

**Proceedings of the  
23rd International**

**MACHINE TOOL  
DESIGN AND RESEARCH  
Conference**

**PROCEEDINGS OF THE  
TWENTY-THIRD INTERNATIONAL  
MACHINE TOOL DESIGN AND RESEARCH  
CONFERENCE**

*held in Manchester  
14th-15th September 1982*

*Edited by*

**B. J. DAVIES**

*Professor of Manufacturing Technology  
Department of Mechanical Engineering  
University of Manchester Institute of Science and Technology  
Manchester*



**DEPARTMENT OF MECHANICAL ENGINEERING  
UNIVERSITY OF MANCHESTER INSTITUTE OF SCIENCE AND TECHNOLOGY  
MANCHESTER**

in association with

**THE MACMILLAN PRESS LIMITED**

**M**

Department of Mechanical Engineering  
University of Manchester Institute of Science and Technology  
1983

All rights reserved. No part of this publication may be reproduced or transmitted, in any form or by any means, without permission

First published 1983 by the  
Department of Mechanical Engineering  
University of Manchester Institute of Science and Technology  
P.O. Box 88, Manchester M60 1QD

in association with

The Macmillan Press Ltd  
London and Basingstoke  
Companies and representatives throughout the world

ISBN 0 333 34353 0

Printed in England by Revell & George Limited, Manchester M4 6JD

**TWENTY-THIRD INTERNATIONAL  
MACHINE TOOL DESIGN AND RESEARCH  
CONFERENCE  
14th-15th September 1982**

**CONFERENCE ORGANIZING COMMITTEE**

**Chairman: Professor B. J. Davies**

**POLICY COMMITTEE**

**Professor S. A Tobias (Birmingham)  
Professor W. Johnson (Cambridge)  
Professor J. M. Alexander (Swansea)**

**ORGANIZING AND REVIEWING COMMITTEE**

**Professor B. J. Davies  
Dr. C. F. Noble (Deputy Chairman)  
Dr. J. Atkinson  
Dr. G. Barrow  
Dr. M. Burdekin  
Dr. A. Cowley  
Dr. M. Edkins  
Dr. W. Graham  
Dr. R. G. Hannam  
Dr. J. Hawkyard  
Dr. S. Hinduja  
Dr. E. W. Smith**

**ORGANIZING SECRETARY**

**Mr. R. Kirk**

# CONTENTS

## MACHINE ELEMENTS AND STRUCTURES

Evaluation of the deflections of radial rolling bearings by nomograms M. M. A. TAHA	1
The behaviour of a total cross flow hydrostatic journal bearing M. E. Mohsin and A. Sharratt	13
A comparison of boundary integral and finite element methods for machine tool structural analysis O. B. ALANKUS, R. D. HIBBERD and C. B. BESANT	25
Development of a machine tool structure using composite synthetic granite S. E. OVEIAWE, T. TAY and A. A. SHUMSHERUDDIN	31

## MACHINE ANALYSIS AND TESTING

The effect of assembly operations on the dynamic acceptance tests for machine tools M. A. EL BARADIE	41
The energetic behaviour of crank presses I. CZINEGE	53
Noise analysis of a C-frame mechanical press T. SANO, K. HATSUKANO, K. AOI and C. SODA	59

## MACHINE CONTROL SYSTEMS

Design of a micro-computer based quality monitoring system for the manufacture of knitting machine cylinders H. S. GILL, N. D. BURNS and K. OLDHAM	69
Interpolation in hardware for numerical control: Evaluation of a new large-scale interpolator T. W. SHAW and I. POON	77
Advanced CNC system using multibus compatible hardware and portable software G. M. AJIT and C. S. SRIDHAR	85

## QUALITY CONTROL

Permanent temperature-control in securing technological processes in machine tools H. WIELE, R. MANGELSDORF and H. HEINKE	97
An assessment of the potential of an industrial robot for use as a flexible drilling machine J. A. G. KNIGHT and P. CHAPMAN	103
Development of in-process sensor for surface roughness measurement I. INASAKI	109
Development of a new measuring method for spindle rotation accuracy by three point method K. MITSUI	117

## TOOLING AND DATA FOR NC

Optimizing tooling for CNC turning M. A. KNIGHT and D. SPURGEON	123
A combined approach to obtaining and using tool life data J. TAYLOR	129
Automation of postprocessors to increase productivity K. G. ADAMS and D. FRENCH	131

## **CAD FOR NC**

A method of defining programming parameters for the manufacture of shoe moulds D. FRENCH and K. G. ADAMS	151
Developments in the duct system of computer aided engineering D. P. STURGE and M. S. NICHOLLS	157
Euklid and Ozelot, the linkage of a CAD and CNC-system M. ENGELI and G. STAUFERT	163

## **CUTTING PROCESSES**

Machining high strength materials: Cobalt-based alloys M. WILSON and M. EL BARADIE	171
Tool life and performance data for various polycrystalline diamond tools including twist drills D. K. ASPINWALL and G. MYATT	183
A study of rotary shaving by die-type cutting tool S. KATO, K. YAMAGUCHI and T. NAKAMURA	193

## **WHEEL DRESSING ASPECTS OF GRINDING**

Applications of continuous dressing in grinding operations T. R. A. PEARCE and T. D. HOWES	203
Comparison of methods for truing and dressing diamond and boron nitride wheels H. K. TONSHOFF and W. GARTNER	211
The significance of crushing roll infeed rate in the grinding wheel crushing operation N. N. Z. GINDY and T. J. VICKERSTAFF	219

## **GRINDING MACHINE PERFORMANCE**

Machine tools designed for borazon CBN P. E. GRIEB	225
Design criteria for CNC tool and cutter grinding L. ACARNLEY, D. A. POWELL, G. J. TRMAL and M. J. TYLER	231
The task of grinding of casting surfaces with industrial robots G. STUTE and K. H. WURST	239

## **ELECTROCHEMICAL MACHINING**

An experimental investigation on the sound speed and "choking" phenomenon of gas-liquid two-phase flow in the electrochemical machining (ECM) gap C. Y. YU, Y. Q. WANG, J. W. XU and D. ZHU	249
Computer aided analysis of ECBD process VIJAY K. JAIN, VINOD K. JAIN and P. C. PANDEY	257
Electro chemical creep feed grinding with, Ni-coated WA wheel Y. KITA, M. IDO, M. KUNO and A. MIGUCHI	265

## **ELECTRO-DISCHARGE MACHINING**

The influence of EDM pulse shape on machined surfaces F. SCALARI and M. VIGNALE	275
Role of dielectric flushing on EDM performance A. ERDEN	283
An approach to identification and multicriterion optimization of EDM process A. OSYCZKA, J. ZIMNY, J. ZAJAC and M. BIELUT	291

## UNUSUAL MANUFACTURING PROCESSES

Study of a thermal energy method of removal of surface irregularities W. E. GEORGE and J. A. McGEOUGH	299
A new die manufacturing process suitable for FMS of press forming in large variety production K. KAWAGUCHI, S. HIRAMOTO, O. HAMADA and W. SHIMADA	307
Electrical discharge compaction technique for welding rod manufacture A. F. DARVIZEH, M. CAN, T. J. DAVIES and S. T. S. AL-HASSANI	315
Production of fine short-length metal fibres using self-excited vibration of an elastic tool	323

## CROPPING METHODS FOR TUBE AND BAR

A survey of methods for producing tubular billets by shearing C. K. CHOONG, M. K. DAS and S. A. TOBIAS	333
The study of precision bar cropping under axial load (Part II) C. JINDU, Z. XIANRU, Z. ZIGONG, Y. DEHONG, T. ZHAOJUE and T. GUOHUA	343

## EXTRUSION AND DRAWING

Theoretical and experimental analysis of tube and hollow profile extrusion T. ABILDGAARD	353
Study of asymmetric extrusion and drawing of tube — analysis of inclined wall-thickness distribution of tube M. KIUCHI and M. ISHIKAWA	361
Hydrostatic extrusion/drawing of tubes with a fixed mandrel B. LENGYEL and M. J. M. B. MARQUES	369
Enhanced hydrodynamic lubrication in hydrostatic extrusion using a modified die S. THIRUVARUDCHELVAN	377

## FURTHER FORMING STUDIES

Cold form tapping of internal threads P. H. H. TRENDLER and T. HODGSON	387
On the warm forming of machining steels E. NEHL and K. POHLANDT	395
Deformation and temperature distribution in hot forging H. SAIKI, S. SHIMIZU, E. TSURU and T. MATSUO	405

## JOB SHOP CONTROL AND PROCESS PLANNING

Single queue management of a job shop as implemented by a data flow architecture W. C. LEWIS, M. M. BARASH and J. J. SOLBERG	415
Tool oriented automatic process planning W. C. LEWIS, E. BARTLETT, I. FINFTER and M. M. BARASH	421
Increasing economy by re-organization the field of cutting tools in computer aided design, planning and manufacturing H. K. TONSHOFF and J. BALBACH	429

## **FACTORY LAYOUT**

Computer-aided plant relayout	
S. E. Z. ABDEL-BARR and M. M. KOURA	439
Simulation study of "unbalanced" flow line production systems	
D. S. HIRA and P. C. PANDEY	445
Optimal plant location	
M. A. ISMAIL and G. A. E. SEWELL	451

## **FLEXIBLE MANUFACTURING SYSTEMS**

FMS software: Data structures of distributed FMS data bases	
P. G. RANKY	459
Use of the cluster-analysis for developing a flexible manufacturing system for rotational parts	
H. K. TONSHOFF, C. FREIST, R. GRANOW and J. BUSSMANN	467
Flexible manufacturing system complex provided with laser (FMC)	
M. KIMURA, C. SODA, S. OZAKI, Y. YOSHIDA, M. KANAI AND Y. ITOH	475
Modelling a multi-product manufacturing system to assist in the selection of CNC machine tools	
B. R. KILMARTIN, D. HUGHES and R. LEONARD	483
Dialogue oriented workshop order scheduling in flexible automated manufacturing	
G. SPUR, G. SELIGER and A. EGGERS	497
Description of machining function in FMS and ITS analysis	
Y. ITO, Y. SAITO and T. KUDAMA	503
Fault analysis in a flexible manufacturing cell	
P. K. WRIGHT, D. A. BOURNE and R. M. MILLIGAN	511

## **AUTOMATION AND ROBOTICS**

Government attitudes toward programmable automation	
J. A. ALIC	521
Economic considerations in industrial robotics	
P. L. WATTS, A. LEWIS and B. K. NAGPAL	529
The design of a DNC system for use in the production of small, prismatic parts	
D. H. J. HANCOCK	533

# FOREWORD

BY CONFERENCE CHAIRMAN

It gives me great pleasure to preface the proceedings of the latest conference in a long line of international forums. The presentation of papers and exchange of information and views by eminent researchers and industrialists has continued unabated for twenty-three years. Many of the young engineers who now attend as delegates or even submit papers were not born when the conference series started in 1960. During the early years the majority of papers came from the UK with countries in Europe a close second. Since those days, design, manufacturing and supply of machine tools and associated equipment has become increasingly international and competitive. Such internationalism is exemplified by the fact that 45 of the 63 contributing papers for 1982 come from 21 different countries outside the UK. This year Japan, and last year the Peoples Republic of China, head the list of overseas contributors whom we are pleased to welcome to Manchester.

The trend from specific design studies to the wider theme of manufacturing engineering has been natural and welcomed by both delegates and those who subsequently read and study the proceedings. It acknowledges that manufacturing industry is the most important creator of wealth within developed countries and that a sound industrial base is a vital element in enhancing the standard of living in countries still developing.

Perhaps because of my own special area of interest, I particularly note the increasing growth of research and development in Integrated Manufacturing Systems. Topics in this area include CAD, FMS and Robotics, all of which are demonstrably efficient and economical ways of manufacturing batches of capital goods. The manufacture of machine tools themselves is utilising these methods and stands as an example and warning to all who are not prepared or able to innovate.

Despite its pioneering role with System 24 the UK has been slow to install Flexible Manufacturing Systems, but it is a pleasure to be able to report that we now have 16 systems at various stages of development, most with the assistance of DoI. The £60M FMS programme announced by the Minister for Information and Technology in June is particularly welcome. Through this programme UK companies can recover 50% of the cost of consultancy from DoI up to £50,000, and up to a third of the total capital cost. Parallel developments in the SERC are assisting universities to carry out essential related research.

This conference contributes to a wider understanding of the problems and opportunities in this field and it does so without losing sight of the fundamental aspects of design and process analysis upon which new technologies depend.



*Professor B. J. Davies*



# **MACHINE ELEMENTS AND STRUCTURES**



# EVALUATION OF THE DEFLECTIONS OF RADIAL ROLLING BEARINGS BY NOMOGRAMS

M. M. A. TAHA  
College of Engineering, King Abdulaziz University  
Jeddah, Saudi Arabia

## SUMMARY

Calculation of rolling bearing deflections is essential in the design of machine tools to ensure their high rigidity. To avoid lengthy calculations the process of evaluating the rolling bearing deflections is commonly simplified, at the expense of accuracy, by making the assumption of zero clearance. This is because the deflection of rolling bearings with play or preload are usually calculated by solving a system of simultaneous non-linear equations using an iterative process. This is a time consuming and laborious process. The convergence also depends on the initial choice of the variables. To save time and to avoid lengthy iterative process a set of nomograms for radial ball and roller bearings has been prepared. The deflections of radial rolling bearings with play or preload can be found easily and quickly by using the nomograms presented in this paper. The paper also shows the effect of the different variables on the deflection of radial rolling bearings, which gives a clear idea about the choice of the best possible operating conditions of the bearings.

## NOMENCLATURE

B	Function of rolling element number and dimensions (equation 16).
c	Radial clearance.
D	Race diameter.
d	diameter of rolling elements.
E	Modulus of elasticity.
$F_*$	Load on bearing.
$F^*$	A function of load on the bearing and B (equations 18 and 19).
J	Load distribution integral (equation 12).
K	A constant.
$l$	Length of rollers.
P	Load on the rolling elements.
$S_f$	Load sharing factor.
t	A constant.
Z	Number of rolling elements.
$\delta$	Deflection of bearing or rolling elements.
$\epsilon$	Extent of loaded zone factor.
$\psi$	Angular position of rolling elements.

## Subscripts

1, 2, j	Rolling element number.
$l$	Refers to the loaded zone.
r	Radial direction.

## INTRODUCTION

Rolling bearing deflections have to be evaluated frequently during the process of optimisation in machine tool design. To save time in this process of evaluating the rolling bearing deflection, it is frequently assumed that they have zero clearance at the expense of accuracy. This is because the calculation of bearing deflection with play or preload involves the solution of a series of simultaneous non-linear equations, using a time consuming iterative process.

When this method has to be repeated frequently for the optimisation of the bearing assembly (e.g. spindle-bearing assembly of a machine tool), the calculation of the bearing deflection takes up the major portion of the time.

The iterative method together with its theoretical back-ground to determine the rolling element load distribution and bearing deflections taking into account the radial play or preload that may be present in the bearing is reviewed in this paper. A method has been presented to prepare a series of nomograms and graphs to solve these equations. Considerable time and labour can be saved by using the nomograms and graphs which are presented for both radial roller and ball bearings.

The nomograms can also be used to examine the effect of play or preload on the radial deflection of bearings at different loads. The effect is shown clearly in this paper by drawing a three dimensional surface for deflection against load and clearance,  $(\delta Fc)_r$ . The choice of best possible operating conditions of the bearings can easily be obtained from this  $(\delta Fc)_r$  surface.

## AN ITERATIVE METHOD FOR BEARINGS WITH PLAY OR PRELOAD.

When a radial rolling bearing is loaded by a radial load  $F_r$  the inner ring will deflect from centered position in the outer ring by  $\delta_r$  and deform the rolling elements in the loaded zone  $\pm \psi_l$  as shown in figure 1. The deflection at the peak rolling element position (for which  $\psi = 0$ ), is given by

$$\delta_l = \delta_r - c \quad (1)$$

The deflection at any rolling element

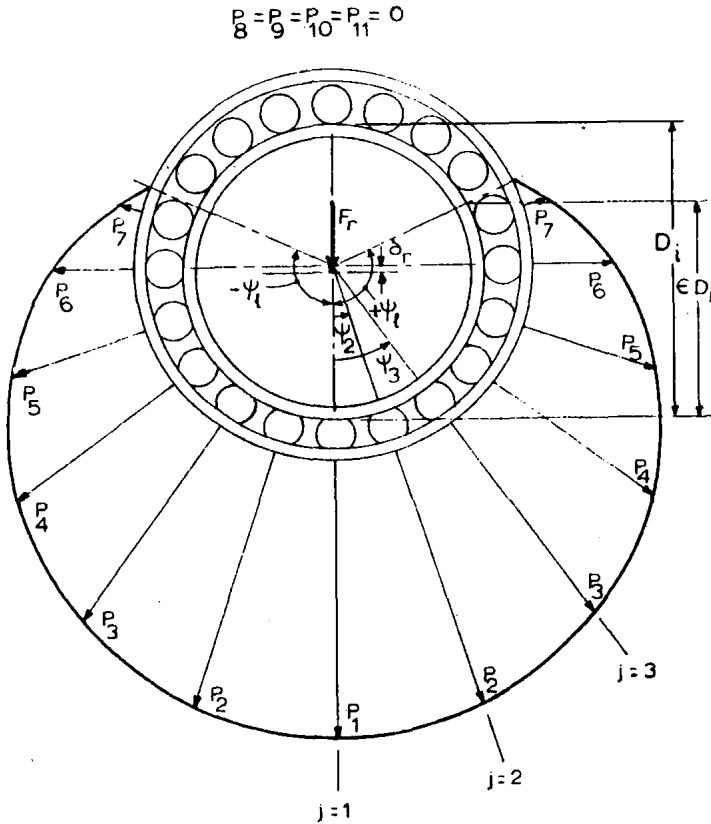


Figure 1. Roller load distribution in a radial rolling bearing.

position  $j$  is given by

$$\delta_j = \delta_r \cos \psi_j - c \quad (2)$$

where  $c$  is the radial clearance in the bearing.

Equation (2) can be rearranged using equation (1) as follows

$$\delta_j = \delta_1 \left[ 1 - \frac{1}{2\epsilon} (1 - \cos \psi_j) \right] \quad (3)$$

$$\text{where, } \epsilon = \frac{1}{2} \left( 1 - \frac{c}{\delta_r} \right) \quad (4)$$

$\epsilon$  depends on the extent of loaded zone as shown in figure 1, and is the ratio of the projected load zone ( $\epsilon D_i$ ) on the inner race diameter to the inner race diameter ( $D_i$ ). For positive clearance bearing  $\epsilon < 0.5$  and for negative clearance bearing  $\epsilon > 0.5$ .

If,  $\epsilon < 1$ , then the loaded zone covers only part of the circumference,  
if,  $\epsilon \geq 1$ , then the whole circumference is loaded,  
if,  $\epsilon = 0.5$ , then half the circumference is loaded (as for zero-clearance bearings).

The angular extent of loaded zone can be found from equation (2) by putting  $\delta_j = 0$

$$\psi_\ell = \cos^{-1} \left( \frac{c}{\delta_r} \right) \quad (5)$$

The load-deflection relationship for a rolling element loaded between the races can be expressed as [1] follows,

$$P = K \delta^t \quad (6)$$

where  $K$  is a constant depending on the material and geometry of the contacting bodies and  $t$  represents the non-linearity of the load vs. deflection curve. For point contact  $t = 1.5$  and for line contact  $t = 1.11$ .

From equation (6)

$$\frac{P_j}{P_1} = \left( \frac{\delta_j}{\delta_1} \right)^t \quad (7)$$

Substituting equation (7) in equation (3)

$$P_j = P_1 \left[ 1 - \frac{1}{2\epsilon} (1 - \cos \psi_j) \right]^t \quad (8)$$

For equilibrium, summation of the vertical components of the rolling element loads must be balanced by the radial load

$$F_r = \sum_{\psi = -\psi_\ell}^{+\psi_\ell} P_j \cos \psi_j \quad (9)$$

Equation (9) can also be written in the integral form

$$\begin{aligned} F_r &= \int_{-\psi_\ell}^{+\psi_\ell} \frac{P_j}{\pi D_i} \times \frac{D_i}{2} \cos \psi d\psi \\ &= Z P_1 \times \frac{1}{2\pi} \int_{-\psi_\ell}^{+\psi_\ell} \left[ 1 - \frac{1}{2\epsilon} (1 - \cos \psi) \right]^t \times \cos \psi d\psi \quad (10) \end{aligned}$$

$$\text{or } F_r = Z P_1 J_r(\epsilon) \quad (11)$$

$$\text{where, } J_r(\epsilon) = \frac{1}{2\pi} \int_{-\psi_\ell}^{+\psi_\ell} \left[ 1 - \frac{1}{2\epsilon} (1 - \cos \psi) \right]^t \times \cos \psi d\psi \quad (12)$$

The values of the load distribution integrals are given in table 1 for line contact ( $t = 1.11$ ) and point contact ( $t = 1.5$ ).

From equation (1), (6) and (11)

$$F_r = ZK(\delta_r - c)^t J_r(\epsilon) \quad (13)$$

When  $F_r$  and  $c$  for a radial rolling bearing are known, the bearing radial deflection  $\delta_r$  can be found by trial as follows:

- assume a value of  $\epsilon$ ,
- find the value of  $J_r(\epsilon)$  from table 1, if necessary, by interpolation.
- substitute these values in equation (13), and calculate the value of  $\delta_r$ ,
- calculate the value of  $\epsilon$  from equation (4) with the new value of  $\delta_r$ ,
- if the new value of  $\epsilon$  is different from that assumed before, repeat the process with the new value of  $\epsilon$  or some other more reasonable value of  $\epsilon$ .

#### EXAMPLE

It is required to find the radial deflection of a bearing having  $Z = 25$  rollers, each  $\ell = 11.4$  mm long, a radial interference of  $c = -0.015$  mm, under a radial load,  $F_r = 17.8$  kN.

$$F_r = ZK(\delta_r - c)^t J_r(\epsilon) \quad (13)$$

Taking the value of  $K$  from equation (20) given later and the data given for  $Z$ ,

TABLE 1. Values of load distribution integral  $J_r(\epsilon)$  for radial rolling bearings with line contact and that with point contact.

$\epsilon$	Line Contact	Point Contact
0	1/Z	1/Z
0.1	0.1268	0.1156
0.2	0.1737	0.1590
0.3	0.2055	0.1892
0.4	0.2286	0.2117
0.5	0.2453	0.2288
0.6	0.2568	0.2416
0.7	0.2636	0.2505
0.8	0.2658	0.2559
0.9	0.2628	0.2576
1.0	0.2523	0.2546
1.25	0.2078	0.2289
1.67	0.1589	0.1871
2.5	0.1075	0.1339
5.0	0.0544	0.0711
$\infty$	0	0

$\epsilon$ ,  $c$  and  $F_r$  the above equation reduces to

$$F_r = 17800 = 7.83 \times 10^6 (\delta_r + 0.015)^{1.11} \quad (13a)$$

$$\epsilon = \frac{1}{2} \left( 1 - \frac{c}{\delta_r} \right) \quad (4)$$

$$= \frac{1}{2} \left( 1 + \frac{0.015}{\delta_r} \right) \quad (4a)$$

#### Iteration number 1

- Assume the initial value for  $\epsilon = 1.0$
- From table 1,  $J_r(1.0) = 0.2523$ ,
- Substituting this value of  $J_r(\epsilon)$  in equation (13a) the value of  $\delta_r$  obtained is

$$\delta_r = -0.000631 \text{ mm}$$

$\delta_r$  cannot be negative. So the process is repeated with a new value of  $\epsilon$ .

#### Iteration number 2

- Assume,  $\epsilon = 2.0$ ,
- From table 1 by interpolation,  $J_r(2.0) = 0.1385$
- From equation (13a) with this value of  $J_r$ ,  $\delta_r = 0.00966 \text{ mm}$
- From equation (4a) with this value of  $\delta_r$ ,  $\epsilon = 1.276$ ; which is not the same as  $\epsilon = 2.0$  assumed earlier.

#### Iteration number 3

- Assume,  $\epsilon = 1.5$ ,
- From table 1 by interpolation,  $J_r(1.5) = 0.1787$
- From equation (13a) with this value of  $J_r$ ,  $\delta_r = 0.00461 \text{ mm}$
- From equation (4a) with this value of  $\delta_r$ ,  $\epsilon = 2.127$ .  $2.127 \neq 1.5$ .

#### Iteration number 4

- Assume,  $\epsilon = 1.8$ ,
- From table 1 by interpolation,  $J_r(1.8) = 0.1508$
- From equation (13a) with this value of  $J_r$ ,  $\delta_r = 0.00784 \text{ mm}$
- From equation (4a) with this value of  $\delta_r$ ,  $\epsilon = 1.457$ .  $1.457 \neq 1.8$ .

#### Iteration number 5

- Assume,  $\epsilon = 1.65$ ,
- From table 1 by interpolation,  $J_r(1.65) = 0.1612$ ,
- From equation (13a) with this value of  $J_r$ ,  $\delta_r = 0.00651 \text{ mm}$
- From equation (4a) with this value of  $\delta_r$ ,  $\epsilon = 1.652$ , which is nearly equal to the assumed value of  $\epsilon = 1.65$ .

Obviously, this method needs a number of iterations for convergence. To save time and labour the deflection of a radial rolling bearing can be found, when  $F_r$ ,  $c$ ,  $Z$  and rolling element dimensions are given, from the nomograms given in the later sections. Two sets of nomograms (one for line contact and the other for point contact) were constructed using procedure given in the next section. Considerable time and labour can be saved by using these nomograms. If very high accuracy is required, the values of  $\epsilon$  obtained from them may be used as the first estimate in the iterative method. This will reduce the number of successive approximations.

After finding  $\delta_r$  the load distribution in the bearing is then evaluated from equations (2) and (6).

#### GRAPHICAL SOLUTION FOR RADIAL ROLLING BEARINGS

The deflection of radial rolling bearings, even with the assumption of rigid races, depends on factors such as bearing geometry, number of rollers, material, external radial load and clearance as shown in the previous section. Construction of nomograms for solving the equations (4) and (13) simultaneously to get the deflection of rolling bearing can be achieved as follows:

Using equation (4), equation (13) can be written for positive clearance as,

$$\frac{F_r}{B} = c^t \left[ \left( \frac{2\epsilon}{1-2\epsilon} \right)^t J_r(\epsilon) \right] \quad (14)$$

and for negative clearance as,

$$\frac{F_r}{B} = (-c)^t \left[ \left( \frac{2\epsilon}{2\epsilon-1} \right)^t J_r(\epsilon) \right] \quad (15)$$

$$\text{where, } B = Z \times K \quad (16)$$

Equation (14) and (15) can be written as,

$$F^* = f_1(c) \times f_2(\epsilon) \quad (17)$$

$$\text{where, } F^* = \frac{F_r}{B} \quad (18)$$

and  $f_1$  and  $f_2$  are functions of  $c$  and  $\epsilon$  respectively.

## DEFLECTIONS OF RADIAL ROLLING BEARINGS

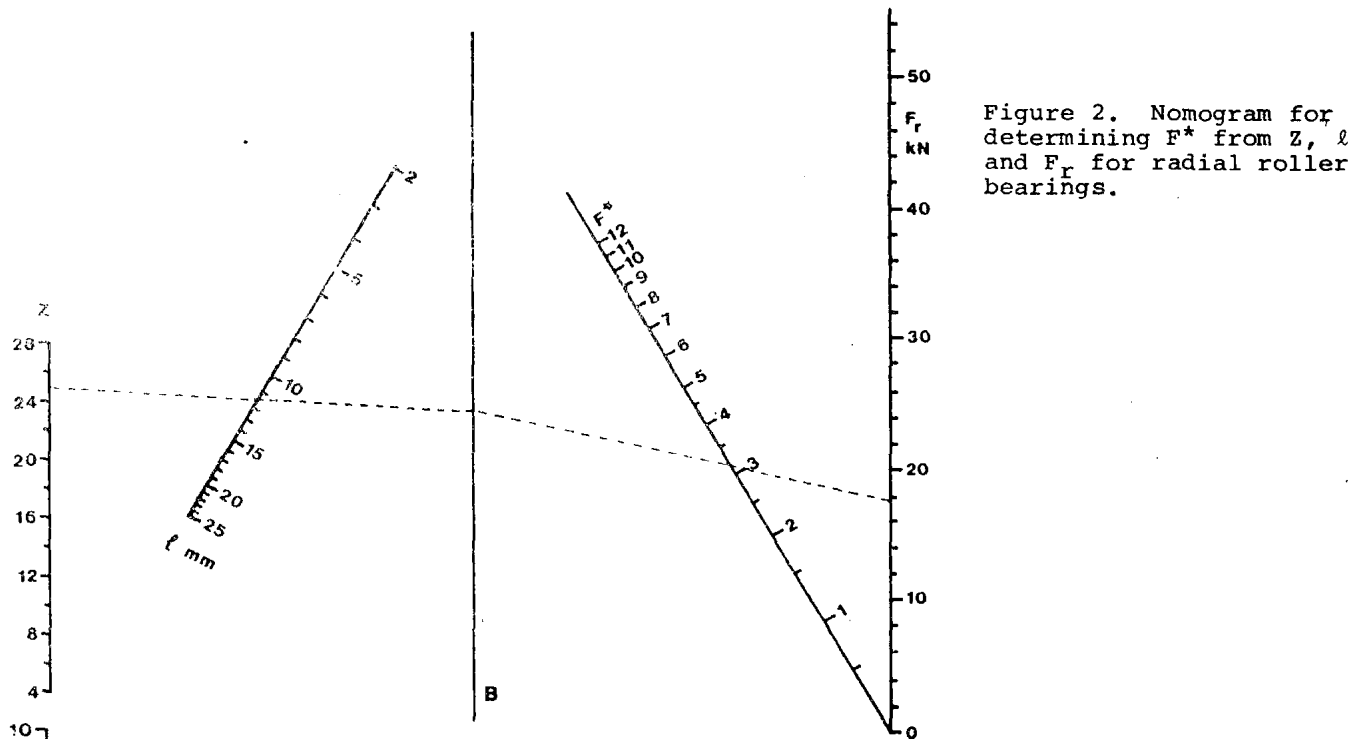


Figure 2. Nomogram for determining  $F^*$  from  $Z$ ,  $l$  and  $F_r$  for radial roller bearings.

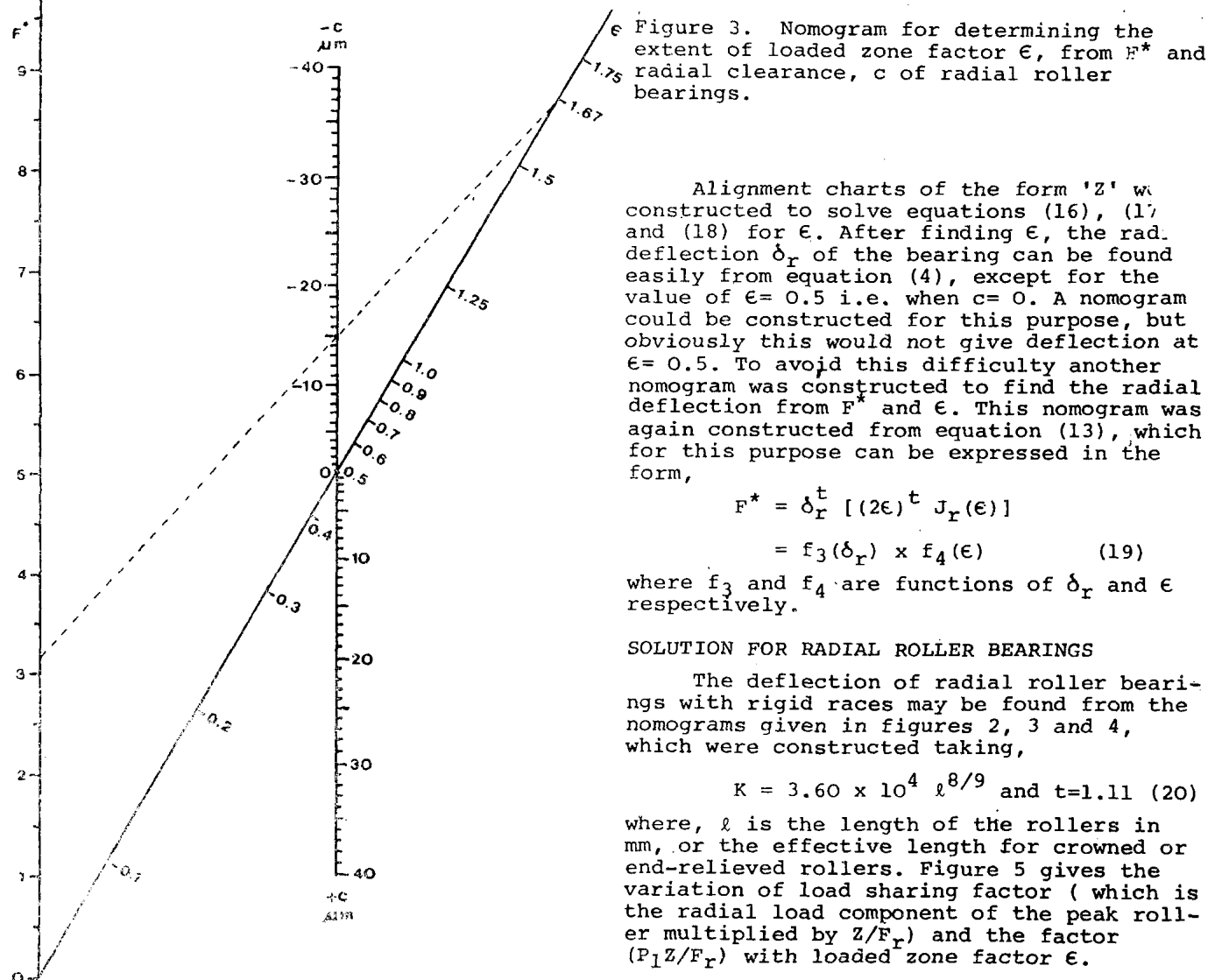


Figure 3. Nomogram for determining the extent of loaded zone factor  $\epsilon$ , from  $F^*$  and radial clearance,  $c$  of radial roller bearings.

Alignment charts of the form 'Z' were constructed to solve equations (16), (17) and (18) for  $\epsilon$ . After finding  $\epsilon$ , the radial deflection  $\delta_r$  of the bearing can be found easily from equation (4), except for the value of  $\epsilon = 0.5$  i.e. when  $c = 0$ . A nomogram could be constructed for this purpose, but obviously this would not give deflection at  $\epsilon = 0.5$ . To avoid this difficulty another nomogram was constructed to find the radial deflection from  $F^*$  and  $\epsilon$ . This nomogram was again constructed from equation (13), which for this purpose can be expressed in the form,

$$F^* = \delta_r^t [(2\epsilon)^t J_r(\epsilon)]$$

$$= f_3(\delta_r) \times f_4(\epsilon) \quad (19)$$

where  $f_3$  and  $f_4$  are functions of  $\delta_r$  and  $\epsilon$  respectively.

## SOLUTION FOR RADIAL ROLLER BEARINGS

The deflection of radial roller bearings with rigid races may be found from the nomograms given in figures 2, 3 and 4, which were constructed taking,

$$K = 3.60 \times 10^4 \ell^{8/9} \text{ and } t=1.11 \quad (20)$$

where,  $\ell$  is the length of the rollers in mm, or the effective length for crowned or end-relieved rollers. Figure 5 gives the variation of load sharing factor (which is the radial load component of the peak roller multiplied by  $Z/F_r$ ) and the factor  $(P_1 Z/F_r)$  with loaded zone factor  $\epsilon$ .

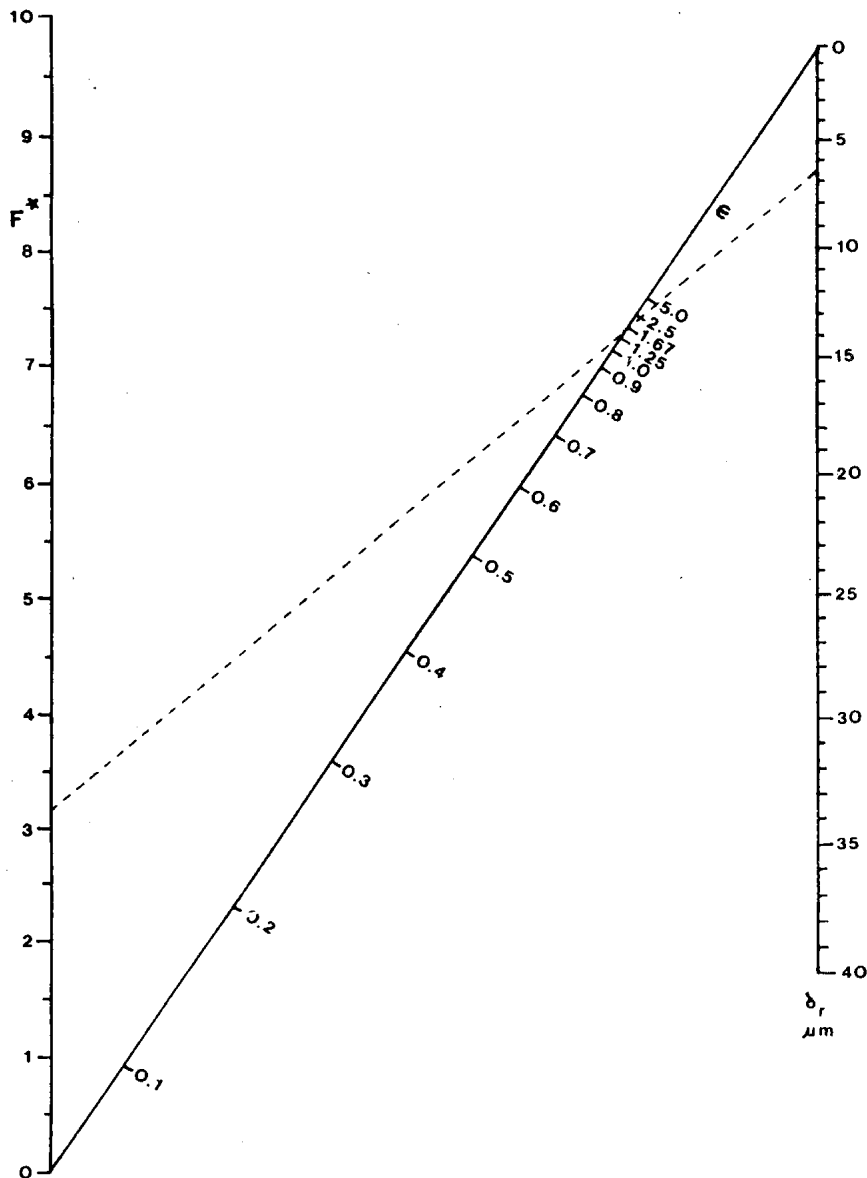


Figure 4. Nomogram for determining the radial deflection  $\delta_r$  of radial roller bearings from  $F^*$  and the extent of loaded zone factor  $\epsilon$ .

Note:  $\delta_r$  is the radial deflection of a bearing given by the deflection of the inner race from centered position in the outer race. For positive clearance bearings, to get the radial deflection after the peak roller is loaded the radial clearance  $c$  is to be subtracted from  $\delta_r$  obtained from the nomogram.

#### Sample Solution

The example involving the radial roller bearing given for the iterative process is solved again using the nomograms.

1. Enter the nomogram in figure 2 with the values of  $Z$ ,  $l$  and  $F_r$ , joining these points with straight lines to meet at a common point on line B, as shown. Obtain  $F^* = 3.2$ , in this case.

2. Enter the nomogram in figure 3 with values of  $F^*$  and  $c$ , obtaining  $\epsilon = 1.65$ .

3. Enter the nomogram in figure 4 at previously determined values of  $F^*$  and  $\epsilon$ , to obtain the radial deflection  $\delta_r = 0.0065$  mm. This deflection is that of the inner race from the centered position within the outer race.

The total peak roller load and the radial load component on the peak roller can now be found. Read the values of  $(P_1 Z / F_r)$  and  $S_f$  corresponding to the value of  $\epsilon$  from figure 5. In this case they are 6.25 and 2.05 respectively.

Thus, total peak roller load =

$$P_1 = \frac{6.25 \times 17.8}{25} = 4.45 \text{ kN}$$

Radial load component on the peak

$$\text{roller} = \frac{2.05 \times 17.8}{25} = 1.46 \text{ kN.}$$

#### SOLUTION FOR RADIAL BALL BEARINGS

The value of  $K$  taken in the construction of nomograms for this case is for bearings with standard groove radii (i.e. inner race groove radius at 52% of the ball diameter, and outer race groove radius not greater than 53% of ball diameter) and is approximated by

$$K = 9.79 \times 10^4 d^{1/2} \text{ and } t = 1.5 \quad (21)$$

where  $d$  is the diameter of the ball.

The graphical solution is presented by nomograms in figures 6 to 8 and curves of figure 5.