

Bernd Jähne



Digital Image Processing

Concepts, Algorithms,
and Scientific Applications

4th edition

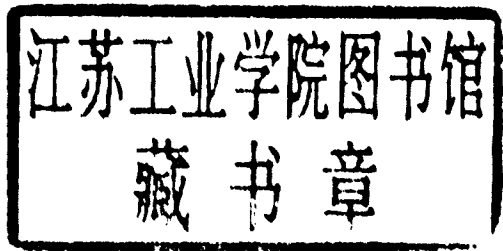


Springer

Bernd Jähne

Digital Image Processing

Concepts, Algorithms, and Scientific Applications
with 221 figures and CD-ROM



Springer.

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La Jolla, CA 92093-0230, USA

ISBN 3-540-62724-3 Springer-Verlag Berlin Heidelberg New York

Die deutsche Bibliothek - CIP Einheitsaufnahme

Digital image processing / Bernd Jähne. - Berlin ; Heidelberg ; New York ; London ; Paris ; Tokyo ;
Hong Kong ; Barcelona ; Budapest :

Springer

Dt. Ausg. u.d.T.:Digitale Bildverarbeitung

ISBN 3-540-62724-3

CD-ROM...[Zur 4., completely rev. and extended ed. -1997]

Buch..-4., completely rev. and extended ed. - 1997

Gb.

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Printed in Germany

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Typesetting: Camera-ready by author
Cover design: Struve&Partner, Heidelberg

SPIN:10573568

61/3020 - 5 4 3 2 1 0 - Gedruckt auf säurefreiem Papier

Preface to the Fourth Edition

In a fast developing area such as digital image processing a book that appeared in its first edition in 1991 required a complete revision just six years later. But what has not changed is the proven concept, offering a systematic approach to digital image processing with the aid of concepts and general principles also used in other areas of natural science. In this way, a reader with a general background in natural science or an engineering discipline is given fast access to the complex subject of image processing. The book covers the basics of image processing. Selected areas are treated in detail in order to introduce the reader both to the way of thinking in digital image processing and to some current research topics. Whenever possible, examples and image material are used to illustrate basic concepts. It is assumed that the reader is familiar with elementary matrix algebra and the Fourier transform.

The new edition contains four parts. Part 1 summarizes the basics required for understanding image processing. Thus there is no longer a mathematical appendix as in the previous editions. Part 2 on image acquisition and preprocessing has been extended by a detailed discussion of image formation. Motion analysis has been integrated into Part 3 as one component of feature extraction. Object detection, object form analysis, and object classification are put together in Part 4 on image analysis.

Generally, this book is not restricted to 2-D image processing. Wherever possible, the subjects are treated in such a manner that they are also valid for higher-dimensional image data (volumetric images, image sequences). Likewise, color images are considered as a special case of multichannel images.

The most significant extension in the fourth edition is the accompanying CD-ROM. The CD contains all the images and artwork of the printed version (and more) as TIFF or encapsulated postscript (EPS) files that can be read and displayed on almost any computer platform. The CD-ROM also contains material in the Adobe PDF format. The directory

Acroread on the CD contains PDF readers for different platforms. Unfortunately, only shortcuts are contained for the operating systems OS/2, Windows 3.1, and Windows 95/NT. If you use one of these operating systems, please make sure that you have installed an Adobe Acrobat reader on your computer (<http://www.adobe.com>) before you use the CD-ROM.

You can use the runtime version of the image processing software *heurisko* to gather your own experience with many of the image operations described in the book. The exercises are organized by chapter and are referred to by corresponding numbers in the written text. The image processing software *heurisko* on the CD runs under Windows 95 and Windows NT. A PC with at least a 486 processor is required.

I would like to thank all individuals and organizations that contributed partly unpublished material for this book:

- Dr. J. P. Burt, David Sarnoff Research Center, Princeton, New Jersey, USA
- Dr. J. Dengler and Dr. M. Schmidt, Department of Medical and Biological Computer Science, German Cancer Research Center Heidelberg
- Dr. B. Schmitt und Prof. Dr. D. Komitowski, Department for Histodiagnosics and Pathomorphological Documentation, German Cancer Research Center Heidelberg
- Dr. J. Steurer, Institute for Communications Technology, Technical University Munich
- Dr. R. Eils, P. Geißler, Dr. H. Haußecker, Dr. F. Hering, M. Lell, Dr. T. Münsterer, U. Schimpf, D. Schmundt, Dr. T. Scholz, Interdisciplinary Center for Scientific Computing, University of Heidelberg
- G. Balschbach, Institute for Environmental Physics, University of Heidelberg
- Dr. U. Schurr, Institute for Botany, University of Heidelberg
- Dr. J. Klinke and J. Dieter, Physical Oceanography Research Division, Scripps Institution of Oceanography, University of California, San Diego
- Dr. T. Scheuermann, Fraunhofer Institute for Chemical Engineering (ICT), Pfinztal
- ELTEC Elektronik GmbH, Mainz
- Dr. Klee, Hoechst AG, Frankfurt
- Stemmer Imaging GmbH, Puchheim
- Dr. J. Weickert, Imaging Center Utrecht, Utrecht University Hospital, Utrecht

- Carl Zeiss Jena GmbH, Jena
- Dr. G. Zinser, Heidelberg Engineering, Optische Meßsysteme GmbH, Heidelberg

I would also like to express my sincere thanks to the staff of Springer-Verlag for their constant interest in this book and their professional advice. Special thanks are due to my friends at AEON Verlag & Studio, Hanau, Germany. Without their dedicated and professional help it would not have been possible to produce this book and the accompanying CD-ROM.

Finally, I would like to thank all readers in advance for their comments on improvements or additions. Likewise, I am grateful for hints on errors, omissions, or typing errors, which, despite all the care taken, may have slipped attention.

Heidelberg, May 1997

Bernd Jähne

From the preface of the first edition

Digital image processing is a fascinating subject in several aspects. Human beings perceive most of the information about their environment through their visual sense. While for a long time images could only be captured by photography, we are now at the edge of another technological revolution which allows image data to be captured, manipulated, and evaluated electronically with computers. With breathtaking pace, computers are becoming more powerful and at the same time less expensive, so that widespread applications for digital image processing emerge. In this way, image processing is becoming a tremendous tool to analyze image data in all areas of natural science. For more and more scientists digital image processing will be the key to study complex scientific problems they could not have dreamed to tackle only a few years ago. A door is opening for new interdisciplinary cooperations merging computer science with the corresponding research areas.

Many students, engineers, and researchers in all natural sciences are faced with the problem of needing to know more about digital image processing. This book is written to meet this need. The author — himself educated in physics — describes digital image processing as a new tool for scientific research. The book starts with the essentials of image processing and leads — in selected areas — to the state-of-the art. This approach gives an insight as to how image processing really works. The selection of the material is guided by the needs of a researcher who wants to apply image processing techniques in his or her field. In this sense,

this book tries to offer an integral view of image processing from image acquisition to the extraction of the data of interest. Many concepts and mathematical tools which find widespread application in natural sciences are also applied in digital image processing. Such analogies are pointed out, since they provide an easy access to many complex problems in digital image processing for readers with a general background in natural sciences. The discussion of the general concepts is supplemented with examples from applications on PC-based image processing systems and ready-to-use implementations of important algorithms.

I am deeply indebted to the many individuals who helped me to write this book. I do this by tracing its history. In the early 1980s, when I worked on the physics of small-scale air-sea interaction at the Institute of Environmental Physics at Heidelberg University, it became obvious that these complex phenomena could not be adequately treated with point measuring probes. Consequently, a number of area extended measuring techniques were developed. Then I searched for techniques to extract the physically relevant data from the images and sought for colleagues with experience in digital image processing. The first contacts were established with the Institute for Applied Physics at Heidelberg University and the German Cancer Research Center in Heidelberg. I would like to thank Prof. Dr. J. Bille, Dr. J. Dengler and Dr. M. Schmidt cordially for many eye-opening conversations and their cooperation.

Then I contacted the faculty for computer science at Karlsruhe University and the Fraunhofer Institute for Information and Data Processing in Karlsruhe. I learnt a great deal from the course of Prof. Dr. H.-H. Nagel and Dr. R. Kories on "Algorithmic Interpretation of Image Sequences" that I attended in the summer term 1986.

I would also like to thank Prof. Dr. K. O. Münnich, director of the Institute for Environmental Physics. From the beginning, he was open-minded about new ideas to apply digital image processing techniques in environmental physics. It is due to his farsightedness and substantial support that the research group "Digital Image Processing in Environmental Physics" could develop so fruitfully at his institute. Many of the examples shown in this book are taken from my research at Heidelberg University and the Scripps Institution of Oceanography. I gratefully acknowledge financial support for this research from the German Science Foundation, the European Community, the US National Science Foundation, and the US Office of Naval Research.

La Jolla, California and Heidelberg, spring 1991

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Part I

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