Second Edition

OPERATIONS RESEARCH

PRINCIPLES AND PRACTICE

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PREFACE

We are deeply gratified by the enthusiastic response given to the first edition of our book by our colleagues and students. We took great care in preparing the second edition. We began by sending a detailed questionnaire describing the planned revisions by chapter to all the schools that used our text. An excellent response was received with many constructive comments. These responses were extremely valuable to us during the preparation of the second edition. We added some new material, included more applications in all the chapters, revised the explanation and presentation of some topics, increased the number of exercises, and achieved a greater independence of chapters as far as is possible.

Examples of new materials include a complete new chapter on *decision analysis* (Chapter 5), the computer solution of linear programming problems, the use of sensitivity analysis output in Chapter 2, the addition of *minimal spanning tree* in Chapter 3, a new section on *goal programming—theory, algorithm and applications* in Chapter 4, the addition of *network of queues* in Chapter 7, a discussion of microcomputer based simulation languages and their applications in Chapter 9, and the inclusion of second order gradient based optimization techniques in Chapter 11.

An example of effective independence of chapters is the treatment of *queueing models* (Chapter 7). In the first edition, this discussion required the knowledge of *Markov Processes* discussed in Chapter 6. In the second edition, queueing models, their steady-state equations and relevant results are derived independently of Markov Processes. This change is in direct response to the comments from the instructors who have used the text.

To make room for the new chapter on decision analysis (Chapter 5), the probability review has been moved to Appendix B. In response to the comments received on the questionnaire, more applications have been added throughout the book, and more "drill-type" exercises have been included. In addition, the exercises at the end of each chapter have been rearranged so that they begin with short-answer review questions on principles, are followed by simple formulation and drill-type numerical problems, and end with more difficult "mind expanding" exercises.

A new feature of the second edition is a companion microcomputer software for several of the OR techniques discussed in the text and a diskette that will be provided free to all the adopters of our book. The software has been written by

Professor Jorge Haddock of Clemson University. Instructors will be given permission to copy the diskette and the manual for student distribution. A detailed solutions manual is also available to the instructor.

Our principal objective is to present the material in a way that would immediately make sense to a beginning student. Often this required a juxtaposition of what might otherwise be regarded as the natural ordering of general theory followed by specific examples. We have observed that beginners rely heavily on examples and will understand the theory far more easily if it is presented as a generalization of one or more specific examples. It seems that, for the most part, students taking operations research courses have had adequate preparation in the mechanics of calculus, linear algebra, and probability. What they have greatest difficulty with is formulation and interpretation. That is, they can "do" the mathematics, but frequently do not understand the meaning of what they are doing. Hence, we have found it helpful to include quite a bit of verbal explanation of mathematical material. Purists may object to the liberties we have taken in providing loose and imprecise statements of perfectly well-defined and precise mathematics, but our embarrassment in so doing is overcome by our conviction that the student needs that kind of help. So although our presentation is far less concise and elegant than we could have made it, we feel strongly that the extra verbiage pays dividends.

Notwithstanding our desire to improve the readability of the material, we felt that it was important to keep the size of the book down. The sheer bulk of some textbooks, particularly when you consider their density, is overwhelming to beginning students. When it takes a student an hour to digest what is written on one page and the book is a thousand pages long, it is not surprising that he or she would feel disheartened. It is a lot easier for students to stay motivated, especially in the early stages, if they can sense that they are making substantial progress. Of course, there *is* a great deal to be learned and, as we assembled this book, we began to understand how our predecessors could have ended up with so much material. We too experienced an almost compulsive urge to include more. Even with merciless editing (which involved removing several major topics that happened to be personal favorites of the authors) the book turned out to be longer than we had originally intended. Nevertheless, we feel that it is possible to cover most of this book in one academic year.

To avoid creating the impression that our treatment of these subjects is conclusive, and to open the door to sources of additional information, we have provided selected references at the end of each chapter. We have also provided a few bibliographic comments to guide readers in their research. Obviously, these brief bibliographies are only starting points, which in turn will lead to further sources.

Throughout the text, we have kept the use of higher mathematics at an intermediate level. For example, Chapters 1 through 4 on linear programming and networks require only linear algebra. Chapters 5 through 10 require a basic knowledge of probability. We have included in Appendix B the basic concepts on probability that are necessary. Although the treatment in Appendix B on probability is reasonably complete, it is probably too concise to serve as an introduction to the topic by itself.

Individual instructors will, of course, exercise their judgment to select and to rearrange certain material. Because we know this will happen, we decided at the outset that a highly integrated text would be undesirable. Thus we have deliberately sought to maintain independence among chapters to permit maximum flexibility in their use. We have used Chapters 2, 3, and 4 for a graduate level, introductory course on linear programming. For a similar undergraduate course, we omit Chapter 4. For a graduate level course on probabilistic models of OR, we use Chapters 6 to 8 and parts of 9 and 10. For the corresponding undergraduate course, we attempt to cover the same topics, but must delete some of the more specialized material in each chapter. Some of our colleagues have told us that they use Chapters 2, 3, 7, 8, 9, and 10 for a survey course in operations research. Some have also used Chapters 2, 3, 4, 10, and 11 for an overview course on optimization techniques. Teachers being an individualistic lot, we are confident that instructors will find unique ways to adapt this book to suit their own needs as they perceive them.

It is perhaps obvious that the authors are indebted to the many researchers who have developed the underlying concepts that permeate this text. Although far too numerous to mention, we have tried to recognize their contributions through bibliographic references at the end of each chapter. In addition to the above, several individuals have directly contributed to the composition of the second edition. We are specifically indebted to Dr. Herbert Moskowitz of Purdue University for writing the chapter on decision analysis and to Dr. Jorge Haddock of Clemson University for developing the companion microcomputer software for the book. Special thanks are owed to Dr. J. W. Schmidt, C.B.M., Inc., for the third example in Chapter 9; Dr. R. S. Schecter of the University of Texas; and to Dr. Charles Beightler, Dr. R. M. Crisp, and Dr. W. L. Meier for the material on geometric programming in Chapter 11. The pleasant personality and excellent typing skills of Patti Coffill at the University of Oklahoma made it easier to revise several parts of the book. Finally, we are grateful to the instructors who have adopted our first edition and for their encouragement and helpful suggestions that made the second edition a reality.

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CHAPTER 1
THE NATURE OF OPERATIONS RESEARCH

1.1 THE HISTORY OF OPERATIONS RESEARCH

In order to understand what operations research (OR) is today, one must know something of its history and evolution. Although particular models and techniques of OR can be traced back to much earlier origins, it is generally agreed that the discipline began during World War II. Many strategic and tactical problems associated with the Allied military effort were simply too complicated to expect adequate solutions from any one individual, or even a single discipline. In response to these complex problems, groups of scientists with diverse educational backgrounds were assembled as special units within the armed forces. Because of the diversity of its membership, one of the earliest groups in Britain came to be known as "Blackett's circus."

Partly because the scientists involved were talented men, partly because of the pressures of wartime necessity, and partly because of the synergism generated from the interactions of different disciplines, these teams of scientists were remarkably successful in improving the effectiveness of complex military operations. Examples of typical projects were radar deployment policies, antiaircraft fire control, fleet convoy sizing, and detection of enemy submarines. By 1941, each of the three wings of the British Armed Forces were utilizing such scientific teams. As the dramatic success of the idea became amply demonstrated, other allied nations adopted the same approach and organized their own teams. Because the problems assigned to these groups were in the nature of military operations, their work became known as operational research in the United Kingdom, and as operations research elsewhere. The American effort, although it began at a later date, produced many fundamental advances in the mathematical techniques for analyzing military problems. For further details of early activities in operations research, an excellent summary is given in Trefethen (5).

After the war, many of the scientists who had been active in the military OR groups turned their attention to the possibilities of applying a similar approach to civilian problems. Some returned to universities and concentrated their efforts on providing a sound foundation for many of the techniques that had been hastily developed earlier, while others devoted renewed efforts to developing new techniques. Many individuals moved into various sectors of the private economy, where they adapted methods developed by others to the unique problems of particular industries.

In terms of applications, the first civilian organizations to seize upon the OR methodology were, generally, large profit-making corporations. For example, petroleum companies were among the first to make regular use of linear programming on a large scale for production planning. It was logical that "big business" would take the lead in adopting OR. To any profit-oriented organization, OR offered a way to obtain a competitive advantage; but in the early years when all OR work was in the nature of basic research, only the large companies could afford it. Later, as researchers began to recognize common categories of problems (inventory, allocation, replacement, scheduling, etc.) and the techniques for dealing with such problems became standardized, smaller companies were able to benefit from the pool of accumulated knowledge without investing heavily in research. With a few notable exceptions—such as the work on traffic control conducted at the New York Port Authority by Leslie Edie and others in the early 1950s—applications of OR in service-oriented industries and in the public sector did not begin to flourish until the mid 1960s. Today, however, service organizations such as banks, hospitals, libraries, and judicial systems recognize that OR can aid in improving the effectiveness with which they deliver their respective services. In addition, federal, state, and local government agencies are using OR, particularly in their planning and policy-making activities. In fact, work on some of these specialized applications of OR has multiplied so rapidly in recent years that subspecialties, based upon the area of application, appear to be developing. Recent operations research conferences have included special sessions on such topics as "OR in community health planning," "OR models of the criminal justice system," "mass transit studies," "travel and tourism," "energy," "education models," and "OR applications in sports."

An important factor in the rapid spread and sustained success of the OR approach to problem solving was the concurrent development of electronic computers. The computer was from the beginning an invaluable tool, enabling the OR analyst to perform otherwise intractable calculations. Indeed, many of the problem-solving methods now regarded as standard would be unthinkably impractical to implement without modern computers. By generating practical uses for increasingly larger and faster machines, OR has both benefited from and contributed to the explosive growth of computer capability that has occurred over the past three and a half decades.

By the early 1950s, civilian OR activities had reached a level of development that began to suggest that a unique discipline was in formation. The Operations Research Society of America (ORSA) was founded in 1952 to serve the professional needs of scientists working in the OR area. A parallel movement resulted, in 1953, in the formation of The Institute of Management Sciences (TIMS). The journals of these two organizations, *Operations Research* and *Management Science*, as well as regular conferences of the members, helped to draw together the many diverse results into some semblance of a coherent body of knowledge.

Beginning about the same time and continuing into the early 1960s, more and more colleges and universities in the United States introduced first individual courses, then whole programs, into their curricula. Graduate programs leading to advanced degrees at both the M.S. and Ph.D level were approved in many major universities. For good or bad, it happened that little uniformity was observed in deciding where within the academic structure OR belonged. Depending on unique development patterns at each institution, OR programs sometimes appeared within departments of industrial engineering, sometimes in business schools, and occasionally in mathematics or economics. In keeping with the original interdisciplinary character of the work, some universities established interdisciplinary committees to administer OR programs; but because such academic "orphans" tend to be inherently unstable, they have usually been either officially or unofficially absorbed into more traditional parts of the university structure. Of course, the parent discipline tends to impart a particular unique characteristic to the OR program, and the consequent lack of uniformity in academic programs has acted against achieving "definition" in the field. Perhaps it is all for the best.

It is interesting to note that the modern perception of OR as a body of established models and techniques—that is, a discipline in itself—is quite different from the original concept of OR as an *activity*, which was performed by interdisciplinary teams. An evolution of this kind is to be expected in any emerging field of scientific inquiry. In initial formative years, there are no experts, no traditions, no literature. As problems are successfully solved, the body of specific knowledge grows to a point where it begins to require specialization even to know what has been previously accomplished. The pioneering efforts of one generation become the standard practice of the next. Still, it ought to be remembered that at least a portion of the record of success of OR can be attributed to its ecumenical nature. It is in the best traditions of the field to adopt the procedures of any discipline that can make a contribution to the solution of the problem at hand. If the initial open-mindedness of OR ever degenerates into orthodoxy, if the methods ever begin to outweigh the objectives, then the field will have lost one of its most vital precepts.

1.2
THE MEANING OF OPERATIONS RESEARCH

From the historical and philosophical summary just presented, it should be apparent that the term "operations research" has a number of quite distinct variations of meaning. To some, OR is that certain body of problems, techniques, and solutions that has been accumulated under the name of OR over the past 30 years, and we apply OR when we recognize a problem of that certain genre. To others, it is an activity or process—something we do, rather than know—which by its very nature is applied. Perhaps in time the meaning will stabilize, but at this point it would be premature to exclude any of these interpretations. It would also be counterproductive to attempt to make distinctions between "operations research" and the "systems approach." While these terms are sometimes viewed as distinct, they are often conceptualized in such a manner as to defy separation. Any attempt to draw boundaries between them would in practice be arbitrary.

How, then, can we define operations research? The Operational Research Society of Great Britain has adopted the following definition:

> Operational research is the application of the methods of science to complex problems arising in the direction and management of large systems of men, machines, materials and money in industry, business, government, and defense. The distinctive approach is to develop a scientific model of the system, incorporating measurements of factors such as chance and risk, with which to predict and compare the outcomes of alternative decisions, strategies or controls. The purpose is to help management determine its policy and actions scientifically.

The Operations Research Society of America has offered a shorter, but similar, description:

> Operations research is concerned with scientifically deciding how to best design and operate man-machine systems, usually under conditions requiring the allocation of scarce resources.

Although both of these definitions leave something to be desired, they are about as specific as one would want to be in defining such a broad area. It is noteworthy that both definitions emphasize the *motivation* for the work; namely, to aid decision makers in dealing with complex real-world problems. Even when the methods seem to become so abstract as to lose real-world relevance, the student may take some comfort in the fact that the ultimate goal is always some useful application. Both definitions also mention *methodology*, describing it only very generally as "scientific." That term is perhaps a bit too general, inasmuch as the methods of science are so diverse and varied. A more precise description of the OR methodology would indicate its reliance on "models." Of course, that term would itself require further elaboration, and it is to that task that we now turn our attention

1.3 MODELS IN OPERATIONS RESEARCH

The essence of the operations research activity lies in the construction and use of models. Although modeling must be learned from individual experimentation, we will attempt here to discuss it in broad, almost philosophical terms. This overview is worth having, and setting a proper orientation in advance may help to avoid misconceptions later.

First, one should realize that some of the connotations associated with the word "model" in common English usage are not present in the OR use of the word. A model in the sense intended here is just a simplified representation of something real. This usage does carry with it the implication that a model is always, necessarily, a representation that it is less than perfect.

Why model? There are many conceivable reasons why one might prefer to deal with a substitute for the "real thing" rather than with the "thing" itself. Often, the motivation is economic—to save money, time, or some other valuable commodity. Sometimes it is to avoid risks associated with the tampering of a real object. Sometimes the real environment is so complicated that a representative model is needed just to understand it, or to communicate with others about it. Such models are quite prevalent in the life sciences, physical chemistry, and physics.

Given that one has something real, which we will call the "real system," and that there is some understandable reason for wanting to deal with it—that is, a "problem" related to the real system which calls for definite "conclusions"—the modeling process can be depicted as in Fig. 1.1. The broken line on the left represents what might be termed the "direct approach," for which we are seeking a substitute.

The first step is construction of the model itself, which is indicated by the line labeled "Formulation." This step requires a set of coordinated decisions as to what aspects of the real system should be incorporated in the model, what aspects can be ignored, what assumptions can and should be made, into what form the model should be cast, and so on. In some instances, formulation may require no