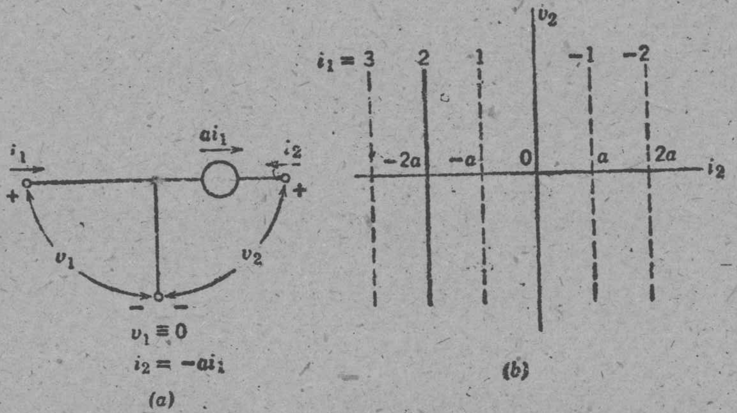
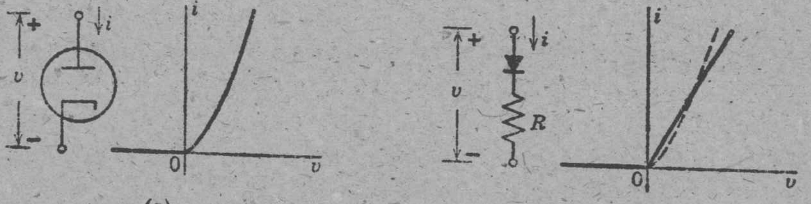


Fig. 1.2. The controlled source. (a) Circuit symbol for a current-controlled current source; (b) Output curve (v_2 vs. i_2) showing the effect of the control signal.

1.3 Circuit Models

The ideal diode allows us to make piecewise-linear models of nonlinear devices. From the current-versus-voltage curve for a typical vacuum diode [Fig. 1.3(a)], we see the general character of diode curves that suggests the ideal-rectifier approximation. By combining just a single resistance with an ideal diode, as in Fig. 1.3(b), we have a piecewise-linear model that matches the actual curve very closely.

Inclusion of a controlled (or dependent) source allows us to devise models that approximate the output curve of a control valve (such as a



(a) (b)

Fig. 1.3. Diode curve and piecewise-linear approximation.

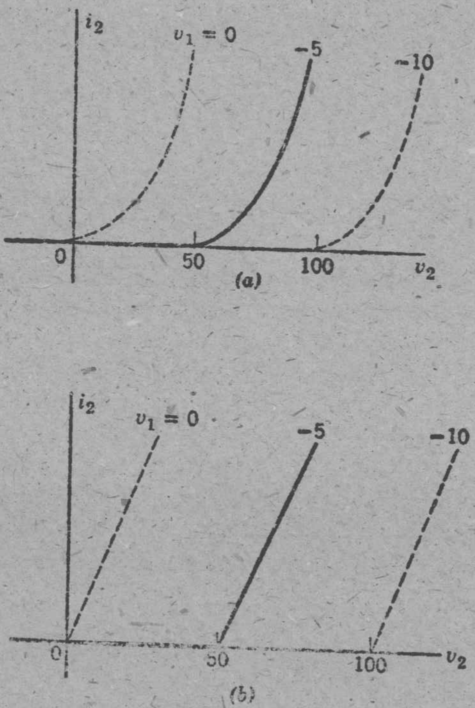


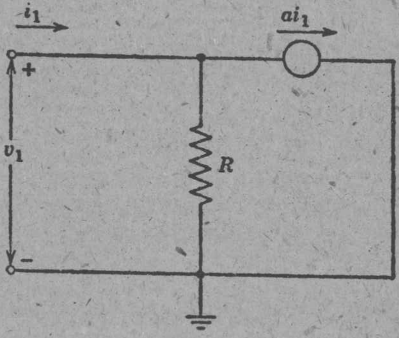
Fig. 1.4. Control valve curve and piecewise-linear approximation. (a) i_2 vs. v_2 for $v_1 = -5$; (b) Approximation.

vacuum triode or transistor). A typical triode curve is shown in Fig. 1.4(*a*) and a simple piecewise-linear approximation is indicated in (*b*).

Many nonlinear electronic devices are fairly linear in certain regions of operation, with rather abrupt transitions connecting the different regions. In other words, a piecewise-linear model is often a very realistic representation.

1.4 Analysis Methods

From the analysis standpoint, piecewise-linear models lead to linear equations with restricted ranges of validity. A piecewise-linear analysis problem consists of a number of linear problems, each one pertinent to a separate "piece" or "range" of the variables involved. The convenience of the piecewise-linear approach lies in the ease of solution of linear equations.



$$v_1 = Ri_1(1 - a)$$

$$R_{\text{input}} = \frac{v_1}{i_1} = R(1 - a)$$

Fig. 1.5. Effect of controlled source on apparent resistance.

Background gained in the study of linear *RLC* circuits is useful since it applies directly to nonlinear electronic circuit problems when the devices are represented by piecewise-linear models. The very fact that piecewise-linear approximations provide a facility for handling nonlinear problems provides a strong stimulus for further study of linear circuit theory. The more we know about linear circuit theory, the better our preparation for handling electronic circuit problems.

The controlled source, though linear, introduces effects not treated in elementary circuit theory. Figure 1.5 offers a specific example. The controlled source *dictates* the current through R, thereby establishing v_1 and determining the apparent input resistance. If we apply Thevenin's theorem and calculate the effective resistance between terminals by "short-circuiting the internal voltage sources and open-circuiting the current sources," we obtain the value R, which is incorrect. The error arises from an incorrect use of Thevenin's theorem. Only *independent* sources may be open-circuited or short-circuited to find effective resistance, since only independent sources contribute to the constant A (open-circuit voltage) in the general linear terminal relation

$$v = A + Bi \tag{1.1}$$

Controlled sources are *dependent* on the control signal. Since they change when the control signal changes, they contribute to the constant B (effective resistance). Thus, controlled sources do not violate the laws of elementary linear circuit theory but they do require us to *understand* the laws, rules, and theorems in order to apply them properly.

In this book we shall describe certain conduction processes, from these deduce the terminal behavior of various electronic devices, make circuit models for the devices, and use the models to study the operation of basic electronic circuits. Attention is given to the important functions performed by each class of circuits. As we proceed, it will become clear that a relatively small number of fundamental ideas, properly interpreted, permit one to understand the operation of a very large variety of electronic devices, circuits, functions, and systems. An integration of such ideas is the basis for circuit and system design.

PROBLEMS

1.1. Which of the following terms apply to the ideal diode? (*a*) linear, (*b*) capacitive, (*c*) resistive, (*d*) inductive, (*e*) nonlinear, (*f*) lossless, (*g*) bidirectional, (*h*) reactive, (*i*) passive, (*j*) active.

1.2. Do the following relations between instantaneous current and voltage specify the behavior of the ideal diode completely?

$$i \leqslant 0$$
$$v \geqslant 0$$
$$vi = 0$$

1.3. Compare the following relations with those of Problem 1.2.

$$v = 0 \qquad \text{for } i > 0$$
$$i = 0 \qquad \text{for } v < 0$$

1.4. What is the circuit model M for a driving-point curve like that shown in Fig. P1.1?

1.5. Sketch the output curve i_2 vs. v_2 for the voltage-controlled voltage source shown in Fig. P1.2. What are the dimensions of μ?

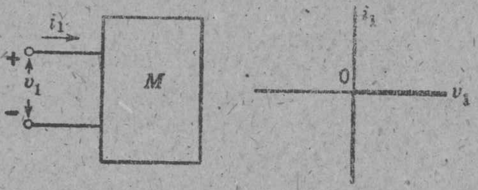


Fig. P1.1

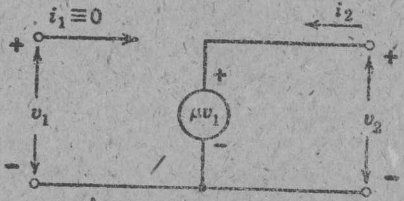


Fig. P1.2

1.6. Sketch the output curve i_2 vs. v_2 for the voltage-controlled current source shown in Fig. P1.3. What are the dimensions of g?

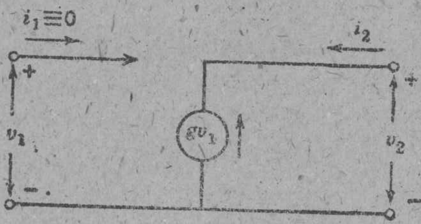


Fig. P1.3

1.7. Sketch the output curve v_2 vs. i_2 for the current-controlled voltage source shown in Fig. P1.4. What are the dimensions of r?

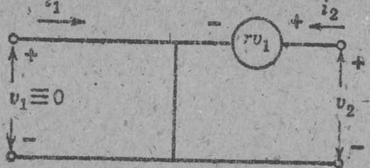


Fig. P1.4