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Edited by G. Goos and J. Hartmanis

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B. S. Garbow J. M. Boyle
J. J. Dongarra C. B. Moler

Matrix Eigensystem Routines –
EISPACK Guide Extension



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PREFACE

This volume supplements the earlier Volume 6 in this series [10]; together they provide guidance for the complete second release of the EISPACK Eigensystem Package. The stress in this book is on four additional problem classes: the symmetric band eigenproblem, the generalized symmetric eigenproblem, the generalized real eigenproblem, and the singular value decomposition of a rectangular matrix and solution of an associated linear least squares problem.

The organization of material in this volume follows closely that of [10]. Several of the newer problems transform to problems covered earlier in [10]; reference should be made there for details that apply after the problem has been transformed. Towards achieving a certain degree of self-sufficiency, however, the documentation for seven earlier subroutines that recur here in the various recommended paths of Section 2 has been recopied in Section 7.1 of this volume.

The EISPAC control program, available with the IBM version of EISPACK, extends to each of the newer problem classes except Singular Value Decomposition; the discussion of its usage, where applicable, is integrated into the various sections of this volume. Its documentation, earlier provided in [10], has been recopied in Section 7.2 of this volume.

EISPACK is a product of the NATS (National Activity to Test Software) Project ([3],[4],[5]) which has been guided by the principle that the effort to produce high quality mathematical software is justified by the wide use of that software in the scientific community. EISPACK has been distributed to several hundred computer centers throughout the world since the package was first released in May, 1972, and now the second release is available as described in Section 5.

Building a systematized collection of mathematical software is necessarily a collaborative undertaking demanding the interplay of a variety of skills; we wish to acknowledge a few whose roles were especially crucial during the preparation of the second release. J. Wilkinson persisted in his encouragement of the project and his counsel was often sought during his frequent visits to North America. Organization and direction came from W. Cowell and J. Pool. B. Smith and V. Klema provided highly valued consultation. The field testing was carried out at the installations listed in Section 5 through the sustained efforts of M. Berg, A. Cline, D. Dodson, B. Einarsson, S. Eisenstat, I. Farkas, P. Fox, F. Fritsch, C. Frymann, G. Haigler, H. Happ, L. Harding, M. Havens, H. Hull, D. Kincaid, P. Messina, M. Overton, R. Raffenetti, J. Stein, J. Walsh, and J. Wang. Additional assistance in providing the timing information for the tables of Section 4 was given by T. Pinter. Appreciation is also expressed for the very important feedback received from users not formally associated with the testing effort. Finally, we acknowledge the skill and cooperation of our typist, J. Beumer.

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Section 1

INTRODUCTION

The subset of the EISPACK package of Fortran IV programs included within this volume is a systematized collection of subroutines which compute the eigenvalues and/or eigenvectors for three special classes of matrix problems and the singular value decomposition of an arbitrary matrix. The three problem classes are real symmetric band, generalized real symmetric, and generalized real. The singular value decomposition, in turn, enables the solution of certain linear least squares problems. The subroutines are based mainly upon Algol procedures published in the Handbook series of Springer-Verlag by Wilkinson and Reinsch [1] and the QZ algorithm of Moler and Stewart [8] as extended by Ward [9]. They have been adapted and thoroughly tested on several different machines, and have been certified and are supported by the NATS project [3,4,5]. The machines for which they are certified include IBM 360-370, CDC 6000-7000, Univac 1110, Honeywell 6070, DEC PDP-10, and Burroughs 6700.

This manual is a user guide to these newer capabilities of EISPACK (complementing [10]) and to a control program EISPAC available with the IBM version of the package. It contains program segments which illustrate each of the basic computations with EISPACK and discusses variants of these that provide mild tradeoffs of efficiency, storage, and accuracy. Other sections of the guide discuss the validation procedures used for testing EISPACK, report execution times of the EISPACK subroutines on several machines, advertise the certified status and availability of EISPACK, and describe the major differences between the published Algol procedures in [1] and their Fortran counterparts. The final section includes detailed documentation with Fortran listings of each EISPACK subroutine referenced herein and the document for the control program.

Section 1.1

ORGANIZATION OF THE GUIDE

This guide is organized for the convenience, hopefully, of the user. Material most pertinent to the basic uses of the package and the control program appears in the early sections and references the more detailed and specific information in later sections. Here follows a brief description of the organization of the guide.

The remaining subsection of this introduction is a general statement with regard to the expected accuracy of the results from EISPACK. This statement is based upon the careful and detailed analyses of Wilkinson and others. Only a brief overview is provided in this subsection and the interested reader is directed to [1] and [2] for more detailed statements of accuracy.

Section 2 is divided into a prologue and four major subsections. The prologue introduces the concept of an EISPACK path, discusses the economies that can be realized with the use of the control program if available, and instructs on the selection among the 10 basic paths of this volume. The first subsection establishes several conventions that are useful in clarifying the discussions of the paths. It then details the 10 basic paths and associated control program calls in the form of program segments. Each program segment is introduced by a brief description of the problem it solves and any specific considerations needed for the path, and is followed by a summary of array storage requirements for the path and sample execution times on the IBM 370/195 computer. The next subsection describes possible variants of the 10 basic paths, focusing on those conditions for which the variants are to be preferred. The third subsection provides further information about specific details of EISPACK and the control program and

suggests several additional applications of the package. Complete sample programs illustrating the use of EISPACK and EISPAC to solve a specified eigenproblem appear at the end of this subsection. The last subsection describes the EISPACK capabilities to compute the singular value decomposition of a matrix and to solve an associated linear least squares problem.

Section 3 outlines the validation procedures for EISPACK that led to the certification of the package. Section 4 reports sample execution times of the individual subroutines and of several of the program segments of Section 2 and also discusses such considerations as the dependence of the execution times upon the matrix and the computer. The statement of certification for EISPACK, the machines and operating systems on which it has been certified, and its availability appear in Section 5. Section 6 itemizes the principal differences between the referenced Fortran subroutines and their Algol antecedents published in [1]. Finally, the documentation and Fortran listing for each subroutine appear in edited form in Section 7.

Section 1.2

ACCURACY OF THE EISPACK SUBROUTINES

The most useful statement that can be made with regard to the accuracy of the EISPACK subroutines is that they are based on algorithms which are numerically stable; that is, for every computed eigenpair (λ, z) associated with a matrix A , there exists a matrix E with norm small compared to that of A for which λ and z are an exact eigenpair of $A+E$. For generalized problems, the corresponding claim can be made after associating also with B a matrix of small relative norm. This backward or inverse approach in describing the accuracy of the subroutines is necessitated by the inherent properties of the problem which, in general, preclude the more familiar forward approach. However, for real symmetric band matrices the forward approach also applies, and indeed is a consequence of the backward analysis. For these problems the eigenvalues computed by EISPACK must be close to the exact ones, but a similar claim for the eigenvectors is not possible. What is true in this case is that the computed eigenvectors will be closely orthogonal if the subroutines that accumulate the transformations are used.

The size of E , of course, is crucial to a meaningful statement of accuracy, and the reader is referred to the detailed error analyses of Wilkinson and others ([1],[2]). In our many tests of EISPACK, we have seldom observed an E with norm larger than a small multiple of the product of the order of the matrix or system, an appropriate norm, and the precision of the machine.

Section 2

HOW TO USE EISPACK

This section is designed to provide, in readily accessible form, the basic information you need to correctly use subroutines from EISPACK to solve an eigenproblem. The way in which this information is presented is influenced by the design of the eigensystem package; hence we will first consider briefly the global structure of EISPACK.

EISPACK is capable of performing 32 different basic computations (22 of them described in [10]), plus several variations of them. If each of these computations (and variations) were performed completely within a single EISPACK subroutine, the package would be unwieldy indeed. It also would be highly redundant, since the same steps appear in many of the computations. To avoid these problems, the subroutines in EISPACK are designed so that each performs a basic step which appears in one or more of the computations. (See [7] for an introduction to the modularization of EISPACK.) Consequently, the redundancy (hence the size) of the package is minimized.

Another consequence is that, in general, more than one subroutine from EISPACK is required to perform a given computation. These subroutines must of course be called in the correct order and with the proper parameters; in addition, some computations also require certain auxiliary actions, e.g., initializing parameters and testing for errors. Throughout the remainder of this book such an ordered set of subroutine calls and associated auxiliary actions will be called an EISPACK *path*.

As a result of this structure the documentation for the use of EISPACK comprises two main parts: a description of the basic paths and their variations, and a description of the individual subroutines in EISPACK. The information about the paths constitutes the remainder of this section

while the subroutine documentation is collected in Section 7.

The path descriptions are divided into three parts. Section 2.1 describes the 10 basic paths and includes a table to facilitate reference to them. Section 2.2 describes some of the variations of these paths and suggests when such variations might be useful. Section 2.3 contains certain additional information about and examples of the use of EISPACK. To keep the descriptions of the basic paths and their variants simple, we have omitted much of the detailed information (e.g., the meanings of non-zero error indicators and the descriptions of certain parameters) and collected it in Section 2.3. Detailed information about each subroutine may be obtained from the documentation for the individual subroutines in Section 7. We hope, however, that the information given in this section will be sufficient to permit you to correctly solve most eigenproblems.

The detail of path information that you must know to solve certain basic eigenproblems can be reduced by using an appropriate driver subroutine to build the desired path from other EISPACK members. Applicability of the driver subroutines is limited to those problems where all eigenvalues and eigenvectors or all eigenvalues only are desired. There is a driver subroutine for each class of problems handled by the package; driver subroutine calls corresponding to six of the 10 basic paths are given as part of the discussion of the paths in this section.

Substantial further reduction in the detail of path information that you must know to solve an eigenproblem, with wider applicability, can be achieved by use of a control program, called EISPAC, which is available with the IBM version of the eigensystem package [6]. (It is only practical to implement this control program on those computing systems which adequately support execution-time loading of subroutines.) EISPAC accepts a relatively straightforward problem description stated in terms of the