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Laurence Wolsey (Eds.)

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Preface

This volume contains the papers presented at CP-AI-OR 2007: The Fourth International Conference on Integration of Artificial Intelligence, Constraint Programming, and Operations Research Techniques for Combinatorial Optimization Problems held May 23–26, 2007 in Brussels, Belgium. More information about the conference can be found at the Web site:

`http://www.cs.brown.edu/sites/cpaior07/Welcome.html`.

There were 80 submissions and each submission was reviewed by at least three Program Committee members. After careful consideration and discussion, the committee decided to accept 28 papers (one of which was withdrawn just before the publication of the proceedings). The papers submitted this year were of high quality and representative of a vibrant, multi-disciplinary community at the intersection of artificial intelligence, constraint programming, and operations research. The program also included three invited talks and two tutorials. We were extremely pleased that Claude Le Pape, George Nemhauser, and Bart Selman accepted our invitation to be invited speakers at the conference. Similarly, we were very fortunate to attract Thierry Benoist and John Chinneck to give tutorials at CP-AI-OR 2007. The conference was also preceded by a Master Class on constraint-based scheduling organized by Amedeo Cesta.

We would like to thank the Program Committee members who worked hard to produce high-quality reviews for the papers under tight deadlines, as well as the reviewers involved in the paper selection. We also would like to acknowledge the contributions of Laurent Michel (Publicity Chair), Barry O'Sullivan (Sponsorship Chair), Susanne Heipcke and Michael Juenger (Academic and Industrial Liaison Chairs), and Etienne Loute (Conference Co-chair) to the success of CP-AI-OR 2007. It is also a great pleasure to thank Fabienne Henry for her tremendous help in organizing the conference. The submissions, reviews, discussions, and the preparation of the proceedings were all handled by the EasyChair system. Finally, we would also like to thank the sponsors of the conference: The Association for Constraint Programming, the Cork Constraint Computation Centre (Ireland), ILOG S.A. (France), the Intelligent Information Systems Institute at Cornell University (USA), the European Commission (through the Marie Curie Research Training Network: ADONET), the Fonds National de Recherche Scientifique in Belgium, the Belgian Science Policy, ORBEL (Master Class), as well as Brown University (USA), the Facultés St. Louis (Brussels, Belgium) and the Université catholique de Louvain (Louvain-la-Neuve, Belgium).

March 2007

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Minimum Cardinality Matrix Decomposition into Consecutive-Ones Matrices: CP and IP Approaches

Davaatseren Baatar[1], Natashia Boland[1], Sebastian Brand[2],
and Peter J. Stuckey[2]

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University of Melbourne, Australia

Abstract. We consider the problem of decomposing an integer matrix into a positively weighted sum of binary matrices that have the consecutive-ones property. This problem is well-known and of practical relevance. It has an important application in cancer radiation therapy treatment planning: the sequencing of multileaf collimators to deliver a given radiation intensity matrix, representing (a component of) the treatment plan.

Two criteria characterise the efficacy of a decomposition: the *beam-on time* (length of time the radiation source is switched on during the treatment), and the *cardinality* (the number of machine set-ups required to deliver the planned treatment).

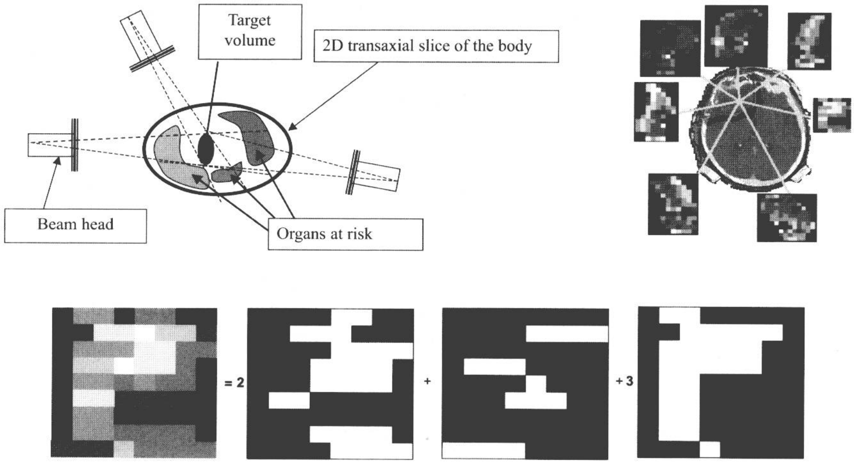
Minimising the former is known to be easy. However finding a decomposition of minimal cardinality is NP-hard. Progress so far has largely been restricted to heuristic algorithms, mostly using linear programming, integer programming and combinatorial enumerative methods as the solving technologies. We present a novel model, with corresponding constraint programming and integer programming formulations. We compare these computationally with previous formulations, and we show that constraint programming performs very well by comparison.

1 Introduction

The problem of decomposing an integer matrix into a weighted sum of binary matrices has received much attention in recent years, largely due to its application in radiation treatment for cancer.

Intensity-modulated radiation therapy (IMRT) has been increasingly used for the treatment of a variety of cancers [17]. This treatment approach employs two devices that allow higher doses of radiation to be administered to the tumour, while decreasing the exposure of sensitive organs (Fig. 1). The first is that the source of radiation can be rotated about the body of the patient: by positioning the tumour at a "focal point", and aiming the radiation beam at this point from various angles, the tumour receives a high dose from all angles, while the surrounding tissue only gets high exposure from some angles.

P. Van Hentenryck and L. Wolsey (Eds.): CPAIOR 2007, LNCS 4510, pp. 1–15, 2007.



$$\begin{bmatrix} 0 & 3 & 3 & 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 5 & 5 & 6 & 4 & 4 & 1 \\ 0 & 3 & 3 & 3 & 5 & 5 & 2 & 2 \\ 0 & 4 & 4 & 6 & 5 & 5 & 2 & 0 \\ 0 & 3 & 3 & 2 & 3 & 2 & 2 & 0 \\ 0 & 5 & 5 & 1 & 1 & 1 & 0 & 0 \\ 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 3 & 2 & 2 & 2 & 2 & 0 \\ 1 & 1 & 1 & 4 & 2 & 2 & 2 & 2 \end{bmatrix} = 2 \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} + 3 \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Fig. 1. Intensity-modulated radiotherapy

The second is more subtle, and involves repeated exposures from the same angle, where the uniform-intensity rectangular field of radiation delivered by the radiation source is "shaped" in a different way for each exposure, and each exposure can be for a different length of time. This process builds up a complex profile of received radiation in the patient's body, effectively converting the uniform radiation field delivered by the machine to an intensity-modulated field. The latter is usually described by discretising the 2-dimensional rectangular field, and specifying a radiation intensity level in each discrete element, representing the total length of time for which that element should be exposed to radiation.

A *treatment plan* for a single IMRT treatment session with a patient thus typically consists of a set of angles, together with a matrix for each angle, known as the *intensity matrix*, which represents the modulated field to be delivered at that angle. Typically the intensity is scaled so that the entries in the intensity matrix are integer. Indeed, they are usually quite small integers. Finding a good treatment plan is a challenging problem in its own right, and has been the subject of a great deal of research. We recommend the reader refer to the papers [13,9,15] and references therein.

In this paper, we assume a treatment plan is given, and focus on the delivery of the modulated field (intensity matrix) at a given angle. IMRT can be delivered by a variety of technologies: here we focus on its delivery via a machine known as a multileaf collimator, operating in "step-and-shoot" mode [7]. This machine delivers a rectangular field of radiation, of uniform intensity, that can be shaped through partial occlusion of the field by lead rods, or "leaves". These are positioned horizontally on the left and right side of the field, and can slide laterally across the field to block the radiation, and so shape the field. The discretisation giving rise to the intensity matrix is taken to be compatible with the leaf widths. In step-and-shoot mode, the leaves are moved into a specified position, the radiation source switched on for a specified length of time and then switched off, the leaves moved to a new position, and so on (Fig. 1).

The shaped radiation field delivered by the leaves in each position can be represented as a binary matrix, with 1's in elements exposed in that position, and 0's in elements covered by the leaves (Fig. 1). The structure of the machinery ensures that all 1's in any row occur in a consecutive sequence: the matrix has the consecutive-ones property. The length of time radiation is applied to the shaped field is called its beam-on time. To correctly deliver the required intensity matrix, the matrices corresponding to the shaped fields, weighted with their beam-on times, must sum to the intensity matrix.

This motivates the following problem specification.

2 Problem Specification and Related Work

Let I be an $m \times n$ matrix of non-negative integers (the intensity matrix). The problem is to find a decomposition of I into a positive linear combination of binary matrices that have the consecutive-ones property. Often the radiation delivery technology imposes other constraints on the matrices, but here we focus on the simplest form, in which only the consecutive-ones property is required. For convenience, we use the abbreviation C1 for a binary consecutive-ones matrix. We also refer to a shaped field, represented by a C1 matrix, as a *pattern.*

Formally, we seek positive integer coefficients b_k (the beam-on times) and C1 matrices X_k (the patterns), such that

$$I = \sum_{k \in \Omega} b_k X_k \tag{1}$$

where Ω is the index set of the binary matrices X_k, and for $k \in \Omega$:

$$X_{k,i,j_L} = 1 \wedge X_{k,i,j_R} = 1 \;\rightarrow\; X_{k,i,j_M} = 1 \tag{2}$$

for all $1 \leqslant j_L < j_M < j_R \leqslant n$ and all $i = 1, \ldots, m$.

Example 1. Consider the matrix

$$I = \begin{pmatrix} 2 & 5 & 3 \\ 3 & 5 & 2 \end{pmatrix}.$$

Two decompositions are

$$D_1 = 1\begin{pmatrix}0\,1\,1\\1\,1\,0\end{pmatrix} + 2\begin{pmatrix}1\,1\,1\\1\,1\,1\end{pmatrix} + 2\begin{pmatrix}0\,1\,0\\0\,1\,0\end{pmatrix} \quad \text{and}$$

$$D_2 = 2\begin{pmatrix}1\,1\,0\\0\,1\,1\end{pmatrix} + 3\begin{pmatrix}0\,1\,1\\1\,1\,0\end{pmatrix},$$

that is, we have $I = D_1 = D_2$. ◊

We denote by B and K the sum of coefficients b_k and the number of matrices X_k used in the decomposition (1), respectively, i.e.

$$B = \sum_{k\in\Omega} b_k \qquad \text{and} \qquad K = |\{\, b_k \mid k \in \Omega \,\}|\,.$$

We call B the *total beam-on time* and K the *cardinality* of the decomposition. The efficacy of a decomposition is characterised by its values for B and K: the smaller these values the better. In Example 1, the decompositions have the values $B_1 = B_2 = 5$, $K_1 = 3$ and $K_2 = 2$; so D_2 is preferred.

The problem of finding a decomposition that minimises B can be solved in polynomial time using linear programming or combinatorial algorithms [1,8,10,3]. However, it is possible for a decomposition to have minimal B but large K; indeed algorithms for minimising B tend to produce solutions with much larger K values than is necessary.

In radiation therapy, clinical practitioners would prefer solutions that minimise B, while ensuring K is as small as possible, i.e. they would prefer a lexicographically minimum solution, minimising B first and then K, written as $lex\_min(B, K)$. Since minimising B is easy, its minimal value, which we denote by B^*, is readily computable. The problem then becomes one of minimising K subject to the constraint that $B = B^*$. Although this problem, too, is NP-hard (it follows directly from the proof of NP-hardness of the problem of minimising K alone, given in [3]), it is hoped that solution methods effective in practice can be developed.

In the last decade, dozens of heuristic algorithms have indeed been developed, for example [1,8,10,6,3]; approximation algorithms are studied in [4]. Some of these attempt to find solutions in which both B and K are "small", while some seek low cardinality solutions while ensuring $B = B^*$ is fixed. An exact algorithm for the $lex\_min(B, K)$ problem has also been developed [10]: it is a highly complex, specialised enumerative algorithm that appears to carry out similar steps to those that might be expected in a constraint programming approach.

However the development of tractable exact formulations has lagged behind. Several exact integer programming models were introduced in [2] and [11] in order to solve the $lex\_min(B, K)$ problem, but these were either not tested computationally or were able to solve only small problems in reasonable CPU time. In this paper we develop a new model, that we refer to as the *Counter Model.* We derive both an integer programming formulation and a constraint programming method, and test both of these computationally against previous integer programming models. Our integer programming formulation performs substantially better than existing formulations, and the constraint programming approach provides the best computational results overall.

In the remainder of this paper, we first briefly review the existing integer programming formulations, and then present the Counter Model, our new integer programming formulation, and our constraint programming method. We then provide a computational comparison of these, and make our conclusions.

3 Existing Integer Programming Formulations

The central issue in modelling the $lex\_min(B, K)$ problem is that K, the cardinality of the decomposition, is unknown, and yet the natural variable indices depend on it. The two integer linear programming models in the current literature that can be used for the $lex\_min(B, K)$ problem take different approaches in tackling this issue. [11] overcome it by indexing according to radiation units; [2] instead calculates an upper bound on K. Here we give descriptions of these models and some additional symmetry breaking constraints.

Notation. The range expression $[a..b]$ with integers a, b denotes the integer set $\{ e \mid a \leqslant e \leqslant b \}$.

3.1 The Unit Radiation Model

The model of [11] focuses on individual units of radiation. It is based on the assumption that the total beam-on time is fixed, in our case to B^*. What is not known is: for each of the B^* units of radiation, what pattern should be used for the delivery of that unit? In the model, binary variables $d_{t,i,j}$ are used to indicate whether the element (i, j) is exposed in the tth pattern corresponding to the tth unit of radiation, for $t \in [1..B^*]$. They are linked to the intensity matrix by

$$I_{ij} = \sum_{t=1}^{B^*} d_{t,i,j}, \qquad \text{for all } i \in [1..m], j \in [1..n]. \tag{3}$$

The leaf structure in the pattern is captured by binary variables:

$$p_{t,i,j} = \begin{cases} 1 & \text{if the right leaf in row } i \text{ of pattern } t \text{ covers column } j, \\ 0 & \text{otherwise,} \end{cases}$$

$$\ell_{t,i,j} = \begin{cases} 1 & \text{if the left leaf in row } i \text{ of pattern } t \text{ covers column } j, \\ 0 & \text{otherwise,} \end{cases}$$

for all $t \in [1..B^*]$, $i \in [1..m]$, $j \in [1..n]$. The relationship between these three sets of binary variables is given by

$$p_{t,i,j} + \ell_{t,i,j} = 1 - d_{t,i,j} \qquad \text{for all } t \in [1..B^*], i \in [1..m], j \in [1..n], \tag{4}$$