ELECTRONIC CIRCUIT ANALYSIS FOR SCIENTISTS

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JOHN WILEY & SONS
New York London Sydney Toronto

This book is dedicated to our wives, Diane McCray and Virginia Cahill

Library of Congress Cataloging in Publication Data

McCray, James A 1932–
Electronic circuit analysis for scientists.

Bibliography: p.
1. Electronic circuits. 2. Electric networks.
I. Cahill, Thomas A., 1937– joint author.
II. Title.

TK7876.M3 621.3815'3 72-8986
ISBN 0-471-58295-6

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

PREFACE

When the fact that the majority of students studying physics, chemistry, and biology will eventually be engaged in experimental work is considered, it becomes evident that some early training in electronics is essential. However, the past attempts to satisfy this need have not produced the desired effect—namely the ability to quickly analyze, understand, and use electronic equipment intelligently and effectively. One of the basic problems, of course, is the limited amount of time that a student may devote to such a study.

Many good courses in electronics are offered in electrical engineering departments. The problem with these courses is that they are often part of a carefully designed program for electrical engineers and require extensive prerequisites. Also, they often involve material not really useful to students not planning a career in electronics.

One way around this problem is to offer courses in electronics as part of the curriculum in the physical or biological sciences. These courses unfortunately often become merely a survey of electronics, which, while introducing a student to a wide variety of circuits and techniques, never really allow him to do the calculations required by his particular research problems. Physicists have their own problems, in that electronics courses often become too involved in the physics of the real devices encountered which, although admittedly interesting, are not apt to help the student in a problem in the laboratory. Occasionally the courses provide a mechanism for introducing a little applied mathematics, which again may or may not help the student solve a circuit or connect two pieces of equipment together without engendering smoke.

It is necessary to resist the above temptations and design an electronics course which concentrates on circuit analysis and the practical use of electronic equipment. The physics of devices may be discussed outside the main lectures, in the laboratory, etc., and references can be given to the many excellent texts on physical electronics.

Such a course has been designed and given for the last eight years by the authors to students of physics, chemistry, biology, and applied science.

Usually the students were seniors or graduate students. The text is designed for and has been used in a one-quarter course with three lecture hours and one three-hour laboratory per week. The course has then been followed by a one-lecture hour, one three-hour laboratory course in the second quarter in which the students design, build, and operate their own circuits and electronics systems.

It has been our experience that most of the students enjoyed the course, and were no longer "afraid" of electronics. Those students going on to graduate school have demonstrated a definite ability to work with electronics, while those terminating at the Bachelor's level find their background in electronics to be a real asset in obtaining a position.

What we have tried to do in this text is to bridge the gap between the more sophisticated science student and the electronics engineer. By "sophisticated" we mean having some familiarity with calculus, some notion of ordinary differential equations, and the kind of physics or chemistry background offered by a reasonable introductory course. We hope that students who have studied this text will be able to read the vast electronics literature available and will be able to communicate profitably with the electronics people with whom they will come into contact.

The authors would like to acknowledge the important role which our students have played in the development of the text. Particular mention must be made of the teaching assistants, Steve Brooks, Gary Smith, Don McCauley, Stan Johnson, Bob Eldred, and Mike Ellison, especially with regard to the laboratories. Another student, George Ellis, drafted all the diagrams and waveforms for the text. We would also like to thank Ginny Cahill, whose extensive editorial assistance helped to prepare the manuscript for publication. Finally, we would like to acknowledge the support of the editors of John Wiley & Sons, especially Gary Brahms and Don Deneck, whose help and encouragement made this book possible.

James A. McCray
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NOMENCLATURE

(1) V, I, Q, P

These quantities are constant or direct current (dc) values except in situations in which an alternating current (ac) is superimposed on top of a direct current. In these cases, they are total values for voltage, current, charge, and power.

(2) v, i, q, p

These quantities represent deviations from dc values, usually in time. $dV/dt = d(V_{DC} + \Delta V)/dt = dv/dt$. Equations in these variables are called "ac" or "signal" equations.

(3) $\bar{V}, \bar{I}, \bar{Q}, \bar{P}$

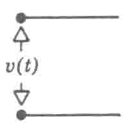
These quantities always represent the Laplace transforms of v, i, q, or p, and may or may not be written explicitly as $\bar{V}(s)$, $\bar{I}(s)$, etc.

(4) 

This symbol represents an ideal voltage source, one whose internal impedance is zero.

(5) 

This symbol represents an ideal current source, one whose internal impedance is infinite.

(6) 

This nomenclature merely indicates the voltage between the two points.

(7) Q

Q is unfortunately used universally for four quantities: 1. Charge. 2. "Q" of a circuit. 3. Operating point Q. 4. Transistor label Q.

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ANALYSIS OF PASSIVE ELEMENTS
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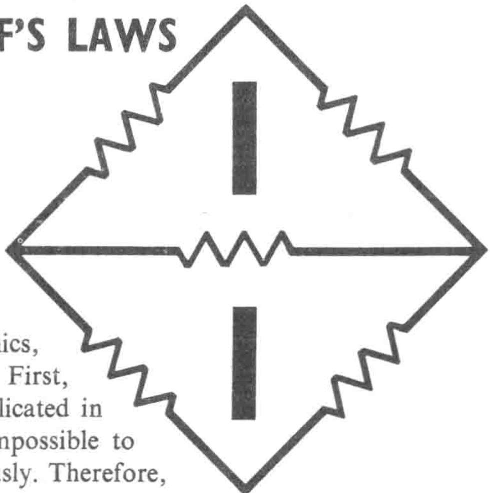

INTRODUCTION

To have a working model of electronics, two problems must be surmounted. First, real devices and circuits are so complicated in an exact analysis that it would be impossible to treat all but the simplest cases rigorously. Therefore, basic models must be assumed for the various components of a real circuit, and these models must be justified on the basis of simplicity and ability to predict that which is observed in the laboratory. Second, once models are assumed for the components, methods must be developed to solve the integro-differential equations that arise when the elements are combined in circuits and subjected to realistic input conditions. A third point might also be raised: that in one book, or even a set of books, only a small fraction of the vast field of electronics can be covered. We feel strongly that the inevitable selection of topics should favor those that can help a scientist in the laboratory make best use of the tremendous resources of electronics.

To handle the first problem, we have used a *lumped parameter model* for passive elements; resistance, capacitance, and inductance. When this model is insufficient, as in the case of RF transmission lines, modifications are made that cover the deviations. For active elements, we have used a linear approximation for the first nine chapters, and have divided active elements into those possessing high input impedance (tubes, FET's, etc.) and low input impedance (transistors). This division introduces simplifications in the analysis and leads naturally to voltage and current devices.

To handle the second problem, we have not relied upon previous knowledge of differential equations beyond a basic familiarity with integrals and differentials. This is not enough, for as circuits become realistic, the equations become horrendous. Therefore, as early as possible in the second chapter, we introduce *Laplace transforms*, which reduce coupled integro-differential equations to simple algebra. We then use the *dc Circuit Scheme* with the Laplace transforms, and the result is a method that allows a scientist to solve real circuits and find output voltages without ever setting up the coupled equations. We do this by avoiding the temptation to do contour integrals (which we admit is fun) by using tables of inverse transforms. Finally, we show that by using *pole-zero plots*, even the inverse transform tables can be avoided in many occasions. We have found that our students have thus gained both a real facility with electronics and an understanding of circuits that would be extremely hard to gain any other way. These methods have, of course, been known to electrical engineers for some time, but they have not been widely used on the level of a physical or biological scientist.

To handle the third problem, we have selected topics both by their utility to a scientist in a modern laboratory and by their contribution to an understanding of circuits. To maintain this book within limits set by a single quarter of instruction, we have deleted almost all discussion of the physics of devices. All that is left is what we feel is a bare minimum required for appreciation of the simple models that we use. However, many good books exist on all levels of sophistication for the physical analysis of real devices, and we refer to these, listed in the Bibliography, for the details. Also, this material can be added by the instructor in his lectures or in the laboratory at exactly the level that suits his class.

KIRCHHOFF'S LAWS

As we have discussed in the introduction, we will use the lumped parameter model for the passive elements: resistance, capacitance, and inductance.

We base our model on the following results, which are assumed valid for ideal lumped parameter components:

1. The energy dissipated in a resistor is

$$W_R = R\int_0^t i^2\,dt,$$

where R is the *resistance* and i is the *current* passing through the resistor.

2. The energy stored in a capacitor (potential energy) is

$$W_C = \frac{1}{2}\frac{q^2}{C},$$

where C is the *capacitance* and q is the *charge* on the capacitor.

3. The energy stored in an inductor ("kinetic energy") is

$$W_L = \tfrac{1}{2}Li^2,$$

where L is the *inductance.*

The physical laws which apply to electronic circuits are the *conservation of charge* and the *conservation of energy*. The first conservation law leads to Kirchhoff's first law, which may be written

At a node
Σ (current in) $=$ Σ (current out).

The second conservation law leads to Kirchhoff's second law, which may be written

Around a closed loop
Σ (voltage sources) $=$ Σ (voltage drops).

To illustrate the second conservation law, we consider the single loop circuit of Fig. 1-1.

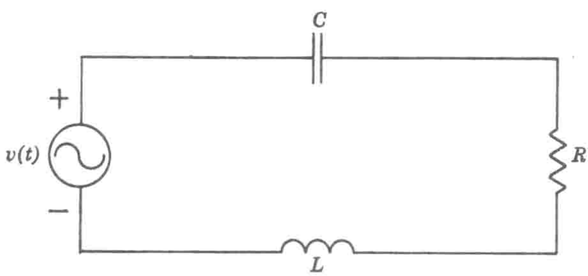

FIGURE 1-1. Series *RCL* circuit.

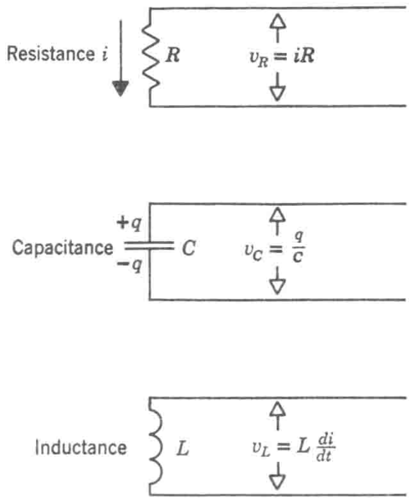

FIGURE 1-2. Voltage drop with resistive, capacitive, and inductive elements.

The energy put into the circuit from the voltage source is given by

$$W = \int_0^q v(t)\, dq = \int_0^t vi\, dt.$$

Conservation of energy requires that

$$W = W_R + W_C + W_L,$$

or

$$\int_0^t vi\, dt = \int_0^t i^2 R\, dt + \frac{q^2}{2C} + \frac{L}{2} i^2.$$

If we differentiate this equation with respect to time and cancel a common factor i, we have

$$v(t) = iR + \frac{q}{C} + L\frac{di}{dt},$$

or

$$v(t) - L\frac{di}{dt} = \frac{q}{C} + iR.$$

$$\begin{pmatrix}\text{External}\\ \text{voltage}\\ \text{source,}\\ \text{or e.m.f.}\end{pmatrix} + \begin{pmatrix}\text{Back}\\ \text{e.m.f.}\\ \text{due to}\\ \text{inductance}\end{pmatrix} = \begin{pmatrix}\text{Voltage}\\ \text{across}\\ \text{capacitor}\end{pmatrix} + \begin{pmatrix}iR \text{ drop}\\ \text{across resistor}\\ \text{(Ohm's law)}\end{pmatrix}.$$

We shall work, then, with passive elements having the "voltage drops" shown in Fig. 1-2.

DIFFERENTIATING AND INTEGRATING CIRCUITS

Now let us consider some very simple but useful passive circuits. The first circuit to be considered is the *differentiating circuit;* however, it is also known by such other names as blocking circuit, clipping circuit, high-pass circuit, and lead circuit. This copious supply of names brings out a very important point about electronic circuits. A given configuration of circuit elements may have several functions depending upon the relative values of the circuit parameters and the parameters of the input wave form. Throughout this book, $v_0(t)$ is assumed to be an ideal voltage source; that is, one having zero internal resistance.

In analyzing the circuit of Fig. 1-3, we first assume that the load current $i_2(t)$ is very small compared with the loop current $i(t)$. Application of Kirchhoff's second law yields

$$v_0(t) - \frac{q(t)}{C} - i(t)R = 0,$$

which may be written as a differential equation for $q(t)$:

$$\frac{dq}{dt} + \frac{1}{RC}\,q = \frac{v_0(t)}{R}.$$

Let us initially solve the above equation by standard techniques of differential equations.* The general solution is the sum of the homogeneous solution and the particular solution:

$$q(t) = q_h(t) + q_p(t).$$

The particular solution will yield the long-term (steady-state) response to an input while the homogeneous solution will give the decaying (transient) behavior.

For the homogeneous part, we have

$$\frac{dq_h}{dt} + \frac{q_h}{RC} = 0$$

with solution

$$q_h(t) = Ae^{-t/RC}$$

where A is a constant determined by initial conditions.

* See any good reference to differential equations, such as I. S. Sokolnikoff and R. M. Redheffer, *Mathematics of Physics and Modern Engineering*, McGraw-Hill, New York (1966), Chapters 2 and 3.

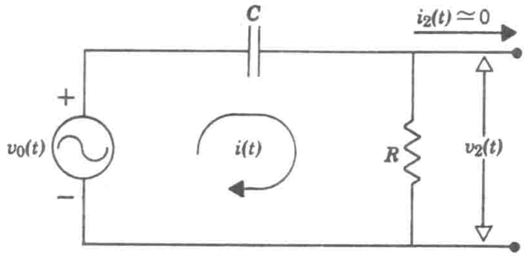

FIGURE 1-3. *RC* differentiating circuit.

The particular solution may be easily found by the *method of undetermined coefficients*, which applies when the right-hand side of the differential equation contains only terms from which a finite number of terms can be obtained by differentiation. We use as a trial function for the particular solution, the inhomogeneous term, plus all of its derivatives. We find it useful to obtain the response $v_2(t)$ of a given circuit to a step input $v_0(t) = Vu(t)$, where

$$u(t) = \begin{cases} 1 & 0 < t \\ 0 & t \leq 0 \end{cases}$$

is the *unit step function*, and to a sinusoidal input $v_0(t) = \text{V} \sin(\omega t + \phi)$, where V is the *amplitude*, ω the angular *frequency*, and ϕ the *phase* of the input sinusoidal waveform. The response to the step input tells us how fast a circuit can respond to a sudden discontinuity in voltage, and the response to a sinusoidal input gives us the circuit's steady-state characteristics.

For a step input, the differential equation is

$$\frac{dq}{dt} + \frac{q}{RC} = \frac{V}{R}.$$

For the particular solution, we have as a trial function

$$q_p(t) = B \text{ (a constant)}.$$

Substitution of this trial solution into the differential equation yields the value of $B = CV$. The solution is

$$q(t) = Ae^{-t/RC} + CV.$$

We now determine A by the initial condition $q(0) = q_0$. Then

$$q(t) = (q_0 - CV)e^{-t/RC} + CV.$$

The response of the differentiating circuit to a step input is

$$v_2(t) = i(t)R.$$

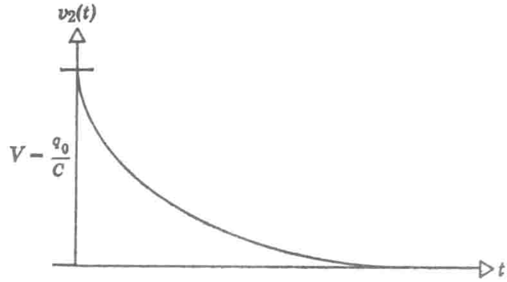

FIGURE 1-4. Output of an *RC* differentiating circuit for an input step function.

Or since $i(t) = dq/dt$,

$$v_2(t) = \left(V - \frac{q_0}{C}\right) e^{-t/\tau},$$

where $\tau = RC$ is the *time constant* of the circuit, the most important circuit parameter. For the case we are considering (Fig. 1-4), the signal falls to about $\frac{1}{3}$ of its initial value in time τ. This is characteristic of an exponential decay.

Actually, we are more interested in the response of the circuit to a pulse, so let us take for $v_0(t)$ the *rectangular pulse* of height V and width T_0 (Fig. 1-5). We may solve this problem by considering different time intervals. Let us assume that initially there is no charge on the capacitor. We have the solution for time interval I:

$$v_2(t) = Ve^{-t/\tau} \qquad 0 \leq t \leq T_0.$$

In time interval II, the differential equation is

$$\frac{dq}{dt'} + \frac{1}{\tau} q = 0,$$

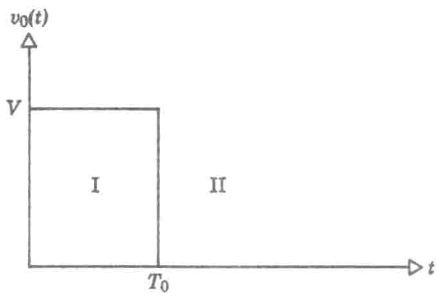

FIGURE 1-5. Rectangular pulse.