MICROWAVE CIRCUIT THEORY AND ANALYSIS

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McGraw-Hill Book Company, Inc.

New York San Francisco Toronto London

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Library of Congress Catalog Card Number 62-17366

23146

PREFACE

The microwave art has progressed considerably during recent years. The complexities of the problems involved and the ever-increasing need for improved civil and military communications employing microwaves have attracted many mathematicians, physicists, and engineers to this new field. As a result, a tremendous store of information about various new theories and techniques in the microwave field is now available which was not incorporated in any textbook written prior to or during World War II. However, there appears to be a gap between the physical and mathematical developments in the microwave field. The object of this text is to attempt to bridge this gap and to present some basic principles of microwave theory and techniques. With Maxwell's equations as a starting point, the analytical treatments for many new microwave techniques which often lead to the design of some microwave components are incorporated in this text.

The complexities of the mathematical problems involved in microwave analysis usually call for various analytical techniques and mathematical tools, all of which may not be familiar to the average engineer. Therefore a brief review of elementary mathematics is presented at the beginning of this text. Perhaps one of the essential purposes of this mathematical review is to introduce unambiguously mathematical symbols and nomenclatures which are used throughout the text. In addition, a set of mathematical identities and formulas often useful for microwave field analyses are included in the review.

For a book of this nature certain prerequisites on the part of the reader have to be assumed. Familiarity with electrostatic and magnetostatic field, Maxwell's equations, and an elementary knowledge of real and complex variables will be desirable for the reader.

The sections on surface-wave lines, discontinuities in transmission lines, nonuniform lines, and nonreciprocal networks as presented in Chapters 7, 11, 12, and 13, respectively, are relatively new in the microwave field, and their inclusion may be considered almost necessary in any modern text since the theories and techniques described in these chapters constitute a major development in the microwave art. The unusual development of many microwave filter theories and techniques during and after World War II necessitates an entire chapter (Chapter 10) on microwave filters.

The rapid and continuous growth of microwave engineering in recent years would seem to discourage any attempt to write a book covering all new materials at a given date. There is always the possibility of encountering in the immediate future new discoveries on theories and techniques that can hardly be conceived today, but will increase in significance in years to come. As mentioned before, attempts have been made to include in the book some of the most recent topics in the microwave field wherever possible. With this object in mind, numerous technical papers and books have been freely consulted while preparing the text. The author realizes that no formal acknowledgment of these sources, of any kind, can be adequate, and he wishes to express his sincere gratitude to the authors and publishers of the sources referred to in various parts of the book. Grateful acknowledgment is also made for the use of some pioneering works in the microwave analysis field, such as "Principles of Microwave Circuits" by Montgomery, Dicke, and Purcell, which essentially formed the nucleus of one chapter.

In addition, the author acknowledges with sincere thanks the help and encouragement of his colleagues at the Ramo-Wooldridge Corporation and the Radio Corporation of America. He is particularly grateful to Drs. Duane Roller, George Jeromson, Thomas Stout, and Mildred Moe and Mr. Alex Stogryn for their many valuable suggestions. Sincere thanks are due to Mr. William Foss and Miss Andrea Haug for their help in organizing the manuscript.

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CHAPTER 1

INTRODUCTION AND EARLY HISTORY

The rapid advancement of microwave theories and techniques in the past two decades and their application in radio communication systems well justify the tremendous research and development effort that went into the improvement of the microwave art. Today microwaves are playing an ever-increasing role in the field of communication. Millions of miles of microwave facilities are now available for voice and television communications all over the world. In addition, microwave systems are being used for such services as teletype, telemeter, radar, radio beacons, remote control, and many others.

The microwave art is often considered to be one of the youngest branches of radio engineering. However, its foundation goes back to the origin of radio. It was perhaps Faraday who first clearly conceived the existence of electric and magnetic fields, but it was left to Maxwell to develop this conception into his all-encompassing electromagnetic field theory. His concept of the displacement current as necessary to account for the existence of electromagnetic waves and his postulate that electromagnetic waves must propagate in free space with the velocity of light may be considered the foundation of electromagnetic field theory and hence of microwave theory. Experimental verification of Maxwell's postulate and theory was first made by Hertz. He demonstrated the existence of electromagnetic waves and showed that their properties, such as reflection, refraction, and interference, are similar to those of light waves.

A tremendous amount of subsequent research and development was, of course, needed to bring the microwave art to its present stage. Brilliant researchers such as Green and Kirchhoff established a firm foundation of the electromagnetic theory at an early date. Later, eminent physicists and mathematicians—Rayleigh, Watson, Sommerfeld, and many others—contributed significantly to the theory of microwaves and the electromagnetic theory. One of the outstanding events in the history of microwave circuit theory was the recognition of the similarities between effects in different electromagnetic field configurations and those in inductive, capacitive, and resistive elements, the circuit elements that are so familiar to electrical engineers. The impedance concept for a traveling or a stationary wave, largely developed by Schelkunoff, enabled electrical engineers to treat a microwave component enclosing such a wave as if it were a circuit element in low-frequency communication systems.

The striking similarities between microwave components and low-frequency circuit components are more than a mere coincidence. When the basic circuit components of electrical engineering are viewed in their relationship to energy, a fundamental physical concept, one sees that the energy of an inductor is always stored in the magnetic field, whereas the energy of a capacitor is always stored in the electric field. Therefore any microwave component or combination of components that stores energy in the magnetic field may be considered to possess an equivalent inductance L. Similarly, any microwave component that stores energy in the electric field may be regarded as having a capacitance C. When the energy in a microwave component is dissipated at the boundary walls in the form of heat or is converted into radiation, the effect becomes similar to that of a resistance R in a low-frequency system. For sinusoidally time-varying circuits, the impedance or admittance encountered in microwave systems can be related uniquely to R, L, and C components in exactly the same way as in a low-frequency system. There is a one-to-one correspondence between the microwave and the low-frequency circuit components.

Unfortunately, a severe restriction is imposed in drawing the analogy between the microwave and low-frequency circuit components. Since the stored energy as well as the dissipated energy of any electromagnetic field depends on the volume enclosing the field or the surface exposed to the field, the R, L, and C parameters defined on the basis of energy become dependent on the ratios of the field dimension to the wavelength. For a low-frequency circuit, these ratios change very little for any reasonable frequency band, and hence a simple relationship can be established between the R, L, and C parameters and the impedance. In any microwave component, however, the dimensions are, in general, comparable to the wavelength. Therefore one can assign an inductance or capacitance value to the microwave component only when the frequency remains constant. Nevertheless, the equivalent-circuit concepts are quite useful, particularly for a relatively small bandwidth, as they enable electrical engineers to make use of a large store of information available in low-frequency-circuit theory.

Another important event in the history of microwaves was the interpretation and utilization of the "discontinuities" in microwave components. Any discontinuity that perturbs the normal field distribution generates an infinite number of modes which can store energy and which consequently give rise to equivalent reactances or susceptances. In many microwave components a discontinuity is a disadvantage since it produces undesirable reflections and thus lowers the transmission efficiency. In some modern microwave components, however, adjustable discontinuities are utilized as tuning devices, matching units, and for compensating for the mechanical irregularities introduced into components during their fabrication.

Still another important event in the history of microwaves was the introduction of nonreciprocal transmitting systems through the use of ferrites and similar ferromagnetic materials. Under certain conditions, electromagnetic waves propagated in opposite directions through a ferromagnetic material may have quite different characteristics. When such a nonreciprocal device is introduced in a transmission system, it produces little attenuation of the "forward" wave but a relatively large attenuation of the reflected wave. The amplitude of the reflected wave thus becomes very small in comparison with that of the incident wave. In other words, the generator experiences no reflection, and, in effect, the result is similar to that obtained when the generator is supplied with a matched termination. The applications of nonreciprocal networks have already proved to be useful for microwave circuits utilizing magnetrons whose frequency tends to drift with a mismatched load. Attempts are being made to utilize nonreciprocal networks in several other microwave circuits, including those of television systems, so as to prevent "ghosts" resulting from reflections.

The tremendous achievements obtained in communication systems through the use of microwaves can hardly be described in a few pages. Furthermore, even a brief description of the developments in all phases of microwave communication is well beyond the scope of the present volume. We shall therefore confine ourselves to a discussion of the general principles involved in microwave circuit theory and to the analysis and synthesis of the various components of passive microwave circuits.

CHAPTER 2

MATHEMATICAL REVIEW

2.1. Dirac's Delta Function

In electromagnetic fields which are excited by a point source, or even a line source, the conditions which the field components must satisfy at the source are, in general, quite different from those required outside the source. Stated differently, some boundary conditions, specified in a field, may exist only at the source. For the purpose of analysis of such problems it is convenient to use an improper function which, by definition, exists in the neighborhood of a point, say r_0, and which vanishes everywhere else on a one-dimensional space. Mathematically,

$$\int_\alpha^\beta \delta(r - r_0)\, dr = 1 \tag{2.1}$$

where r_0 is included in the interval (α, β), and δ is the improper function called the delta function. The existence of such a function can be conceived only through a limiting process.

When the source is located at the origin $r_0 = 0$,

$$\int_\alpha^\beta \delta(r)\, dr = 1 \tag{2.2}$$

where $\alpha < 0 < \beta$

From the definition of $\delta(r - r_0)$ it follows that for a function $f(r)$ which is continuous at $r = r_0$,

$$\int_\alpha^\beta f(r)\delta(r - r_0)\, dr = f(r_0) \tag{2.3}$$

Some important relations involving the delta function are noted below without proof.

$$\begin{aligned} f(r)\delta(r - r_0) &= f(r_0)\delta(r - r_0) \\ f(r)\delta'(r) &= -f'(r)\delta(r) \\ f(r)\delta^n(r) &= (-1)^n f^{(n)}(r)\delta(r) \\ \delta(r - r_0) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-j\omega(r-r_0)}\, d\omega \end{aligned} \tag{2.4}$$

where $j = \sqrt{-1}$ and δ' and δ^n denote the first and nth derivative of δ, respectively.

It should be noted that the delta function is not limited to one variable and that it can be extended to problems involving both surface and volume. For instance,

$$\begin{aligned} \delta(x - x_0)\delta(y - y_0) &= 0 \qquad x \neq x_0,\ y \neq y_0 \\ \iint \delta(x - x_0)\delta(y - y_0)\,dx\,dy &= 1 \\ \delta(x - x_0)\delta(y - y_0)\delta(z - z_0) &= 0 \qquad x \neq x_0,\ y \neq y_0,\ z \neq z_0 \\ \iiint \delta(x - x_0)\delta(y - y_0)\delta(z - z_0)\,dx\,dy\,dz &= 1 \end{aligned} \tag{2.5}$$

Specific applications of the delta function will be discussed in Sec. 2.4.

2.2. Matrices

Matrices and determinants serve as convenient mathematical tools for the solution of algebraic equations involving several variables of the type encountered in many circuit problems. Through the efforts of the

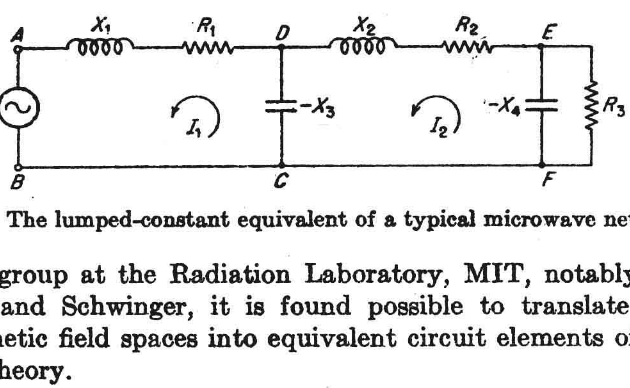

FIG. 2.1. The lumped-constant equivalent of a typical microwave network.

theoretical group at the Radiation Laboratory, MIT, notably those of Marcuvitz and Schwinger, it is found possible to translate bounded electromagnetic field spaces into equivalent circuit elements of the low-frequency theory.

Consider a lumped-constant equivalent of a microwave network as shown in Fig. 2.1. The voltage equations for the loop $ABCD$ and $DCFE$ can be written as

$$\begin{aligned} V &= I_1(R_1 + jX_1 - jX_3) + jI_2X_3 \\ 0 &= I_1(jX_3) + I_2\left(R_2 - \frac{jR_3X_4}{R_3 - jX_4} + jX_2 - jX_3\right) \end{aligned} \tag{2.6}$$

For such a system of linear equations, it is apparent that all investigations of the system may be carried out, perhaps with greater efficiency, by working only with an array of coefficients a_{ij}, with $i, j = 1, 2, \ldots,$ such that

$$\begin{bmatrix} V \\ 0 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{2.7}$$

where

$$
\begin{aligned}
a_{11} &= R_1 + j(X_1 - X_3) \\
a_{12} &= jX_3 \\
a_{21} &= jX_3 \\
a_{22} &= R_2 + j\left(X_2 - \frac{R_3X_4}{R_3 - jX_4} - X_3\right)
\end{aligned}
$$

The rectangular arrays of numbers in (2.7), which are called matrices, can be written symbolically as

$$\hat{V} = \hat{Z}\hat{I} \tag{2.8}$$

where $\hat{Z}$ is the impedance matrix of the system and is a square array of coefficients, and $\hat{V}$ and $\hat{I}$ are, respectively, the voltage and current matrices.

Each $\hat{V}$ and $\hat{I}$ matrix has one column. This type of matrix is often called a vector. Thus

$$\hat{V} = \begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ \cdot \\ V_n \end{bmatrix} \qquad \hat{I} = \begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ \cdot \\ I_n \end{bmatrix} \tag{2.9}$$

are vectors.

Diagonal Matrices. A matrix $\hat{Z}$ is called diagonal if it is a square, say $n \times n$, and if its off-diagonal elements are zero; that is,

$$\hat{Z} = \begin{bmatrix} Z_{11} & 0 & 0 & \cdots & 0 \\ 0 & Z_{22} & 0 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & 0 & \cdots & Z_{nn} \end{bmatrix} \tag{2.10}$$

Such a matrix is obtained in the network theory to represent the impedance of a circuit consisting of several loops so that there is no mutual impedance between any two loops. If the elements $Z_{11}, Z_{22}, \ldots, Z_{nn}$ are equal, the matrix is called a scalar matrix. If $\hat{Z}$ is a scalar matrix with each diagonal element equal to Z_0, and B is another $n \times n$ matrix, by multiplication, we obtain ZB, an $n \times n$ matrix with elements Z_0B_{11}, $Z_0B_{22}, \ldots, Z_0B_{nn}$, etc. This type of scalar multiplication of matrices results when the impedance matrix of a circuit includes one or more transformers. The scalar matrix whose diagonal elements are equal to unity is called an identity matrix and is denoted by $\hat{1}$. The product of any matrix with $\hat{1}$ yields the matrix itself. The concept of this identity matrix is necessary in defining the inverse matrix, which will be shown later.

The multiplication of two matrices is possible when and only when the number of columns in the first matrix is the same as the number of rows in the second. Consider, for example, the product of two matrices $\hat{A}(m \times n) \times \hat{B}(n \times s)$. This operation is defined to yield a matrix $\hat{C}$ given by

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1s} \\ b_{12} & b_{22} & \cdots & b_{2s} \\ \cdots & \cdots & \cdots & \cdots \\ b_{n1} & b_{n2} & \cdots & b_{ns} \end{bmatrix}$$

$$= \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1s} \\ c_{21} & c_{22} & \cdots & c_{2s} \\ \cdots & \cdots & \cdots & \cdots \\ c_{m1} & \cdots & & c_{ms} \end{bmatrix}$$

where

$$c_{11} = a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} + \cdots + a_{1n}b_{n1}$$
$$c_{12} = a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} + \cdots + a_{1n}b_{n2}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$c_{ms} = a_{m1}b_{1s} + a_{m2}b_{2s} + a_{m3}b_{3s} + \cdots + a_{mn}b_{ns}$$

or

$$c_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj} \tag{2.11}$$

EXAMPLE 1. The matrix product given below will further illustrate (2.11):

$$\begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} (2+9+20) & (4+12+24) \\ (1+6+15) & (2+8+18) \\ (4+15+30) & (8+20+36) \end{bmatrix} = \begin{bmatrix} 31 & 40 \\ 22 & 28 \\ 49 & 64 \end{bmatrix}$$

Inverse Matrix. The inverse matrix $\hat{Z}^{-1}$ is defined as the matrix which when multiplied by $\hat{Z}$ yields an identity matrix. In order that the matrix $\hat{Z}$ have an inverse, it is necessary, in general, that $\hat{Z}$ be a square matrix. If Z is a square matrix, $\hat{Z}^{-1}$ is also a square matrix. To obtain $\hat{Z}^{-1}$ we:

1. Obtain $\tilde{Z}$, the transpose of Z, that is, write $\hat{Z}$ with the rows and columns interchanged.
2. Find $Z^{(a)}$, that is, the adjoint matrix where each element of $\tilde{Z}$ is replaced by its cofactor.
3. Divide each element of $Z^{(a)}$ by the determinant of Z, denoted by $|Z|$.

The resultant matrix thus obtained is $\hat{Z}^{-1}$.

EXAMPLE 2. For a ferromagnetic material the permeability is not a simple scalar but can be expressed in the form of a matrix $\hat{\mu}$, given by

$$\hat{\mu} = \begin{bmatrix} \mu & -jk & 0 \\ jk & \mu & 0 \\ 0 & 0 & \mu_z \end{bmatrix}$$

Suppose it is desired to find $\bar{\mu}^{-1}$. We first find the transposed matrix:

$$\bar{\mu} = \begin{bmatrix} \mu & jk & 0 \\ -jk & \mu & 0 \\ 0 & 0 & \mu_z \end{bmatrix}$$

Replacing each element of $\bar{\mu}$ by its cofactor yields

$$\begin{bmatrix} \mu\mu_z & jk\mu_z & 0 \\ -jk\mu_z & \mu\mu_z & 0 \\ 0 & 0 & \mu^2 - k^2 \end{bmatrix}$$

Finally, the determinant of $\bar{\mu} = |\mu| = \mu_z(\mu^2 - k^2)$; hence

$$\bar{\mu}^{-1} = \begin{bmatrix} \dfrac{\mu}{\mu^2 - k^2} & \dfrac{jk}{\mu^2 - k^2} & 0 \\ -\dfrac{jk}{\mu^2 - k^2} & \dfrac{\mu}{\mu^2 - k^2} & 0 \\ 0 & 0 & \dfrac{1}{\mu_z} \end{bmatrix}$$

The use of matrices, usually in the form of scattering matrices, has proved very useful in the analysis and synthesis of microwave circuits, particularly for the multiport junctions in microwave networks. Such matrices for typical microwave circuits are discussed in detail in Chap. 9.

2.3. Vector Analysis

Physical quantities such as force, velocity, field intensity, etc., can be defined uniquely only when their magnitudes and directions are specified. These quantities are, by definition, vectors. The common laws of the algebraic operations such as addition, multiplication, and division which are applicable to scalars or quantities which can be specified by their magnitudes only are not applicable to vectors. These must therefore be redefined for vectors. In order to distinguish vectors from scalars, a boldface type will be used to denote the vectors in this text. The magnitude of the vector **F** will be represented by $|F|$ or simply F.

A complete analysis of the vector algebra is outside the scope of this book. We shall, however, for the purpose of review, discuss briefly those vector operations which occur frequently in microwave theory.

Scalar and Vector Products. Unlike scalars, it has proved convenient to define two independent types of vector multiplication, namely, the "scalar product" and the "vector product."

Let **A** and **B** be any two vectors. The scalar product of **A** and **B** is the quantity $|A|\,|B| \cos \theta$, where θ is the angle between the vectors **A** and **B**. $|A|$ and $|B|$ are the moduli of the vectors **A** and **B**, respectively.

Symbolically,

$$\mathbf{A} \cdot \mathbf{B} = |A|\,|B| \cos \theta \tag{2.12}$$

If the vectors are expressed in terms of their components in cartesian coordinates such that

$$\mathbf{A} = \mathbf{u}_x A_x + \mathbf{u}_y A_y + \mathbf{u}_z A_z$$
$$\mathbf{B} = \mathbf{u}_x B_x + \mathbf{u}_y B_y + \mathbf{u}_z B_z$$

where $\mathbf{u}_x$, $\mathbf{u}_y$, and $\mathbf{u}_z$ are unit vectors in the x, y, and z directions, respectively, then

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z \tag{2.13}$$

The scalar product of two vectors is also called the "dot product," or "inner product."

The "vector product," or "cross product," of **A** and **B** is, by definition, a vector directed normal to the plane containing **A** and **B** and is equal in magnitude to $|A|\,|B| \sin \theta$, θ being the acute angle between **A** and **B**. Symbolically,

$$\mathbf{A} \times \mathbf{B} = |A|\,|B| \sin \theta \, \mathbf{u}_n \tag{2.14}$$

where $\mathbf{u}_n$ is a unit vector normal to the plane containing **A** and **B**. The sense of the vector $\mathbf{u}_n$, that is, whether it is directed upward or downward from the plane containing **A** and **B**, is specified such that if a right-hand screw is turned in the same direction as if rotating **A** into **B** (through the smaller angle between them), $\mathbf{u}_n$ coincides with the direction in which the screw is driven.

From the definition of the vector product, one derives the following identities:

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A} \tag{2.15}$$

$$\mathbf{A} \times \mathbf{B} = \mathbf{u}_x(A_y B_z - A_z B_y) + \mathbf{u}_y(A_z B_x - A_x B_z) + \mathbf{u}_z(A_x B_y - A_y B_x) \tag{2.16}$$

where **A** and **B** are any two vectors.

Space and Time Derivatives of Vectors. Like scalar quantities, the differentiation of a vector can be carried out with respect to one or more of the variables involved in the vector. The differentiation of a vector (in, for example, cartesian coordinates) with respect to a scalar, say time, is expressed simply as

$$\frac{\partial}{\partial t} \mathbf{A} = \frac{\partial}{\partial t} (\mathbf{u}_x A_x + \mathbf{u}_y A_y + \mathbf{u}_z A_z) = \mathbf{u}_x \dot{A}_x + \mathbf{u}_y \dot{A}_y + \mathbf{u}_z \dot{A}_z \tag{2.17}$$

where a dot at the top denotes a differential operator $\partial/\partial t$.

Another differential operator very widely used in physical problems is known as the divergence. Symbolically,

$$\text{div } \mathbf{A} = \nabla \cdot \mathbf{A} \tag{2.18}$$