PLANE TRIGONOMETRY WITH TABLES

SECOND EDITION

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By DONALD H. BALLOU, PH.D.
Middlebury College

and FREDERICK H. STEEN, PH.D.
Allegheny College

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PREFACE

Finding that many instructors would welcome material on graphs and complex numbers for possible inclusion in their courses in trigonometry, the authors have prepared chapters on these topics for the second edition of this text. These chapters have been inserted where it was felt they would most naturally occur in the development of the subject. They are, however, completely independent of the other material and may be used or not entirely according to the individual instructor's needs and wishes.

As this new material was being added, the authors also took the opportunity of changing the exercises. Except in those cases where it was felt that there would be a definite loss if particular exercises were altered or omitted, either entirely new exercises have been prepared or new numerical data have been included. In making these changes the authors have tried to retain the best features of the old exercises while at the same time introducing such variety and improvements as seemed desirable.

As was stated in the preface of the first edition, this text was written for college and engineering-school courses in trigonometry. Most students in these courses are preparing themselves for later work, either in more advanced mathematics, or in physics, engineering, navigation, or other sciences. The need of these students for an understanding of the principles of trigonometry and for a mastery of the techniques has directed the preparation and presentation of the subject matter of the text.

The aim has been to develop the theory and to illustrate its applications as clearly and effectively as possible. Care has been taken to make the explanations sufficiently detailed so that they can be easily followed by the student. Numerous examples illustrate the principles and methods of the theory.

A large number of carefully graded exercises have been provided throughout the text. Along with these written exercises are numerous sets of "Orals." These will help the student in learning new methods and formulas as they are first taken up in class, and they will provide material for rapid drill on topics previously studied. Many practical problems serve to acquaint the student with the situations in which trigonometry is directly applied. These problems, taken from various fields, illustrate many of the uses of trigonometry, particularly in physics, surveying, navigation, and aviation.

In developing the theory, the general-angle definitions of the trigonometric functions have been given first. This is to emphasize from the beginning the general nature of the functions. Because of its extensive use in the calculus, radian measure has been introduced at the very first and used along with degree measure throughout the book. The proving of trigonometric identities has been considered not as an end in itself but as a procedure for simplifying a trigonometric expression or as a means of deriving an equivalent expression with certain different characteristics. This treatment serves to familiarize the student with the fundamental relations and at the same time to give him skill in the type of manipulation used in later work.

Logarithms have been discussed in a separate chapter. As this chapter is independent of the others, it may be taken at the instructor's convenience. It is suggested that it be studied either at the beginning of the course or in connection with the solution of triangles.

The text was used in lithoprinted form at the Georgia School of Technology for several years prior to publication. The authors wish to express their sincere thanks to their former colleagues in the Department of Mathematics there for their interest and cooperation when teaching the material in its preliminary form. Their criticisms and suggestions were most helpful in determining the present form of the text. The authors are particularly grateful to Dr. D. M. Smith, Head of the Department of Mathematics at that time, for his sympathetic advice and encouragement. They also wish to express their sincere appreciation for the suggestions made by the many users of the text since it first made its appearance.

D. H. B.
F. H. S

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PLANE TRIGONOMETRY

CHAPTER I

ANGLES AND COORDINATES

1 · Introduction

The beginnings of trigonometry are to be found in the work of certain Greek and Egyptian astronomers, notably Hipparchus of Nicaea (about 140 B.C.) and Ptolemy of Alexandria (about 140 A.D.). Its name, meaning triangle measurement, indicates the use for which it was originally developed. The solution of triangles is still an important application of trigonometry: the solution of plane triangles in surveying and of spherical triangles in navigation and astronomy. However, since the days of the ancient Greeks, mathematics and science have been extended in manifold directions. Each extension has meant increased use and importance for trigonometry, entirely apart from its use in the solution of triangles. Today the formulas and relationships of trigonometry permeate the higher branches of mathematics, and these branches of mathematics are the language and tools of the physical and engineering sciences.

2 · Angles

Specifically, trigonometry consists of the study of certain ratios associated with an angle, the properties of these ratios, their interrelationships, and their applications to various problems. The angle, rather than the triangle, occupies the central position in modern trigonometry.

When the angle plays this fundamental role, its concept is a more general one than that employed in plane geometry. An angle AOB (Fig. 1) is considered as having been generated by a half-line rotating about

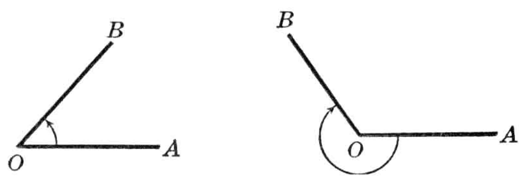

FIG. 1

its end point O from an initial position OA to a final position OB. OA is called the **initial side** of the angle, OB the **terminal side**, and O the **vertex.** The size of the angle is determined by the amount of rotation, and the angle is considered **positive** or **negative** according as the rotation is *counterclockwise* or *clockwise.* The direction of rotation is often indicated by an arrowhead. Thus in Fig. 1 the first angle is positive and the second negative. Since the rotation may be of any amount and in either direction, this concept enables us to consider angles of any size whatever, positive or negative.

It is seen that two angles may have the same initial and terminal sides and yet differ from each other by an integral number of complete revolutions. Such angles are said to be **coterminal.** There are an unlimited number of angles coterminal with any given angle. We shall use most frequently angles which are positive and which are generated by less than one complete revolution of the rotating line. We shall call such angles **primary angles.** Thus in Fig. 2 the angles α and β are (positive) coterminal angles, and β is a primary angle.

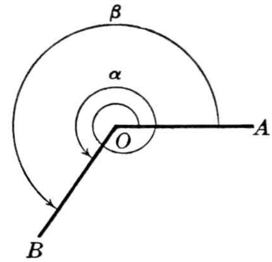

FIG. 2

3 · Measurement of Angles: Degrees and Radians

Two units for the measurement of angles are in common use in trigonometry. The first of these is the **degree**, which is $\frac{1}{360}$ of the angle formed by one complete revolution of the generating line. The degree is divided into 60 equal parts called **minutes**, and a minute into 60 equal parts called **seconds**. These units are already familiar to the student.

The second unit of measurement commonly used is the **radian**,* *an angle which when placed with its vertex at the center of a circle intercepts an arc equal in length to the radius of the circle* (Fig. 3). The number of radians in an angle is thus determined by the number of times the radius is contained in the intercepted arc. If θ is the measure of an angle in radians, s the length of the intercepted arc, and r the radius of the circle, then

$$\theta = \frac{s}{r}, \quad or \quad s = r\theta,$$

or, in words,

$$\textit{The measure of an angle in radians} = \frac{\textit{length of arc}}{\textit{length of radius}}.$$

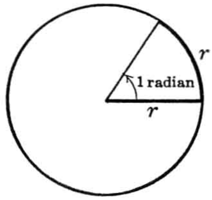

FIG. 3

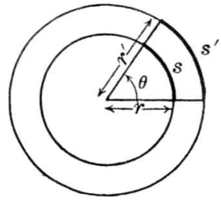

FIG. 4

*Radian measure is used extensively in the calculus.

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Since central angles in a circle are proportional to their intercepted arcs, it is logical to use this ratio of the arc to the radius as a measure of angles. Moreover, the definition of a radian is independent of the circle used, for if θ is a central angle intercepting an arc of length s in a circle of radius r, and an arc of length s' in a circle of radius r' (Fig. 4), then

$$\frac{s}{2\,\pi r} = \frac{s'}{2\,\pi r'},$$

and this gives

$$\frac{s}{r} = \frac{s'}{r'}.$$

An angle of 360° intercepts the entire circumference, an arc $2\,\pi r$ in length. The radian measure of the angle is therefore $2\,\pi$, and hence $2\,\pi$ radians = 360°, or

$$\boldsymbol{\pi} \textbf{ radians} = \textbf{180°}.$$

From this we see that

$$1 \text{ radian} = \frac{180}{\pi} \text{ degrees} = 57.3° \text{ (approx.)},$$

and

$$1 \text{ degree} = \frac{\pi}{180} \text{ radians} = 0.017 \text{ radian (approx.)}.$$

The fundamental relation, π radians = 180°, is frequently used to obtain a simple expression for the number of radians in an angle whose measure is given in degrees, and vice versa. For example, $\frac{\pi}{2}$ radians $= \frac{180°}{2} = 90°$; and 60°, being one third of 180°, equals $\frac{\pi}{3}$ radians. For more complicated problems, such as changing 3.736 radians to degrees or 127° 14.7′ to radians, it is simpler to use conversion tables. For reference we list just a few of these conversion factors:

1 radian = 57.29578 degrees	1 degree = 0.017453 radian
= 57° 17.7′	1 minute = 0.000291 radian

When using radian measure we often omit the word "radian." Thus if we write $\frac{\pi}{3}$ for an angle, we mean an angle of $\frac{\pi}{3}$ radians.

4 · Area of a Sector

One important use of radian measure is in the formula for the area of the sector of a circle. If K denotes the area of the sector AOB (Fig. 5) and s is the length of the arc AB, then

$$\frac{K}{\pi r^2} = \frac{s}{2\,\pi r}.$$

From this,

$$K = \tfrac{1}{2}\, rs.$$

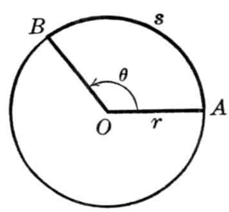


FIG. 5

This formula was proved in plane geometry and is easily remembered by its similarity to that for the area of a triangle. If now the angle θ is measured in radians so that $s = r\theta$, this takes the more useful form

$$K = \tfrac{1}{2} r^2 \theta.$$

5 · Motion on a Circle

Radian measure is also useful in considering the motion of a particle on a circle. If a point moves from A to B around the circumference of a circle (Fig. 5), the distance traveled is given by the radian formula

$$s = r\theta.$$

Furthermore, if the point moves with constant velocity, v, in its path, then

$$v = r\omega,$$

ω being the rate at which the angle θ is changing — the **angular velocity.** If the velocity is not constant, but v and ω represent the average of the velocities over a period of time, this same relation holds between them.

6 · Rectangular Coordinates

In order to define the trigonometric ratios of a given angle, use is made of a standard rectangular coordinate system. As the **coordinate axes** of this system we draw $X'X$ horizontal and $Y'Y$ vertical, intersecting in the point O (Fig. 6). Then $X'X$ is called the ***x*-axis**, $Y'Y$ the ***y*-axis**, and O the **origin.**

Any point P in the plane is now located by means of its distances from the coordinate axes, measured in terms of some convenient unit. Its distance from the y-axis is called the ***x*-coordinate**, or **abscissa**, and is taken as positive if P is to the right of $Y'Y$, and negative if to the left. Its distance from the x-axis is called the ***y*-coordinate**, or **ordinate**, and is taken as positive if P is above $X'X$, and negative if below. The coordinates of a point P whose abscissa is x and ordinate y are written (x, y), the abscissa always being written first. Thus (Fig. 6) if a point is 2 units to the right of the y-axis and 5 units below the x-axis, its coordinates are $(2, -5)$. To any point P in the plane there corresponds a unique pair of numbers for its coordinates, and conversely, to any pair of numbers considered as its coordinates there corresponds a unique point in the plane.

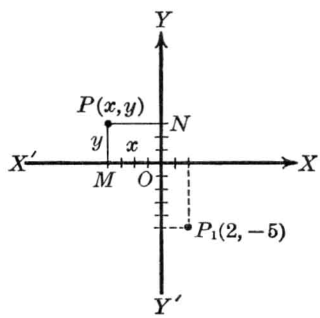


FIG. 6

The coordinates of a point are sometimes represented by **directed line segments**, segments whose direction and length both have a meaning assigned to them. The length of the segment gives the magnitude of the coordinate, and its direction, indicated by the order in which the end points are written, gives the sign. Segments which represent coordinates are positive when directed to the right or upward and negative when directed to the left or downward. Thus in Fig. 6 the abscissa of the point $P(x, y)$ is represented by the directed line segment NP, or OM. Each of these is negative, since it is directed to the left. The ordinate of P is represented by either of the directed line segments MP or ON. These are positive, since they are directed upward.

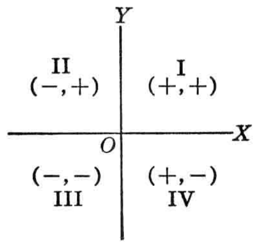


FIG. 7

Each of the four parts into which a plane is divided by the coordinate axes is called a **quadrant**. These are numbered I, II, III, IV, in a counterclockwise direction beginning at the upper right (Fig. 7). Also indicated in Fig. 7 are the signs of the abscissa and of the ordinate of points in each of the four quadrants.

In addition to the coordinates of a point, use is also made of its distance, OP, from the origin. This is called simply the **distance** of P and is denoted by r. When P is different from O, r is always taken as positive. For any point $P(x, y)$ in the plane (Fig. 8) we have from the Pythagorean theorem that

$$r^2 = x^2 + y^2.$$

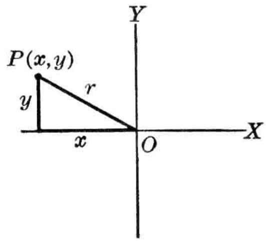
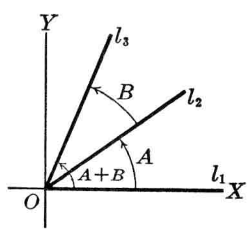


FIG. 8

In our study of angles it is often convenient to consider an angle placed so that its vertex is at the origin and its initial side lies along the positive x-axis. When it is so placed, an angle is said to be in **standard position**. Furthermore, an angle is said to lie in a certain quadrant if, when in standard position, its terminal side falls in that quadrant.

ORALS

1. In the figure: (*a*) locate the vertex of angle A; (*b*) locate the initial side of angle A, angle B, angle $A + B$; (*c*) locate the terminal side of each of these angles; (*d*) state which of these angles are in standard position.



2. Name the quadrant in which each of the following points lies:

(*a*) (1, 3), (− 5, 2), (4, − 3)
(*b*) (− 2, 1), (− 3, − 1), (2, − 6)

3. Give the distance of each point in the preceding exercise.

4. Give the distance of each of the following points:

$$(3, 0), \quad (0, 4), \quad (0, 0), \quad (-1, 0), \quad (-5, 0)$$

5. Where are all the points for which (*a*) the ordinate is 3; (*b*) the abscissa is -4; (*c*) the distance is 2; (*d*) the abscissa is 3 and the distance is 5?

6. Express in degrees the angles of the following number of radians:

(*a*) $\frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{6}, \frac{\pi}{4}$ (*c*) $\frac{4\pi}{3}, \frac{5\pi}{6}, 3\pi, \frac{-7\pi}{4}$

(*b*) $\frac{3\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{4}, \frac{-5\pi}{3}$ (*d*) $\frac{2\pi}{3}, \frac{5\pi}{4}, \frac{11\pi}{6}, -2\pi$

7. Express in radians, in terms of π, the angles of the following number of degrees:

(*a*) 180°, 90°, 45°, 30° (*c*) 120°, 210°, 315°, $-270°$
(*b*) 60°, 150°, 225°, $-90°$ (*d*) 135°, 240°, 330°, $-540°$

8. Each of the following numbers is the radian measure, correct to two decimals, of some common angle, such as 30° or 90°. Give the corresponding degree measure of each of these angles.

(*a*) 3.14, 0.52, 1.57, 6.28 (*b*) 2.09, 12.57, 0.79, 4.71

9. Name the quadrant in which each of the following angles lies:

(*a*) 225°, 160°, 660°, $-100°$ (*b*) $\frac{2\pi}{3}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{-5\pi}{4}$

10. Give the primary angle coterminal with each of the following:

(*a*) 480°, 690°, $-45°$, $-210°$ (*b*) $3\pi, \frac{5\pi}{2}, \frac{-\pi}{3}, \frac{-3\pi}{4}$

11. How many radians are there in the central angle which intercepts an arc of 24 inches on a circle of radius 4 inches?

12. What is the length of arc on a circle of radius 4 which is subtended by a central angle of (*a*) 1 radian, (*b*) $\frac{5}{2}$ radians, (*c*) $\frac{\pi}{4}$ radians?

13. What is the length of the arc on a circle of radius 12 which subtends a central angle of (*a*) 30°, (*b*) 135°, (*c*) 240°?

14. The minute hand of a clock is 5 inches long. What is the distance moved by its outer end when the hand has moved 20 minutes?

15. What is the area of the sector cut from a circle of radius 4 by a central angle of 1 radian?

16. A fan makes 1200 revolutions per minute. What is its angular velocity in radians per second?

EXERCISES

1. Locate on graph paper the points of Oral 2.

2. Locate on graph paper the points of Oral 4.

3. Draw in standard position each of the angles of Oral 9.

4. Change (*a*) 30°, (*b*) 76° 52.1′ to radians, correct to four decimals.

5. Change 2.57 radians to degrees and minutes, correct to the nearest tenth of a minute.

6. If the radius of the earth is taken as 3960 miles, find the distance from the equator to the Tropic of Cancer, latitude $23\frac{1}{2}°$ N.

7. Find, to the nearest 10 miles, the greatest north-south extent of the United States, given that the latitude of Sand Key (off Key West, Florida) is 24° 27′ N and that the latitude of the tip of the northwest angle of Minnesota is 49° 24′ N. The radius of the earth is approximately 3960 miles.

8. The mean radius of the earth is 3959 miles. Using this radius find, to the nearest 10 feet, the length of 1 minute of arc of a great circle on the earth.

NOTE. This is approximately the length of the **nautical mile**, which in the United States is taken as 6080.27 feet.

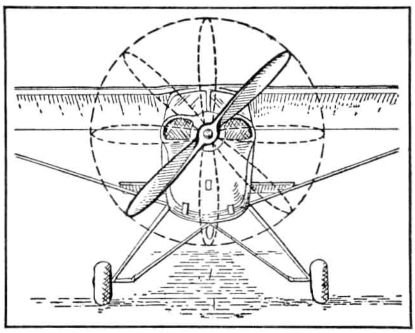
9. Find the area of a circular sector of radius 6 inches and angle 40°.

10. The minute hand of a clock is $1\frac{1}{2}$ inches long and the hour hand 1 inch long. Find the total area swept over by the hands from 8 : 00 A.M. to 8 : 20 A.M.

11. A flywheel 1 foot in diameter is rotating at the rate of 2400 revolutions per minute. What is its angular velocity in radians per second? What is the linear velocity of a point on its rim in feet per second?

12. What is the velocity (feet per second) of a belt which passes over a pulley 20 inches in diameter turning at the rate of 90 radians per minute?

13. A boy is riding a bicycle down the street at 12 miles per hour. If the outer diameter of the tires is 26 inches, what is the angular velocity in radians per second of the wheels?



14. The propeller drive of the Aeronca 65 is direct from the engine. What is the tip speed in feet per second of the 72-inch (diameter) propeller if the engine is turning at 2400 revolutions per minute? What is the angular velocity? (See sketch.)

15. The diameter of a Boeing Clipper propeller is 14 feet 10 inches. The gear-ratio is 16 to 9, that is, the gears convert 16 revolutions of the engine into 9 revolutions of the propeller. What is the tip speed in feet per second at 2000 engine revolutions per minute? What is the angular velocity?

7 · The Mil

Not used in ordinary trigonometry, but employed extensively by the United States Army, is an angular unit known as the **mil**. In artillery measurements one complete revolution is divided into 6400 mils; that is,

$$\textbf{1600 mills} = \mathbf{90°}.$$

The mil, defined in this way, is also *approximately the angle subtended by one yard at a distance of a thousand yards, or by one foot at a distance of a thousand feet,* and so forth; from this fact the mil derives its name.

If the yard (or other unit) is measured along the circular arc, it follows that the mil is approximately $\frac{1}{1000}$ of a radian. Then the formula $s = r\theta$ of Sec. 3 becomes

$$s = \frac{r\theta}{1000}, \ \theta \text{ in mils.}$$

In actual practice in the field, however, the yard is usually thought of as measured along a straight line perpendicular to the line of sight. Thus it would be considered either as the chord AB of the arc (Fig. 9 a), perpen-

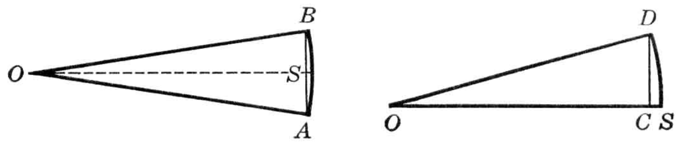


FIG. 9 a FIG. 9 b

dicular at its mid-point to the line of sight OS, or the "half-chord" CD (Fig. 9 b), perpendicular at its end point to the line of sight.* For small angles the differences between the lengths of the chord, the half-chord, and the arc are small. Hence, when θ is small, s in the above formula may be thought of as representing any one of these.

The army uses the mil to make rapid estimates of angles and distances, and only approximate results are expected. It is possible to make these estimates in terms of mils because of the particularly usable relation between the length of the subtended line and the distance from the observer. Soldiers, knowing how many mils their fingers and hands subtend when held at arm's length, are trained in these approximate calculations. Both

* With this last interpretation θ mils equals approximately 1000 tan θ (see Secs. 8 and 12 for definitions of tan θ).

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