

Quantum Field Theory and Quantum Statistics  
Essays in Honour of the Sixtieth Birthday of E S Fradkin

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Volume 1: Quantum Statistics and Methods of  
Field Theory

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Birthday of E S Fradkin

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## Volume 1: Quantum Statistics and Methods of Field Theory

Edited by

**I A Batalin**

**C J Isham**

and

**G A Vilkovisky**



Adam Hilger, Bristol

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# List of Contributors

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- S L Adler**, The Institute for Advanced Study, Princeton, NJ 08540, USA
- V de Alfaro**, Istituto di Fisica Teorica, Universita di Torino, 10125 Torino, Italy
- A S Babichenko**, Lebedev Physical Institute, Academy of Sciences of the USSR, Leninskii Prospect 53, Moscow 117924, USSR
- A O Barvinsky**, Department of Mathematics, Moscow State Pedagogical Institute, Davidovskii 4, Moscow 107140, USSR
- I A Batalin**, Lebedev Physical Institute, Academy of Sciences of the USSR, Leninskii Prospect 53, Moscow 117924, USSR
- I Bialynicki-Birula**, Institute of Theoretical Physics, Polish Academy of Sciences, Lotnikow 32/46, 02-668 Warsaw, Poland
- D G Boulware**, Department of Physics FM-15, University of Washington, Seattle, WA 98195, USA
- L Brink**, Institute of Theoretical Physics, Chalmers University of Technology, University of Göteborg, S-412 96 Göteborg, Sweden
- A Cabo**, Instituto de Matematica, Cibernetica y Computacion, Calle O No 8 Vedado, La Habana, Cuba
- E R Caianiello**, Dipartimento di Fisica Teorica, Facolta di Scienze, Universita di Salerno, 84100 Salerno, Italy
- D Chang**, Physics Department, Northwestern University, Evanston, IL 60201, USA
- L-N Chang**, Physics Department, Virginia Polytechnic Institute & State University, Blacksburg, VA 24601, USA
- N-P Chang**, Physics Department, City College of the City University of New York, New York, NY 10031, USA
- S M Christensen**, National Center for Supercomputing Applications, and Department of Physics, University of Illinois, Champaign, IL 61820, USA

- S Deser**, Department of Physics, Brandeis University, Waltham, MA 02254, USA
- B S DeWitt**, Physics Department, The University of Texas at Austin, Austin, Texas 78712, USA
- S D Drell**, Stanford Linear Accelerator Center (SLAC), Stanford University, Stanford, CA 94305, USA
- M J Duff**, CERN, CH-1211 Geneva 23, Switzerland *and* The Blackett Laboratory, Imperial College, London SW7 2BZ, UK
- S F Edwards**, Cavendish Laboratory, University of Cambridge, Cambridge CB3 0HE, UK
- G V Efimov**, Joint Institute for Nuclear Research (JINR), Dubna, Head Post Office, PO Box 79, Moscow 101000, USSR
- U Esposito**, Dipartimento di Fisica Nucleare, Struttura della Materia & Fisica Applicata, Facoltà di Ingegneria, Piazzale Tecchio, 80125 Napoli, Italy
- V Ya Fainberg**, Lebedev Physical Institute, Academy of Sciences of the USSR, Leninskii Prospect 53, Moscow 117924, USSR
- E L Feinberg**, Lebedev Physical Institute, Academy of Sciences of the USSR, Leninskii Prospect 53, Moscow 117924, USSR
- S Ferrara**, CERN, CH-1211 Geneva 23, Switzerland
- P H Frampton**, Physics Department, Boston University, 147 Bay State Road, Boston, MA 02215, USA
- H M Fried**, Physics Department, Brown University, Providence, RI 02912, USA
- S Fubini**, CERN, CH-1211 Geneva 23, Switzerland
- G Furlan**, Istituto di Fisica Teorica, Università di Trieste, Strada Costiera 11, 34014 Trieste, Italy
- A Galperin**, Joint Institute for Nuclear Research (JINR), Dubna, Head Post Office, PO Box 79, Moscow 101000, USSR
- I M Gel'fand**, Laboratory of Molecular Biology and Bioorganic Chemistry, Moscow State University, Moscow 117234, USSR
- J-L Gervais**, Laboratoire de Physique Theorique, Ecole Normale Supérieure, 24 Rue Lhomond, 75231 Paris Cedex 05, France
- V L Ginzburg**, Lebedev Physical Institute, Academy of Sciences of the USSR, Leninskii Prospect 53, Moscow 117924, USSR
- D M Gitman**, Department of Mathematics, Moscow Institute of Radio Engineering, Electronics and Automatics, Vernadsky Prospect 78, Moscow 117454, USSR
- Yu A Golfand**, Lebedev Physical Institute, Academy of Sciences of the USSR, Leninskii Prospect 53, Moscow 117924, USSR
- M B Green**, Department of Physics, Queen Mary College, University of London, London E1 4NS, UK
- M T Grisaru**, Department of Physics, Brandeis University, Waltham, MA 02254, USA

- J B Hartle**, Department of Physics, University of California at Santa Barbara, Santa Barbara, CA 93106, USA
- S W Hawking**, Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge CB3 9EW, UK
- M Henneaux**, Faculté des Sciences, Université Libre de Bruxelles, Campus Plaine CP231, B-1050 Brussels, Belgium
- C Itzykson**, Service de Physique Theorique, Centre d'Etudes Nucleaires de Saclay, BP No 2, 91191 Gif-sur-Yvette, France
- E Ivanov**, Joint Institute for Nuclear Research (JINR), Dubna, Head Post Office, PO Box 79, Moscow 101000, USSR
- R Jackiw**, Center for Theoretical Physics and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, MA 02139, USA
- D R T Jones**, Department of Applied Mathematics and Theoretical Physics, University of Liverpool, PO Box 147, Liverpool L69 3BX, UK
- M Kaku**, Physics Department, City College of the City University of New York, New York, NY 10031, USA
- O K Kalashnikov**, Lebedev Physical Institute, Academy of Sciences of the USSR, Leninskii Prospect 53, Moscow 117924, USSR
- R E Kallosh**, CERN, CH-1211 Geneva 23, Switzerland *and* Lebedev Physical Institute, Academy of Sciences of the USSR, Leninskii Prospect 53, Moscow 117924, USSR
- D A Kirzhnits**, Lebedev Physical Institute, Academy of Sciences of the USSR, Leninskii Prospect 53, Moscow 117924, USSR
- J R Klauder**, AT & T Bell Laboratories, Murray Hill, NJ 07974, USA *and* RIMS, Kyoto University, Kyoto 606, Japan
- H Leutwyler**, Institute of Theoretical Physics, University of Bern, Sidlerstrasse 5, CH-3012 Bern, Switzerland
- F E Low**, Massachusetts Institute of Technology (MIT), Cambridge, MA 02139, USA
- R Marnelius**, Institute of Theoretical Physics, Chalmers University of Technology, University of Göteborg, S-412 96 Göteborg, Sweden
- R A Minlos**, Department of Mechancis and Mathematics, Moscow State University, Moscow 117234, USSR
- R N Mohapatra**, Department of Physics & Astronomy, University of Maryland, College Park, MD 20742, USA
- Y Nambu**, Department of Physics, and Enrico Fermi Institute, University of Chicago, Chicago, IL 60637, USA
- Y Ne'eman**, Center for Theoretical Physics, The University of Texas at Austin, Austin, Texas 78712, USA *and* Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv 69978, Israel
- R I Nepomechie**, Department of Physics, University of Miami, Coral Gables, FL 33124, USA

- P van Nieuwenhuizen**, Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, NY 11794, USA
- K Nishijima**, Research Institute for Fundamental Physics, Kyoto University, Kyoto 606, Japan
- V Ogievetsky**, Joint Institute for Nuclear Research (JINR), Dubna, Head Post Office, PO Box 79, Moscow 101000, USSR
- M Olvera de la Cruz**, Cavendish Laboratory, University of Cambridge, Cambridge CB3 0HE, UK
- M Ya Palchik**, Institute of Automation and Electrometry, Siberian Division of the USSR Academy of Sciences, Novosibirsk 630090, USSR
- G Parisi**, Dipartimento di Fisica, Università Roma II, Tor Vergata, 00173 Roma, Italy
- L Parker**, Physics Department, University of Wisconsin–Milwaukee, Milwaukee, WI 53201, USA
- J Przesowski**, Institute of Theoretical Physics, Polish Academy of Sciences, Lotnikow 32/46, 02-668 Warsaw, Poland
- S Randjbar-Daemi**, Institute of Theoretical Physics, University of Bern, Sidlerstrasse 5, CH-3012 Bern, Switzerland
- V I Ritus**, Lebedev Physical Institute, Academy of Sciences of the USSR, Leninskii Prospect 53, Moscow 117924, USSR
- B Sakita**, Physics Department, City College of the City University of New York, New York, NY 10031, USA
- Abdus Salam**, International Centre for Theoretical Physics, PO Box 566-Miramare, Strada Costiera 11, 34100 Trieste, Italy
- K Schleich**, Enrico Fermi Institute, University of Chicago, Chicago, IL 60637, USA
- A S Schwarz**, Department of Theoretical Nuclear Physics, Moscow Physics Engineering Institute, Kashirskoye Shosse 1, Moscow 115409, USSR
- A E Shabad**, Lebedev Physical Institute, Academy of Sciences of the USSR, Leninskii Prospect 53, Moscow 117924, USSR
- D M Shteingradt**, Institute for High Temperatures, Academy of Sciences of the USSR, Moscow 127412, USSR
- E Sokatchev**, Joint Institute for Nuclear Research (JINR), Dubna, Head Post Office, PO Box 79, Moscow 101000, USSR
- K S Stelle**, The Blackett Laboratory, Imperial College, London SW7 2BZ, UK
- J Strathdee**, International Centre for Theoretical Physics, PO Box 566-Miramare, Strada Costiera 11, 34100 Trieste, Italy
- J G Taylor**, Department of Mathematics, King's College, University of London, The Strand, London WC2R 2LS, UK
- C Teitelboim**, Center for Theoretical Physics, The University of Texas at Austin, Austin, Texas 78712, USA *and* Centro de Estudios Científicos de Santiago, Casilla 16443, Santiago 9, Chile

- H Terazawa**, Institute for Nuclear Study, University of Tokyo, Midori-cho, Tanashi, Tokyo 188, Japan
- I T Todorov**, Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Boul. Lenin 72, Sofia 1184, Bulgaria
- E T Tomboulis**, Department of Physics, University of California at Los Angeles, Los Angeles, CA 90024, USA
- D J Toms**, Department of Theoretical Physics, University of Newcastle upon Tyne, Newcastle upon Tyne NE1 7RU, UK
- P K Townsend**, Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge CB3 9EW, UK
- A A Tseytlin**, Lebedev Physical Institute, Academy of Sciences of the USSR, Leninskii Prospect 53, Moscow 117924, USSR
- Yu S Tyupkin**, World Data Centre B2, Molodezhnaya 3, Moscow 117296, USSR
- I V Tyutin**, Department of Physics, Moscow Institute of Electronic Engineering, Bolshoi Vuzovskii 3/12, Moscow 109028, USSR
- M A Vasiliev**, Lebedev Physical Institute, Academy of Sciences of the USSR, Leninskii Prospect 53, Moscow 117924, USSR
- G A Vilkovisky**, Lebedev Physical Institute, Academy of Sciences of the USSR, Leninskii Prospect 53, Moscow 117924, USSR
- J Wess**, Institut für Theoretische Physik, Universität Karlsruhe, D-7500 Karlsruhe, FRG
- P C West**, Department of Mathematics, King's College, University of London, The Strand, London WC2R 2LS, UK
- B de Wit**, Institute for Theoretical Physics, University of Utrecht, PO Box 80.006, 3508 TA Utrecht, The Netherlands
- V N Zaikin**, Lebedev Physical Institute, Academy of Sciences of the USSR, Leninskii Prospect 53, Moscow 117924, USSR
- A Zee**, Institute for Theoretical Physics, University of California at Santa Barbara, Santa Barbara, CA 93106, USA
- S C Zhang**, Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, NY 11794, USA



# Preface

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By publishing this book theoretical physicists are commemorating the life and work of Efim Samoilovich Fradkin who was sixty on 30 November 1984. Fradkin's scientific biography and the biography of modern quantum theory are inseparable as, in the past 35 years, Efim Samoilovich has continuously figured among the leaders of this very dynamic science. It is safe to say that quantum field theory has nowadays entered into its 'inflationary' stage when each new idea appears only to get rapidly replaced by a yet newer one. Each new wave brings to the surface a new generation of talented young people, not many of whom manage to ride along the crest of this wave. The phenomenon that is Fradkin is unique in this context; he appears to be reborn with every new idea in the field and, moreover, every new generation of physicists adopts him very easily. Much is said in this book of his striking and unfading creative activity, of the broad scope of his scientific interests and of his fundamental contribution to quantum field theory and statistical physics.

As wide-ranging as have been Fradkin's studies, this book forms an encyclopaedia of modern high-energy theoretical physics. There is hardly any trend in quantum field theory and statistical physics that falls outside its scope. The enthusiasm with which theoreticians of all generations responded to our proposal to publish a commemorative collection was such that two volumes were required to accommodate all the papers received. In volume 1 we have placed articles of a general nature; this volume is devoted to quantum statistical physics and methods of field theory that are in one way or another independent of specific models. Volume 2 is totally devoted to modern models of quantum field theory. This is the domain of gravitation, supersymmetry and strings, but note that the first essay in volume 2 sounds a warning.

The creation of this book came about with the help and encouragement of many people whom we would now like to thank. This relates in the first

place to Vitaly Lazarevich Ginzburg, head of the theoreticians at the Lebedev Physical Institute, who supported our project at an early stage, and who not only contributed an article but also offered much organisational assistance. When we had just started working on the book and found our experience to be rather inadequate, we enjoyed lavish assistance from Steven Christensen. When we experienced difficulties in the translation of the historical and biographical matter we were helped by the considerable literary skills of K P Lukyanenko, and the photographs in which Efim Samoilovich is so eminently recognisable we owe to O D Beznisko. All Efim Samoilovich's friends and former students were very enthusiastic and forward in doing any job that was required. In particular A O Barvinsky, A E Shabad, A A Tseytlin and V N Zaikin spent many hours reading manuscripts, translating and checking texts, compiling all sorts of lists, and so forth. Organisationally so complex a project as this book could not have been possible without the attention, benevolence and tolerance of Jim Reville, Commissioning Editor for Adam Hilger.

This book is international in a true sense—by its authors, the editors and, what is most important, by the subject which forms its content and motivation of the life of Efim Samoilovich Fradkin. The noble spirit of the international scientific community, which we felt so warmly throughout our project, is perhaps the strongest assertion of the book in which 94 theoretical physicists from 16 countries have united to express their respect for one of their outstanding colleagues.

**I A Batalin**  
**C J Isham**  
**G A Vilkovisky**  
Moscow–London, June 1985

# ПРЕДИСЛОВИЕ

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Изданием этой книги физики-теоретики отмечают юбилей Ефима Самойловича Фрадкина, которому 30 ноября 1984 г. исполнилось 60 лет. Научная биография Фрадкина – это биография современной квантовой теории, ибо в течение 35 лет Ефим Самойлович бесценно входит в число лидеров этой стремительно развивающейся науки. Без преувеличения можно сказать, что квантовая теория поля вступила теперь в свою “инфляционную” стадию, когда идеи сменяют друг друга со все большей скоростью, каждая новая волна выносит на поверхность новое поколение талантливых и молодых людей, и мало кому удастся удержаться на гребне этой волны. Тем более уникальным кажется феномен Фрадкина, который как будто бы заново рождается с каждой новой идеей, и каждое новое поколение физиков считает его своим. О его поразительной неослабевающей творческой активности, широте научных интересов и фундаментальном вкладе, который он внес в квантовую теорию поля и статистику, много говорится на страницах этой книги.

В полном соответствии с диапазоном исследований Фрадкина эта книга представляет собой энциклопедию современной теоретической физики высоких энергий. Вряд ли найдется такое направление в квантовой теории поля и статистике, которое не было бы здесь представлено. Энтузиазм, с которым теоретики всех поколений откликнулись на наше предложение создать юбилейный сборник, был таков, что потребовалось два тома, чтобы вместить все полученные рукописи. В первом томе мы поместили статьи общего характера. Этот том посвящен квантовой статистике и методам теории поля, которые в той или иной мере не зависят от конкретных моделей. Второй том целиком посвящен современным моделям квантовой теории поля. Здесь господствуют гравитация, суперсимметрия и струны, но первый очерк этого тома звучит как предупреждение.

Созданию этой книги способствовало немало людей, которым мы теперь хотим принести свою благодарность. Это прежде всего относится к Виталию Лазаревичу Гинзбургу, главе теоретиков Физического института им П Н Лебедева, который поддержал наш проект на одном из ранних этапов, и не только написал статью для этой книги, но и оказал нам большую организационную помощь. Когда в начале

работы над книгой нам не хватало опыта, им с нами щедро делился Стивен Кристенсен. Когда мы испытывали трудности при переводе историко-биографического материала, нас выручало высокое литературное мастерство К П Лукьяненко, а фотографиями, на которых Ефим Самойлович такой, каким мы все его знаем, мы обязаны О Д Безниско. Все друзья и ученики Ефима Самойловича с энтузиазмом и инициативой брались за любую работу по этому сборнику. В частности, А О Барвинский, В Н Зайкин, А А Цейтлин и А Е Шабад провели много часов за чтением рукописей, переводом и проверкой текстов, составлением различных списков и так далее. Осуществление столь сложного в организационном отношении проекта вряд ли было бы возможно, если бы не внимательность, благожелательность и терпимость Джима Ревилла, ответственного редактора издательства Адам Хилгер.

Эта книга по-настоящему интернациональна – и по составу авторов, и по составу редакторов и, главное, по тому предмету, который является её содержанием и главным смыслом жизни Ефима Самойловича Фрадкина. Высокий дух международного сообщества ученых, который мы так хорошо ощутили, реализуя наш проект, – это, быть может, самое главное утверждение книги. В ней 94 физика-теоретика из 16 стран объединились, чтобы выразить уважение одному из своих выдающихся коллег.

**К Дж Айшем**  
**И А Баталин**  
**Г А Вилковийский**

Москва – Лондон, июнь 1985 г.

# Letter from S D Drell

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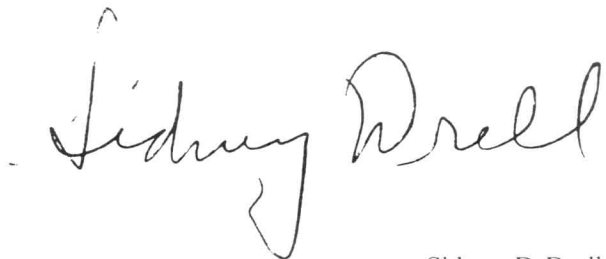
It is a great pleasure and privilege for me to have this opportunity to express my appreciation of Professor E S Fradkin on his sixtieth birthday. Lacking a scientific manuscript that is appropriate for this volume, I wish simply to convey a personal message.

For many years I have known Efim Fradkin as a brilliant theoretical physicist who has made profound and lasting contributions to our understanding of quantum field theory. His research has combined mathematical elegance and skill with a deep understanding of the significance of his work as applied to physical problems. He is a great physicist.

I am especially proud to know Efim as a friend because I admire his warm personality and high principles. When we have been together, in Moscow or Stanford, I have always found Efim a person of great warmth and culture—and have enjoyed and valued his company.

We physicists enjoy the great privilege of being members of a world-wide community of scholars together with whom we strive to deepen our knowledge of the workings of nature and with whom we often share the same fundamental understandings of the human condition. My bonds of friendship and respect for Efim are strengthened by our common values and principles.

I wish him continued great success and happiness in the years ahead.

A handwritten signature in black ink that reads "Sidney Drell". The signature is written in a cursive, flowing style with a large initial 'S' and a distinct 'D'.

Sidney D Drell

# Letter from I M Gel'fand

---

I have known Efim Samoilovich since 1940 when he was a freshman at Minsk University. An outstanding student, I should say, and I had not a single doubt that he would one day become a brilliant scholar. He was equal to none save his own sister who, two years later, was murdered by the Nazis. They were two small Jewish kids from a poor family, called  $\varepsilon$  and  $\delta$  and loved by everybody. Both had extraordinary talent and were already entirely devoted to science at that time.

My lecturing at Minsk came to an end in 1941 with the beginning of the war and I lost sight of Efim Samoilovich until he reappeared in 1947, in Moscow. This time I saw a demobilised lieutenant and an extramural undergraduate of Lvov University. Since I had no doubts about Efim Samoilovich's talent, I told I E Tamm and V L Ginzburg that if they took him at the Lebedev Institute they would have no regrets. I didn't want to lose him to other hands, but he was a physicist and time has shown how right I was.

Each time I turned to physics—whether introducing functional integrals into field theory (being unaware of Feynman's original studies on quantum mechanics), or even before that when involved in relativistic equations, or at the seminars in the 1960s when we debated the approaches to quantum field theory, suggested by Symanzik, Fradkin, myself and Minlos, or in many other cases—Efim Samoilovich was of great help to me, and he continues to be very helpful today as I learn not only from him but also from his younger colleagues representing this fascinating school of physics.

I hope that I have still more to learn from the Fradkin school. Specifically, his studies on conformal theory and supergravity appear to be very inspiring. While rubbing shoulders with him and his disciples, I cannot but feel that Efim Samoilovich is the youngest of them. There is no need for me to wish any success to Efim Samoilovich in his scientific work—this will

always come of its own accord. But I can wish him good health and the skill of being patient with us all while presenting his ideas.

A handwritten signature in black ink, reading "I. M. Gel'fand". The signature is written in a cursive style with a prominent initial "I" and a stylized "M".

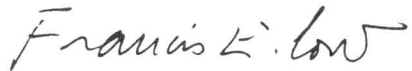
Israil M Gel'fand

# Letter from F E Low

---

I am delighted to send warm greetings to Professor E S Fradkin on the occasion of his sixtieth birthday. We first met when I visited the Soviet Union in 1957, and through the years since then we have met at scientific meetings in the Soviet Union as well as in Cambridge.

It has always been a pleasure to see him, and an honor to have his friendship. I wish him many more years of productive and enjoyable work.

A handwritten signature in black ink that reads "Francis E. Low". The script is fluid and cursive, with the first letters of "Francis" and "Low" being capitalized and prominent.

Francis E Low



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