# Color Science: Concepts and Methods, Quantitative Data and Formulae

SECOND EDITION

GÜNTER WYSZECKI

W.S. STILES

# COLOR SCIENCE

## Concepts and Methods, Quantitative Data and Formulae,

2nd Edition

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# PREFACE

The apparent success of the first edition of this book, published in 1967, and the persistent encouragement of many of our colleagues was necessary and sufficient persuasion for us to write the second edition. Our basic approach to color science has not changed. The second edition is again a collection of concepts and methods, quantitative data, and formulae bearing on color science and directed to the advanced student and research worker in color—physicist, psychologist, and physiologist—and those actively engaged in color engineering projects—illuminating engineers, designers, and industrial consultants. Our aim has again been to assemble the quantitative tools for work on color: details of light sources, color filters, monochromators, and photon detectors; in depth accounts of the working concepts in color matching, color discrimination, and chromatic adaptation; logical presentations of the mathematical algorithms used in the development of these concepts; international standards and procedures of the measurement of light and color.

Much of the material selected for the book is organized in tabular or mathematical form, stressing data and formulae that have gained acceptance as standard or hard data applicable to color calculations or further developments in color-vision research. Some of the contents are presented in considerable detail so as to make the book as much as possible a unique source of reference material for the research worker and serious student of color science. Again, trichromatic principles and their application loom large in this book and, in fact, have been expanded somewhat to include Maxwell's method of trichromatic matching. Color matching is here deployed from first principles and in later sections has been linked to issues of color-vision mechanisms.

A great deal of new and expanded material has been included in the second edition, reflecting the considerable growth of knowledge that has taken place since the 1960s. This has particularly been the case in color matching, increment-threshold work, chromatic adaptation, and the theoretical modeling of color discrimination and other color-vision phenomena. We have attempted to keep pace with the new developments, but this has not been an easy task. For the most part, descriptive and qualitative material on color phenomena which would properly find a place in a textbook or introductory treatise on color has not been included. We have also refrained from delving too deeply into highly speculative issues of color-vision modeling, which have attracted a number of research workers, particularly in recent years. Our account on theories and models of color vision is only a brief review of the basic principles and approaches that have emerged.

We wish to acknowledge with thanks our indebtedness to our many associates.

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Numerous illustrations and tables in this book are taken from, or based on, the work of many different authors published in a variety of journals and books. In each instance we have cited the names of these authors in the captions of the illustrations and legends of the tables, and the complete reference to the original publication is given in the Reference Section. The illustrations and tables listed below have been reproduced in detail, and permission to do so has been received from the following publishing bodies who are the holders of the copyrights. We wish to express our grateful acknowledgment to all the authors and publishing bodies for their kind cooperation.

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*Instituto de Biofisica, Rio de Janeiro*: Fig. 1(2.4.7)
*John Wiley & Sons, Inc.*: Figs. 1(1.4), 2(1.4), 1(3.3.9), 2(3.3.9), 1(3.3.10), 8(3.8.2), 10(3.8.6), 11(3.8.6), 12(3.8.6), 2(3.12.2), 3(3.12.3), 4(3.12.3), 5(3.12.3), 2(3.12.5), 4(2.12.5), 3(5.4.1), 7(5.12.2), 8(5.12.2), 9(5.12.2), 11(5.12.2), 1(5.12.3), 2(5.12.3), 3(5.12.3), 4(5.12.3), 5(5.12.3), 6(5.12.3), 2(5.14.1), 1(5.14.2), 3(5.15), 4(6.3), 5(6.3), 1(6.6.1), 2(6.6.1), 3(6.6.1), 1(7.10.5), Tables 1(3.3.5), I(3.3.8), II(3.3.8), III(3.3.8), IV(3.3.8), 1(3.3.8), 2(3.3.8), 3(3.3.8), 1(3.3.10), 2(3.3.10), I(3.9), II(6.5.1A, B, C), I(5.5.2)
*Junk BW Publishers*: Figs. 2(7.3.2), 1(7.4.1) and 2(7.4.1)
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*National Physical Laboratories, Teddington, Middlesex*: Figs. 4(5.4.1), 1(5.5.4), 1(5.10), 5(5.12.2)
*Optical Society of America*: Figs. 2(1.2.1), 3(1.3.7), 7(1.3.7), 3(1.4), 1(1.4.1), 11(1.6.2), 1(2.4.3), 7(2.4.6), 1(3.6), 1(3.8.6), 2(3.8.6), 3(3.8.6), 4(3.8.6), 5(3.8.6), 6(3.8.6), 7(3.8.6), 8(3.8.6), 1(3.11), 2(3.11), 3(4.3.5), 7(4.3.5), 5(4.3.6), 1(5.4.3), 2(5.4.3), 1(5.4.4), 1(5.4.5), 2(5.4.5), 3(5.4.5), 4(5.4.5), 1(5.6.1), 8(5.7.2), 10(5.7.2), 5(5.8.1), 6(5.8.1), 1(5.12.2), 1(5.13), 2(5.13), 3(6.4), 2(6.5.1), 5(7.9.3), 6(7.9.3), 9(7.9.3), 12(7.9.3), 13(7.9.3), 2(7.10.2), 3(7.10.2), 1(7.10.6), 2(7.10.6), 3(7.10.6), 4(7.10.6), 5(7.10.6), 2(8.3.3), 3(8.3.3), 1(8.3.4), 2(8.3.4), 3(8.3.4), 4(8.3.4), 5(8.3.4), 6(8.3.4), 7(8.3.4), 8(8.3.4), Tables I(3.8.2), II(3.8.2), IV(3.8.2), 2(5.4.1), I(5.4.2), II(5.4.2), I(5.4.3), II(5.4.4), I(6.6.1), I(6.6.4)

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*Physical Society*: Fig. 2(5.5.6)

*Mrs. Stephen Polyak*: Figs. 1(2.2.4) and 1(2.2.6)

*The Royal Society (London)*: Figs. 2(2.2.4), 1(5.11), 1(7.3.2)

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# CONTENTS

# CHAPTER 1

# PHYSICAL DATA

## 1.1 BASIC RADIOMETRIC QUANTITIES AND UNITS

A selection has been made of the radiometric quantities that are regarded as basic and of most general value. These quantities are displayed in Table I(1.1) of the Appendix. The definitions used here are substantially those adopted in the *International Lighting Vocabulary* (third edition, and an early draft of the fourth edition) of the Commission Internationale de l'Eclairage (CIE, 1970). This vocabulary may also be referred to for equivalent definitions in French, German, and Russian, and the translations of the terms (without the definitions) in five additional languages (Spanish, Italian, Dutch, Polish, and Swedish).

The reader is also referred to the *Self-Study Manual on Optical Radiation Measurements* published by the U.S. National Bureau of Standards (NBS, 1976 to 1979) which includes a rather detailed and advanced treatise of the radiometric concepts. Also recommended is the introductory booklet by Bauer (1965) and the monograph by Grum and Becherer (1979).

Table 1(1.1) is a supplement to Table I(1.1). It contains the defining equations of the basic radiometric quantities and the corresponding SI units of measurement (SI = Système Internationale d'Unités). Figure 1(1.1) illustrates the explanatory notes given in Table 1(1.1) and will be found useful in interpreting the defining equations. Figure 2(1.1) illustrates the concept of solid angle and its unit, the steradian.

## 1.2 SOURCES OF RADIANT ENERGY

The data on sources of radiant energy that are considered of basic interest to the colorimetrist and visual research worker are relative spectral radiant power distributions. The spectral range of these distributions is usually confined to the visible. However, some distributions extend to the ultraviolet and near infrared regions of the spectrum to allow for applications to the colorimetry of fluorescent materials and to assess the potential effects of nonvisible radiant power on radiometric, photometric, and colorimetric parameters. For some sources, indications are given of the radiant power emitted in terms of absolute radiometric quantities, for example, irradiance ($W \cdot m^{-2}$).

The spectral distribution of a radiometric quantity is commonly given in terms of that quantity per unit wavelength interval. This practice is followed in this book. However, occasionally it is advantageous to consider the radiometric quantity per unit frequency interval, frequency being in some respects the more fundamental parameter of radiant energy. The conversion of a spectral radiant power distribution based on wavelength to one based on frequency is carried out as follows:

By definition [see Table 1(1.1)] it is

$$\lambda = c\nu^{-1} \qquad [1(1.2)]$$

thus

$$d\lambda = -c\nu^{-2}d\nu \qquad [2(1.2)]$$

Over a spectral region $d\lambda$ the radiant power may be given by $P_\lambda d\lambda$, which on the frequency basis must be equal to $-P_\nu d\nu$. The minus sign allows for the decrease in frequency with an increase in wavelength. From Eqs. 1(1.2) and 2(1.2), it fol-

**Table 1(1.1) Basic Radiometric Quantities and Units [Supplement to Table I(1.1)]**

| Term | Symbol | Defining Equation | Explanatory Notes and Formulae [see Figs. 1(1.1) and 2(1.1)] | | Units |
|---|---|---|---|---|---|
| Radiant energy | | | | | J (joule) |
| Radiant power | $P_e$ | | $d^2P_e = L_e \dfrac{dA_1 \cos \varepsilon_1 \, dA_2 \cos \varepsilon_2}{r^2}$ | | $J \cdot s^{-1} = W$ (watt) |
| Radiant exitance | $M_e$ | $M_e = \dfrac{dP_e}{dA_1}$ | $dA_1$ = surface element of source | | $W \cdot m^{-2}$ |
| Irradiance | $E_e$ | $E_e = \dfrac{dP_e}{dA_2}$ | $dA_2$ = surface element of receiver | | $W \cdot m^{-2}$ |
| Radiant intensity | $I_e$ | $I_e = \dfrac{dP_e}{d\omega_1}$ | $d\omega_1$ = element of solid angle with apex (1) at surface of source | | $W \cdot sr^{-1}$ |
| Radiance | $L_e$ | $L_e = \dfrac{d^2P_e}{dA_1 \cos \varepsilon_1 \, d\omega_1}$ $= \dfrac{d^2E_e}{dA_2 \cos \varepsilon_2 \, d\omega_2}$ $= \dfrac{d(E_e)_n}{d\omega_2}$ | $\varepsilon_1$ = angle between given direction (1)–(2) and normal $n_1$ of $dA_1$; $\varepsilon_2$ = angle between given direction (1)–(2) and normal $n_2$ of $dA_2$; $d\omega_2$ = element of solid angle with apex (2) at surface of receiver | $dA_1 \cos \varepsilon_1 = dA_1$ orthogonally projected on plane perpendicular to given direction (1)–(2); $dA_2 \cos \varepsilon_1 = dA_2$ orthogonally projected on plane perpendicular to given direction (1)–(2); $d(E_e)_n = \dfrac{dE_e}{dA_2 \cos \varepsilon_2}$ | $W \cdot m^{-2} \cdot sr^{-1}$ |
| Solid angle | $\omega$ | $\omega = \dfrac{S}{r^2}$ | $S$ = portion of sphere surface; $r$ = radius of sphere, also distance between (1) of $dA_1$ and (2) of $dA_2$ | $d\omega_1 = \dfrac{dA_2 \cos \varepsilon}{r^2}$; $d\omega_2 = \dfrac{dA_1 \cos \varepsilon_1}{r^2}$ | sr (steradian) |
| Frequency | $\nu$ | | | | $s^{-1}$ |
| Wavelength[a] | $\lambda$ | $\lambda = \dfrac{c}{\nu}$ | $c$ = velocity of radiant energy *in vacuo* | | m |
| Wavenumber[a] | $m$ | $m = \dfrac{1}{\lambda}$ | | | $m^{-1}$ |

[a] In virtually all colorimetric applications, the unit for wavelength is the nanometer (1 nm $= 10^{-9}$ m). In more vision oriented work, either wavelength in terms of nm or wavenumber in terms of $cm^{-1}$ are used.

*Note*: If the spectral concentration of a radiometric quantity $X_e$ is considered, it is usually designated by the name of the quantity preceded by the adjective *spectral*, and by the same symbol for the quantity with the subscript $\lambda$ (or $\nu$, or $m$): $X_{e\lambda} = dX_e/d\lambda$. For example, if $X_e$ is radiant power, the spectral concentration of radiant power is simply referred to as spectral radiant power and is denoted by $P_{e\lambda} = dP_e/d\lambda$.

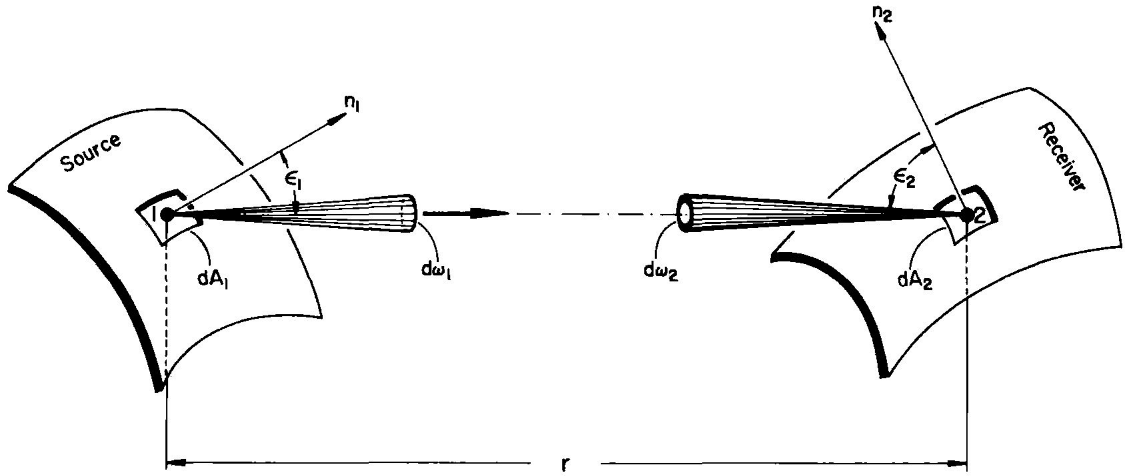


Fig. 1(1.1). Illustration of explanatory notes given in Table 1(1.1) concerning basic radiometric quantities and units.

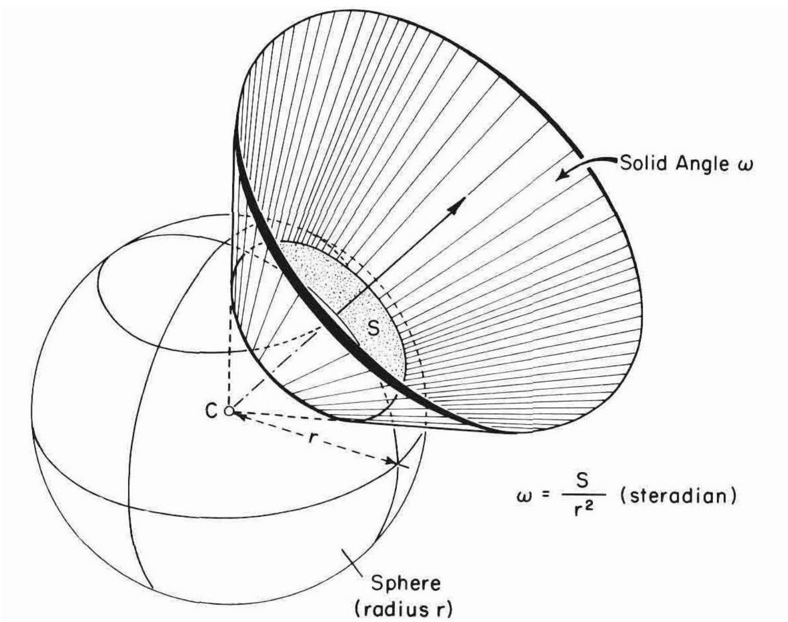


Fig. 2(1.1). Illustration of a solid angle $\omega$ and its measurement in terms of the unit of solid angle, the steradian (sr). The apex of the solid angle is located at $C$. The solid angle cuts off an area $S$ on the surface of a sphere centered at $C$ and of radius $r$. The size of the solid angle $\omega$ is then given by the quotient of $S$ over $r^2$. In the case illustrated, $\omega$ is approximately equal to one steradian. The concept of solid angle is not confined to right-circular cones of the kind depicted in the illustration. Almost any shape of cone, generated by the straight lines emerging from the apex to the points of a closed curve, can represent a solid angle. If the closed curve is a polygon (e.g., a square), the cone and thus the solid angle takes on the shape of a pyramid.