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Preface and acknowledgements

Mechanical systems may be considered as an example of nonlinear science, since the nonlinear effects can arise from a number of sources such as:

(a) geometric nonlinearities,
(b) nonlinear body forces,
(c) constitutive relations,
(d) kinematics and
(e) boundary conditions.

Research in classical nonlinear dynamics has had a long history. However, it is now recognised that, in addition to classical nonlinear behaviour represented by limit cycle, quasiperiodic motion, jump phenomena, etc., a deterministic system may exhibit aperiodic motion reminiscent of the random one. It has been suspected that examples of chaotic oscillations in mechanical systems were observed before the current era of chaos but were either ignored, were described as random or could not be explained at all.

This book presents the general methods of investigation of chaotic behaviour such as Lyapunov exponents, the Melnikov method, Poincaré maps, etc. These methods are then applied to nonlinear mechanical systems, where the chaotic oscillations are present. Throughout this book I have tried to emphasise the equal importance of both mathematical preciseness as well as mechanical systems applications. I hope that this material will be useful for mathematicians who are interested in applications as well as for mechanical engineers with an interest in the theory of oscillations.

I am deeply indebted to W. -H. Steeb, M. S. El Naschie and H. Isomäki for their valuable suggestions during discussions of the many problems treated in this book.

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Very special thanks go to A. V. Holden for inviting me to contribute with this monograph to the series and suggesting the revisions and additions that I hope have made this book more clear.

Finally, I would like to thank J. Brindley, A. P. Fordy and D. Knapp for their hospitality during my visit to the Centre for Nonlinear Studies at Leeds, where I had the opportunity to finish this work.

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Lodz, Johannesburg, Rosanow, Leeds
1989–90

Contents

1 Introduction

1.1 General introduction

The great interest in chaotic systems began after Lorenz's work on the model of atmospheric convection. Edward Lorenz 1963, in his famous work 'Deterministic nonperiodic flow', which was published in the *Journal of Atmospheric Science*, presents a system of three differential equations, which are deterministic but show very irregular (random-like) behaviour. It should be mentioned here that Poincaré (1898) considered the possibility of such systems, and many of the modern ideas and developments in chaos theory can be traced back to his classical work on celestial mechanics. He was the first to point out that the unstable motion of apparently simple systems can be extraordinarily complicated, but unfortunately the methods of his day could not solve the equations of motion of the solar system and other similar dynamical systems that he considered. He developed a new branch of mathematics, topology, which has turned out to be a powerful tool for describing chaotic systems. Later, Birkhoff (1927) proved what Poincaré had conjectured: the existence in the restricted planar three-body problem of infinitely many periodic orbits. These important results of Poincaré and Birkhoff are not readily applicable to problems in applied science, and so the recent explosive growth of chaotic dynamics and its applications follows from Lorenz's work. Of course, the response of deterministic differential equations describing a chaotic system cannot be random in the sense of the theory of stochastic processes. The most important reason is that a system of nonlinear deterministic differential equations has a unique solution for a given set of parameters and initial conditions. After the work of Lorenz, many other deterministic equations showing chaotic behaviour have been obtained, both as simple, analytically theoretical mathematical systems and as models of real physical, biological, or chemical systems. These include both systems of nonlinear ordinary differential equations and maps.

One of the simplest maps that exhibits chaotic behaviour is the logistic equation:

$$x_{n+1} = rx_n(1 - x_n) \qquad 0 < x_n < 1$$

The logistic map is described in detail in Chapter 4, and sample periodic and chaotic behaviours can be illustrated on a pocket calculator. For $r = 3.25$ after a short transient iteration we obtain two values of x_n in iteration; for $r = 3.5$ four values; and for $r = 4$ an irregular, nonperiodic sequence that approximates a chaotic solution is obtained.

Many more simple deterministic models with chaotic behaviour have been found (for example: May, 1976; Feigenbaum, 1978, 1980; Greborgi *et al.*, 1982; Jeffries & Perez, 1983; Yamaguchi & Sakai, 1983; Collet & Eckmann, 1980a,b; Nauenberg & Rudnick, 1981). Scientists from many different branches of science, including mathematics, physics, chemistry, biology, and mechanical, electrical and civil engineering, are working on chaotic experimental systems and models, and hence developing a new way of modelling the phenomena in the real world.

In this book we describe chaotic oscillations based on some examples from mechanical engineering. First, we will give fundamental information about chaotic systems and later describe a number of chaotic systems with applications in mechanics.

In §1.2 we explain the connection between stochastic and chaotic processes, pointing out that the theory of stochastic processes has been developed to describe irregular phenomena in deterministic systems that are too complicated, or have too many variables, to be fully described in detail. Poincaré maps, a powerful tool in the description of chaotic oscillations, are introduced in §1.3.

In Chapter 2 we present the definition of an attractor and describe its main properties. We define a chaotic attractor as one for which at least one Lyapunov exponent is positive. We also present the example of a hyperchaotic attractor (one with two positive Lyapunov exponents) for a system of two coupled oscillators.

The methods of identifying and quantifying chaotic behaviour such as the calculation of Lyapunov exponents from explicitly known equations of motion or experimental time series are presented in Chapter 3. Also, a new method of estimating Lyapunov exponents based on the symbolic dynamics method is presented. The Melnikov method of transverse homoclinic points is described as one that allows us to obtain a necessary condition for chaos. We describe it for systems with periodic and almost periodic perturbations and also describe a new method of estimation of power spectra of near-homoclinic motion. We also show here how spectral analysis can be useful in the investigation of chaotic systems.

Chapter 4 describes some typical routes to chaos: period doubling with its universal properties, intermittency and the quasiperiodic route associated with the breaking of a torus.

One of the most popular oscillators considered among others in mechanical systems, Duffing's oscillator, is presented in Chapter 5. We describe the first example of chaotic behaviour of this system that was found (Ueda's Japanese attractor) and later describe chaotic behaviour of the buckled beam equation, quasiperiodically forced Duffing's equation and systems with time delay. The analytical conditions for chaotic behaviour, including a new one based on Feigenbaum's universal properties of period doubling cascade, are presented.

The possibility of chaotic behaviour of another well known oscillator, Van der Pol's, is discussed in Chapter 6. First we present the condition for the existence of a limit cycle in this system and later we show the route to chaos in such systems.

Another classical oscillatory system, a mathematical pendulum, is described in Chapter 7. We show that the chaotic behaviour of this system is obtained after breaking the symmetry of motion. Finally, we compare the behaviour of a pendulum with the behaviour of a circle map.

In Chapter 8 we present examples of chaotic behaviour of systems with direct application in mechanical engineering, such as a rotor system, Freud's pendulum, oscillators with dry friction, a piecewise linear oscillator and oscillations of structures such as shells and arches. Irregular machine vibrations, which are present in manufacturing processes, may be caused by chaotic phenomena, as we have shown on a model of the cutting process. Finally, we show how to adopt the methods of stochastic processes to investigate the stability of a system forced by chaotic input.

In Chapter 9 we describe the properties of a new type of attractor, which is strange but not chaotic, as its largest Lyapunov exponent is not positive. We show that this type of attractor can occur in quasiperiodically forced systems and discuss the route to chaos via strange nonchaotic attractors.

The properties of basin boundaries are discussed in Chapter 10. We show that even when the system is not chaotic we cannot always predict its behaviour, as small changes of system parameters can move the system to another attractor.

Finally, in the Appendix we give more fundamental information on stability theory, averaging and Hopf bifurcation.

After a decade of extensive research in chaotic dynamics, its successes are well documented in a great number of papers and several books (see references).

The books can be divided into three groups:

(a) collections of papers or invited reviews,
(b) conference proceedings, and
(c) monographs.

Research in physics and fluid dynamics is well documented in two collections of papers (Hao, 1984; Cvitanovic, 1984). Collections of invited reviews, mainly by physicists, can be found in the series *Directions in Chaos* (Hao, 1988). Reviews by mathematicians, physicists and biologists can be found in Holden (1986).

In the last few years a great number of conferences and workshops devoted to chaotic dynamics have been organised. After some of them, the proceedings have been published (for example: Salam & Levi, 1988; Helleman, 1980, Kawakami, 1990, Christiansen & Parmentier, 1989). In most of them papers by researchers from various branches of science can be found.

Most of the monographs on chaos are written by mathematicians: Guckenheimer & Holmes (1983), Sparrow (1982), Steeb & Louw (1986), Lichtenberg & Lieberman (1982) and Ruelle (1989); or physicists: Berge *et al.* (1984), Schuster (1984), Kaneko (1986), Hao (1990) and Lee (1990). When they consider mechanical examples (for example: Guckenheimer & Holmes, 1983) these are discussed from a mathematical point of view. The only two books that specially consider the chaotic dynamics in mechanical and engineering problems are Moon (1987) and Thompson & Stewart (1986), where a number of examples of applications are presented. Examples of applications of the methods of chaotic dynamics in noisy mechanical systems can be found in Kapitaniak (1988c), which considers the effect of random noise on chaotic behaviour.

In this book equal attention is paid to the precise mathematical and mechanical applications and that is why it is different from the others.

1.2 Chaotic and stochastic processes

First, consider the following example of tossing a fair coin. Most of the texts on probability theory begin with it (Feller, 1964; Borowkov, 1972; Gihman & Skorohod, 1974). Assume that we are tossing a fair coin three times. The possible outcomes of this experiment are HHH, HHT, HTH, THH, HTT, THT, TTH, and TTT, where H denotes heads and T denotes tails. Each possible outcome of the experiment is called an elementary event. Thus, there are eight elementary events and it is by definition that the probability of obtaining, for example, HHT is 1/8, but we cannot obtain HHT repeatedly. Each event H or T occurs 'at random' with equal probability. Even if there are no external perturbations such as those produced by the circulation of the air during our experiment, the response is still random. The reason is that the response is 'random' because we cannot guarantee the same initial conditions of the coin and the same direction and value of impulse acting on the coin. So there is no mystery in our experiment and what we observe is a system that has a sensitive dependence on the initial condition and system parameter values.

Another system that seems to be typically stochastic is Brownian motion –

the perpetual irregular motions of small grains or particles of colloidal size immersed in a fluid, which were first noticed by the British botanist Brown in 1826. The irregular perpetual motion of a Brownian particle is the result of its collisions with molecules of the surrounding fluid. The colloidal particle is much bigger and heavier than the colliding molecules of the fluid, so that each collision has a negligible effect, but the superposition of many small interactions produces an observable effect. The molecular collisions of a Brownian particle occur in very rapid succession and their number is tremendous. This frequency is too high and the small changes in the particle's path caused by each single impact are too fine to be discerned by the observer. Thus, the exact path of the particle cannot be followed in any detail and this is the only reason why we have to consider this problem in a stochastic way, so we again have a deterministic system that we cannot fully describe.

Let us now turn to mechanical systems. The vertical vibration of a vehicle moving on a nonsmooth surface is usually given as a motivation of using stochastic methods in mechanical systems. But why is this system stochastic? The answer is that we cannot drive twice on exactly the same line with identically the same velocity, and it is much simpler to consider a stochastic model.

Where else do we use stochastic processes in mechanical system? One case is in modelling the responses of structures to earthquake, wind, etc., but again we do not know the full mechanism of these processes and its complete deterministic descriptions.

In all of the above examples of stochastic processes the system has been deterministic, and so in principle could be completely described. In practice we are using stochastic processes as an approximate description of a deterministic system that has unknown initial conditions and may have high sensitivity to initial conditions.

When we try to identify and model real systems sometimes we obtain as the result of the modelling process a model that shows very regular behaviour while the real system has very irregular behaviour. In that case we add random noise to our model and this noise represents no more than our lack of knowledge of system structure or inadequacy of the identification procedure – see Fig. 1.1.

This introduces the probability of the stability of a solution of a model to the addative noise term. Let us describe one of the methods of stochastic stability analysis that we will use in our future investigations.

We introduce a criterion pertaining to the almost sure stability (that is, stability with probability one) of a linear system described by the following equation:

$$\ddot{q} + 2\xi\dot{q} + [\omega^2 + \phi(t)]q = 0 \tag{1.1}$$

in which $\phi(t)$ is a stochastic (random) process.

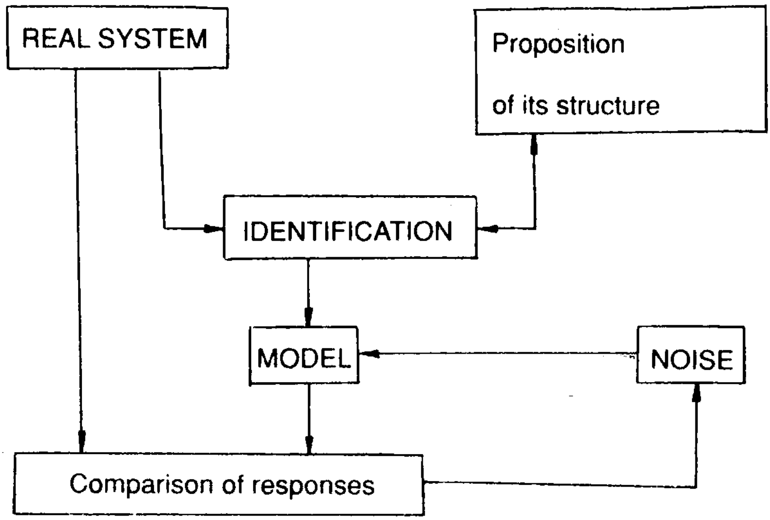

Fig. 1.1 Noise as a component of modelling.

The following description is based on Infante (1968) and Ariaratnam & Xie (1988). It will be assumed that the function $\phi(t)$

(a) is continuous on the interval $0 < t < \infty$ with probability one,
(b) is weakly stationary (that is, the first and second moments are stationary), and
(c) satisfies an ergodic property that guarantees the equality of time averages and ensemble averages with probability one.

The substitution of $y = q \exp(\xi t)$ into equation (1.1) results in the following governing equation:

$$\ddot{y} + [\omega^2 - \xi^2 + \phi(t)]y = 0 \qquad (1.2)$$

Denoting $y_1 = y$ and $y_2 = \dot{y}$, consider the positive definite function

$$v = \alpha^2 y_1^2 + y_2^2 \qquad (1.3)$$

which is the square of the norm of the vector $(\alpha y_1, y_2)$. The parameter α is to be determined. The derivative of $v(t)$ along the solution trajectories of equation (1.2) is given by

$$\dot{v} = -2[\alpha^2 + \xi^2 - \omega^2 - \phi(t)]y_1 y_2$$

and can be bounded as follows:

$$\dot{v} \leq [|\alpha^2 + \xi^2 - \omega^2 - \phi(t)|(1/\alpha)]v(t) \qquad (1.4)$$

After integration along the time axis, we have

$$v(t) \leq v(0) \exp\left(\int_0^t [|\alpha^2 + \xi^2 - \omega^2 - \phi(t)|(1/\alpha)] \, dt \right) \tag{1.5}$$

Also, by the given assumptions:

$$\lim_{t \to \infty} (1/t) \int_0^t [|\alpha^2 + \xi^2 - \omega^2 - \phi(t)|/\alpha] \, dt = E[|\alpha^2 + \xi^2 - \omega^2 - \phi(t)|/\alpha] \tag{1.6}$$

with probability one. Therefore, equation (1.5) will take the form:

$$v(t) \leq v(0) \exp\{tE[|\alpha^2 + \xi^2 - \omega^2 - \phi(t)|/\alpha]\} \tag{1.7}$$

Employing the inverse transformation from $y(t)$ back to $q(t)$:

$$E[|\delta - \phi(t)|] \leq 2\xi(\delta + \omega^2 - \xi^2)^{\frac{1}{2}} \tag{1.8}$$

for the almost sure asymptotic stability of the trivial solution $q = 0$ in equation (1.1). The Schwarz inequality is utilised in equation (1.8), resulting in the following condition:

$$E[\phi(t)] < 4\xi^2(\delta + \omega^2 - \xi^2) - \delta^2 + 2\delta E[\phi(t)] \tag{1.9}$$

Since $\delta = \alpha^2 + \xi^2 - \omega^2$ is yet to be determined by our choice of δ, the right-hand side of equation (1.9) can be optimised with respect to δ to obtain the largest region of stability. This procedure gives the sufficient stability condition

$$E[\phi^2(t)] - \{E[\phi(t)]\}^2 < 4\xi^2\{\omega^2 + E[\phi(t)]\} \tag{1.10}$$

as determined by Infante (1968). Equation (1.10) provides a sufficient criterion for almost sure stability of equation (1.1) and can be computed explicitly if the mean-square value (variance) of $\phi(t)$ is known. Morever when $E[\phi(t)] = 0$, then $E[\phi^2(t)]$ is the mean-square value of the function $\phi(t)$ and can be calculated by

$$E[\phi^2(t)] = \int_0^\infty S(f) \, df \tag{1.11}$$

where $S(f)$ is the power spectral density function of $\phi(t)$.

This stability analysis will be used in Chapter 8.

1.3 Poincaré map

The theoretical base for Poincaré maps was introduced by Poincaré (see Poincaré, 1890; Marsden & McCracken, 1976). The widespread use of computers with graphics facilities to examine chaotic behaviour in dynamical systems (Lorenz, 1963; Ueda, 1979) has led to the method of Poincaré maps becoming one of the most popular and the most illustrative method of describing 'strange attractors'.