HIGH-FREQUENCY AMPLIFIERS

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PREFACE

This book presents principles and techniques useful in the analysis and design of high-frequency amplifiers. It is intended primarily as a textbook for a single-semester course for beginning graduate and senior undergraduate students in electrical engineering, but it can also be used by practicing engineers as a source of information about high-frequency amplifiers. It emphasizes the use of the Smith chart and scattering parameters, both of which are fundamental to this area of analysis and design.

This textbook is unusual compared to other books on electronic circuits because it develops the Smith chart into a useful design aid. The Smith chart is part of the graphical heritage in electrical engineering, but students have had little opportunity to discover the potential and insight that it offers in electronic circuit applications. Preparing this material on applications of the Smith chart has been one of the most exciting and rewarding aspects of writing the book. The material has been collected from many sources scattered throughout the literature; it has been integrated and explained with numerous examples.

Chapter 1 reviews two-port transistor parameters and their conversion relationships. The indefinite admittance matrix is discussed to take advantage of its usefulness for calculating parameters when changing transistor configuration between common base, common emitter, and common collector. The calculus of deviations is introduced only as an additional method for calculating parameters when changing configuration. Criteria for activity and passivity are derived in terms of admittance parameters; then the maximum frequency of oscillation is determined for the high-frequency hybrid-π model of the transistor.

Chapter 2 reviews the impedance and gain properties of a linear active transistor with terminations at its two ports and expresses input and output immittances in terms of generalized parameters. Stability and instability are discussed and conditions for inherent stability derived and also expressed in terms of generalized parameters. Unilateral power gain, tunability, and bandwidths are discussed.

Chapter 3 introduces the Smith chart and develops its use in designing various types of immittance matching networks. How to find transmission line input and output impedances, reciprocals of immittances, and how to locate negative resistances are described. The design of lumped-constant matching networks, microstrip and some of its properties, and the design of microstrip matching networks are described in detail.

Chapter 4 is devoted to unilateral amplifiers, and the cascode circuit is discussed. Some history of the origins of Q puts that important quantity in perspective. Mathematical series-parallel transformations are applied to the design of matching networks to supplement the Smith chart. Obtaining optimum terminations for a specified bandwidth is discussed. The chapter also explains how power, voltage, and current gains can be shown on the Smith chart as families of circles. A major advantage of using the Smith chart in such cases is that the gains expected for all possible passive terminations are displayed completely.

Chapter 5 deals with nonunilateral amplifiers. Power-gain circles are drawn on the Smith chart and used with a given tunability factor to determine the output termination. It is shown how a rectangular grid can be drawn on the Smith chart to determine input admittance of a transistor for any passive output termination. Design procedures for potentially unstable transistors are discussed. A new mismatching technique is presented in detail. Circuit instability caused by the emitter circuit is described.

Chapter 6 introduces scattering parameters, their conversion relationships with other parameters, their physical meanings, and methods for calculating and measuring them. Criteria for activity and passivity, potential instability, the indefinite scattering matrix, and directional couplers are discussed.

Chapter 7 is devoted to amplifier design using scattering parameters. It is shown how generalized scattering parameters for arbitrary terminations are calculated and used in the design of unilateral and nonunilateral amplifiers. Power-gain circles are again drawn on the Smith chart, and stable and unstable regions identified. The conditions for inherent stability and for simultaneous conjugate matching are derived and used. The chapter ends with a design procedure for use with potentially unstable transistors.

Appendixes A and B provide further details of derivations that are not found anywhere else in the literature. General references consisting of the original papers and alternative procedures are given in the bibliography.

Many other important aspects of high-frequency amplifiers had to be omitted from a book of this size. Topics such as noise, neutralization, biasing, multistage circuits, and computer-aided design are adequately covered elsewhere.

It is assumed the reader has a general knowledge of circuit theory, transistor circuits, and of basic matrix manipulations. Some background in transmission lines at high frequencies is helpful but not essential. The book contains a large number of worked examples to illustrate techniques and provides problems at the end of each chapter. All of the material, except for last-minute revisions, has been used with gratifying results in various graduate and senior courses at University of Missouri–Rolla and at the Graduate Engineering Center of UMR in St. Louis.

I thank the many students who have suggested improvements in earlier versions of these chapters. In particular, I thank David Mundis, Russell Woirhaye, and Frank Giannoti for investigating some of the design details in their senior seminar papers. Most of all, I thank Dr. K. Kurokawa for giving me insight into some of the more elusive properties of scattering parameters.

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Rolla, Missou.
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CONTENTS

CHAPTER ONE

TRANSISTOR PARAMETERS

1-0 INTRODUCTION

Transistors are three-terminal devices. In use, one of the terminals is common to both the input and output circuits. This leads to the familiar configurations known as common base (CB), common emitter (CE), and common collector (CC). The common terminal also is often connected to the reference ground.

The common terminal can be paired with one or the other of the two remaining terminals. Each pair is called a port, and two pairs are possible for any of the basic CB, CE, CC configurations. Such circuits are properly called two-port networks. The two ports are usually identified as an input port and as an output port. The collector is not used with an input terminal except in certain special applications. The usual connections are summarized in Fig. 1-1.

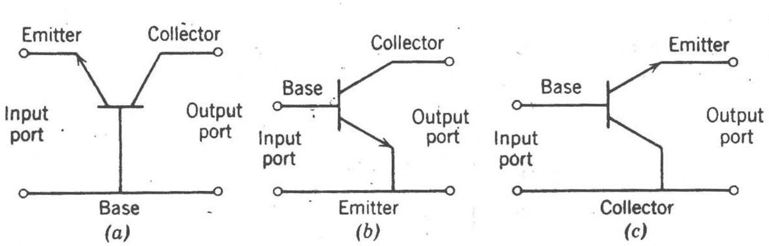

Figure 1-1. Basic transistor configurations: (a) common base; (b) common emitter; (c) common collector.

1-1 TWO-PORT TRANSISTOR PARAMETERS

Basically, electronic circuits are used to process information. Originally, the information may be nonelectrical in nature such as heat, sound, pressure, or humidity. In such cases, a transducer is required to convert the information into its electrical equivalent before the transistor can do its job. The electrical information is called a signal, and it exists in the forms of signal currents, signal voltages, and signal powers.

The signal-handling capability of a transistor depends on how large the signal is. Even though the input-to-output signal relationship of a transistor is inherently nonlinear, a transistor can be adequately represented as a linear two-port device for very small signals. Large signals can cause the transistor to enter cutoff and saturation regions. Therefore, it is usual to assign certain small-signal parameters to the transistor when it is considered to be a linear two-port network.

Several sets of small-signal parameters are possible for the transistor two-port, represented in Fig. 1-2, depending on which of the signal voltages and currents are considered to be the independent variables and which are considered dependent variables. The voltages and currents can be expressed as phasors and the parameters as functions of frequency for small-signal sinusoids or everything can be expressed in terms of Laplace transforms.

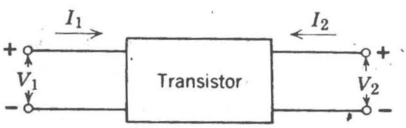

Figure 1-2. Transistor two-port.

The parameter sets are summarized in Table 1-1, in which Φ_{i1} and Φ_{i2} represent the two independent quantities, Φ_{d1} and Φ_{d2} represent the two dependent quantities, and k_i, k_r, k_f, and k_o represent the parameters in a particular set.

The relationship between the dependent variables, the independent variables, and the parameters can be expressed in matrix form as

$$\begin{bmatrix} \Phi_{d1} \\ \Phi_{d2} \end{bmatrix} = \begin{bmatrix} k_i & k_r \\ k_f & k_o \end{bmatrix} \begin{bmatrix} \Phi_{i1} \\ \Phi_{i2} \end{bmatrix} \qquad (1)$$

or as

$$[\Phi_d] = [k][\Phi_i] \qquad (2)$$

where $[\Phi_d]$ and $[\Phi_i]$ are column matrices and $[k]$ is the square parameter matrix.

TABLE 1-1

Φ_{i1}	Φ_{i2}	Φ_{d1}	Φ_{d2}	k_i	k_r	k_f	k_o
I_1	I_2	V_1	V_2	z_i	z_r	z_f	z_o
V_1	V_2	I_1	I_2	y_i	y_r	y_f	y_o
I_1	V_2	V_1	I_2	h_i	h_r	h_f	h_o
V_1	I_2	I_1	V_2	g_i	g_r	g_f	g_o
V_2	$-I_2$	V_1	I_1	A	B	C	D
V_1	$-I_1$	V_2	I_2	$\mathcal{A}$	$\mathcal{B}$	$\mathcal{C}$	$\mathcal{D}$

Only three of the six parameter sets in Table 1-1 have been used extensively to describe two-port transistor circuits. These are the z, y, and h parameters. Scattering parameters, not given in Table 1-1 because they are related to voltages and currents indirectly, are becoming increasingly useful above about 100 MHz. An extensive treatment of scattering parameters is given in Chapter 6.

1-11 Open-Circuit Impedance Parameters $[z]$

If the currents are the independent variables, the dependent voltages are given by

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_i & z_r \\ z_f & z_o \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad (3)$$

or

$$\begin{aligned} V_1 &= z_i I_1 + z_r I_2 \\ V_2 &= z_f I_1 + z_o I_2 \end{aligned} \qquad (4)$$

The z parameters are constants for a particular transistor and are determined at its operating point with a certain signal frequency. They are independent of signal amplitudes provided the amplitudes are small enough to insure linear operation. This constancy allows measurements to be made when one or the other independent current is set equal to zero.

The value of z_i may be found by opening the output circuit to make the *signal* current $I_2 = 0$ and then determining the current I_1 produced by a signal voltage V_1. Then $z_i = V_1/I_1$, and it is called the open-circuit input impedance. If the voltage V_2 produced at the open output terminals by the signal current I_1 is determined, then $z_f = V_2/I_1$, and it is called the open-circuit forward-direction transfer impedance.

Similarly, the value of z_r may be found by opening the input circuit to make the *signal* current $I_1 = 0$ and then determining the voltage V_1 produced at the open input terminals by a signal current I_2. Then $z_r = V_1/I_2$, and it is called the open-circuit reverse-direction transfer impedance. If the voltage V_2 produced at the output terminals by the current I_2 is determined, then $z_o = V_2/I_2$, and it is called the open-circuit output impedance.

Laboratory measurements of open-circuit impedance parameters require open circuits for the *signal* currents, not the DC currents. The transistor must still be biased properly. Effectively, an open circuit for signal current is obtained by inserting a large inductance in series with the circuit to be opened. This technique is adequate if the circuit impedance is already small before the inductance is added, but this condition is hardly satisfied for a reverse-biased collector-base junction. However, it is more nearly satisfied for a forward-biased emitter-base junction. Obtaining accurate measurements of the open-circuit impedance parameters over a wide frequency range can present some formidable problems.

The open-circuit impedance parameters were used to some extent in the early days of transistor development. Signal frequencies up to a few megahertz only were encountered, and the parameters had small reactive components. Advances in transistor modeling and the availability of other parameters have made the open-circuit impedance parameters obsolete as transistor specifications.

1-12 Short-Circuit Admittance Parameters [y]

If the voltages are the independent variables, the dependent currents are given by

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_i & y_r \\ y_f & y_o \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{5}$$

or

$$\begin{aligned} I_1 &= y_i V_1 + y_r V_2 \\ I_2 &= y_f V_1 + y_o V_2 \end{aligned} \tag{6}$$

where

$$y_i = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \text{short-circuit input admittance}$$

$$y_r = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \text{short-circuit reverse transfer admittance}$$

$$y_f = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \text{short-circuit forward transfer admittance}$$

$$y_o = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \text{short-circuit output admittance}$$

The admittance parameters may be real or complex numbers depending on the transistor type and the signal frequency. When making laboratory measurements of the admittance parameters, the short circuits required to make V_1 and V_2 equal to zero are effectively obtained by placing a large capacitance across the terminals in question. This technique is particularly effective if the circuit impedance is already high, such as for a reverse-biased collector-base circuit, but is not so good if the circuit impedance is low, such as for a forward-biased emitter-base circuit.

When measurements are to be made over a wide range of frequencies extending to several hundred megahertz, a broadband short circuit is difficult to obtain. At such high frequencies, tuning stubs adjusted to the proper lengths can provide the required short circuits. Each change in signal frequency requires readjusting the stub lengths. Precise measurements of the short-circuit admittance parameters are sometimes difficult and tedious to obtain, but the y parameters are quite useful. The y-parameter variations with frequency are sometimes specified by the transistor manufacturer.

1-13 Hybrid Parameters [h]

It has been pointed out that an open circuit is most effectively obtained in a circuit whose impedance is normally quite small, and that a short circuit is most effectively obtained in a circuit whose impedance is normally quite high. For the transistor, this means that for the CB and CE configurations, the impedance of the output port is high so that parameter measurements involving a short circuit at the output should be accurate, and the impedance of the input port is small so that parameter measurements involving an open circuit at the input should be accurate. These advantages can be achieved using a set of hybrid parameters.

If I_1 and V_2 are selected to be the independent variables, then the dependent variables V_1 and I_2 are given by

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_i & h_r \\ h_f & h_o \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \tag{7}$$

or

$$\begin{aligned} V_1 &= h_i I_1 + h_r V_2 \\ I_2 &= h_f I_1 + h_o V_2 \end{aligned} \tag{8}$$

where

$$h_i = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \text{short-circuit input impedance}$$

$$h_r = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \text{open-circuit reverse voltage gain}$$

$$h_f = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \text{short-circuit forward current gain}$$

$$h_o = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \text{open-circuit output admittance}$$

These are called hybrid-*h* parameters because they all have different dimensions. For the CB and CE transistor configurations, the required short circuit that makes $V_2=0$ occurs in the high-impedance collector or output circuit and the required open circuit that makes $I_1=0$ occurs in the low-impedance emitter-base or input circuit. The hybrid-*g* and *ABCD* parameters do not satisfy these circuit-impedance conditions, and have not been widely used as transistor specifications.

The *h* parameters may be real or complex numbers depending on the transistor and signal frequency. However, there is no standard and convenient notation for identifying the real and imaginary parts because of the different dimensions. The *z* parameters are conveniently represented by $r+jx$ because z, r, and x are all measured in ohms. The *y* parameters are conveniently represented by $g+jb$ because y, g, and b are all measured in mhos. About the best that can be done with the *h* parameters in this regard is to represent their parts by Re and Im, where Re and Im stand for "real part of" and "imaginary part of," respectively.

The *h* parameters are one of the most widely used parameter sets. Conversions between the z, y, and h parameters are given in Table 1-2.

TABLE 1-2 CONVERSIONS BETWEEN PARAMETERS*

To \ From	z	y	h
z	$\begin{matrix} z_i & z_r \\ z_f & z_o \end{matrix}$	$\begin{matrix} \frac{y_o}{D_y} & \frac{-y_r}{D_y} \\ \frac{-y_f}{D_y} & \frac{y_i}{D_y} \end{matrix}$	$\begin{matrix} \frac{D_h}{h_o} & \frac{h_r}{h_o} \\ \frac{-h_f}{h_o} & \frac{1}{h_o} \end{matrix}$
y	$\begin{matrix} \frac{z_o}{D_z} & \frac{-z_r}{D_z} \\ \frac{-z_f}{D_z} & \frac{z_i}{D_z} \end{matrix}$	$\begin{matrix} y_i & y_r \\ y_f & y_o \end{matrix}$	$\begin{matrix} \frac{1}{h_i} & \frac{-h_r}{h_i} \\ \frac{h_f}{h_i} & \frac{D_h}{h_i} \end{matrix}$
h	$\begin{matrix} \frac{D_z}{z_o} & \frac{z_r}{z_o} \\ \frac{-z_f}{z_o} & \frac{1}{z_o} \end{matrix}$	$\begin{matrix} \frac{1}{y_i} & \frac{-y_r}{y_i} \\ \frac{y_f}{y_i} & \frac{D_y}{y_i} \end{matrix}$	$\begin{matrix} h_i & h_r \\ h_f & h_o \end{matrix}$

*D represents the value of the determinant formed by the parameters, for example, $D_h = h_i h_o - h_r h_f$.

1-2 INDEFINITE ADMITTANCE MATRIX

It is not absolutely essential for either the emitter, base, or collector of the transistor to be the common reference terminal. It is entirely appropriate to select some other external point as the common reference terminal as shown in Fig. 1-3. The terminals 1, 2, and 3 correspond to the base, emitter, and collector. The entire circuit acts like a three-port network. If the signal voltages are the independent variables, the dependent currents are given by

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \tag{9}$$

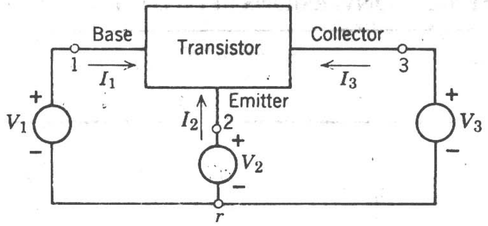

Figure 1-3. Transistor with external reference node r.

The indefinite admittance matrix is useful because the sum of any row and of any column equals zero.

To prove that the sum of any column equals zero, apply Kirchhoff's current law at reference node r in Fig. 1-3. Then

$$I_1 + I_2 + I_3 = 0 \tag{10}$$

Since (9) is valid for any small values of signal voltages, let $V_2 = V_3 = 0$. Then (9) becomes

$$I_1 = y_{11}V_1 \qquad I_2 = y_{21}V_1 \qquad I_3 = y_{31}V_1 \tag{11}$$

Substitute (11) into (10) to obtain

$$(y_{11} + y_{21} + y_{31})V_1 = 0 \tag{12}$$

Since V_1 was not assumed zero, then

$$y_{11} + y_{21} + y_{31} = 0 \tag{13}$$

which proves that the sum of the first column equals zero. Similarly, for the second column let $V_2 \neq 0$ with $V_1 = V_3 = 0$, and for the third column let $V_3 \neq 0$ with $V_1 = V_2 = 0$.

To prove that the sum of any row equals zero, let all three signal voltages be equal to V_o. Since all transistor terminals are at the same voltage relative to node r, there can be no current. Hence, for $V_1 = V_2 = V_3 = V_o$, the currents $I_1 = I_2 = I_3 = 0$. From (9),

$$I_1 = y_{11}V_1 + y_{12}V_2 + y_{13}V_3$$

or

$$0 = (y_{11} + y_{12} + y_{13})V_o \tag{14}$$