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PREFACE

The National Symposium on Recent Developments in Theoretical Physics, co-sponsored by the Institute of Mathematical Sciences, Madras and the George Sudarshan Centre for Physics and Computer Science, C. M. S. College, Kottayam, was held at the latter Centre, from September 15 to 19, 1986. The Symposium brought together about 40 participants from leading research centres in the country besides a few from abroad.

The theme of the Symposium pertained to the recent advances made in the fields of theoretical physics, like high energy physics, condensed matter physics, general relativity and optics. The speakers reviewed recent advances made in frontier areas of physics such as: mathematical aspects of group theory, topology, geometry and gauge theories, regarding the theory of strings which offer the possibility of unifying the theory of gravitation with the strong and electroweak interactions; condensed matter physics, optics and general relativity. In the field of condensed matter physics, topics discussed included the quantum Hall effect, topological defects in solids, the behaviour of quantum fluids at very low temperatures, order-disorder phenomena, the rapidly developing field of chaos, and the interrelationship between statistical mechanics and particle physics.

The outstanding contributions to quantum mechanics made by Dirac and the mathematical style in modern physics, are the subject matter of two special lectures delivered. A special evening lecture on "Quantum Phases in Classical Optics" showed novel features that arise in the quantum mechanical study of the phase of a general optical system and in particular an optical wave propagating through an optical fibre.

The present volume is a collection of the detailed write-ups of most of the lectures delivered at the Symposium.

An afternoon boat trip on the backwaters of the Vembanad lake and a classical Carnatic music flute recital provided pleasant interludes to the serious deliberations.

The Symposium was largely funded by the Department of Science and Technology, Government of India. The successful conduct of the Conference was mainly due to the unstinted cooperation of the faculty of the C. M. S. College, Kottayam, which is gratefully acknowledged.

September 1987
Madras – 600 113

E. C. G. Sudarshan
K. Srinivasa Rao
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EDITORS' NOTE

The papers have been reproduced from the manuscripts submitted by the authors with only a very small number of corrections. The order of papers in this volume is almost the same as in the Conference sessions.

The Manuscripts of the talks delivered by J. Pasupathy on "Chiral Symmetry" and V. P. Nair on "Introductory remarks on String Theory" have not been received.

We record our appreciation and thanks to Mr. T. V. Vasudevan and Mrs. Gayathri Ekambaram for the excellent typing of most of the manuscripts.

E. C. G. Sudarshan

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CONTENTS

THE MATHEMATICAL STYLE OF MODERN PHYSICS

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What I wish to present under the title 'The Mathematical Style of Modern Physics' will not be the latest technical advances in this field, but instead some characteristic features it has acquired over the past few decades, and which are of course shared by recent developments.

A certain well-known book on mechanics describes physics as the science of measurement and change. In physics, as in other natural sciences, particular phenomena are isolated far enough to make precise observations and measurements, then models and theories are constructed in our minds to explain them and predict new phenomena. This involves relying on refined instruments of observation to aid our limited human senses, especially as we explore phenomena far removed from the human scale. Such instruments are of course based on previously understood phenomena, and can be regarded as extensions of ourselves. The important point is that as we look at processes taking place at the microscopic or the macroscopic level, far smaller or far larger than ourselves, intuition gathered from everyday experience often fails as a guid to understanding. In its place we have to develop and rely on mathematics as our guide, and make it into a sixth sense.

Mathematics is of course used, and most effectively, also to describe phenomena on our own scale, and it is easy to underestimate the difficulties faced in the past in the creation of new concepts. Be that as it may, it is generally agreed that with the developments of relativity and quantum theory the texture of theoretical physics has become much more subtle and abstract than might have been anticipated. This situation was described by Dirac in 1931 in these words:

"The steady progress of physics requires for its mathematical formulation a mathematics that gets continually more advanced. This is only natural and to be expected. What, however, was not expected by the scientific workers of the last century was the particular form that the line of advancement of the mathematics would take, namely it was expected that the mathematics would get more and more complicated, but would rest on a permanent basis of axioms and definitions, while actually the modern physical developments have required a mathematics that continually shifts its foundations and gets more abstract. Non-euclidean geometry and non-commutative algebra, which were at one time considered to be purely fictions of the mind and pastimes for logical thinkers, have now been found to be very necessary for the description of general facts of the physical world. It seems likely that this process of increasing abstraction will continue in the future and that advance in physics is to be associated with a continual modification and generalisation of the axioms at the base of the mathematics rather than with a logical development of any one mathematical scheme on a fixed foundation".

This passage conveys most eloquently the changing relationship between mathematics and physics at the fundamental level. It can well be contrasted with, say, the situation in fluid dynamics where the basic equations of Navier and Stokes have been known for a very long time, and the problem lies in solving them under various conditions.

As parts of this changing style in which mathematical structures are used in physical theories, I would like to describe two sets of ideas here. One is the increasing importance of the ideas of symmetry and invariance; the other is the often unavoidable use of unobservable quantities in physical theories.

On the eve of his retirement from the Institute for Advanced Study, Hermann Weyl gave a set of lectures on Symmetry which have since become a classic. In it he says: 'Symmetry, as wide or as narrow as you may define its meaning, is one idea by which man through the ages has tried to comprehend and create order, beauty, and perfection'. The subject of Weyl's discourse was symmetry in the static sense, the most immediate sense in which we all at first appreciate this notion. To say that an object is symmetric - such as a beautiful building or a well-grown crystal - is to say that it presents the same appearance before and after the application of certain transformations to it. These transformations are geometrical in character, being made up of rotations, reflections and translations; and the symmetry of an object is conveyed by the set of all transformations that leave it unchanged. The mathematical language to handle such static symmetry - static because time is not involved - is developed in Weyl's book, and is the theory of finite and of discrete groups. But the fotus of the present discussion is not the static symmetries of objects in space; rather it is the symmetries of physical laws describing processes taking place in space and time, and to appreciate this requires some amount of abstraction. In Bargmann's words, ".... those laws of physics which express a basic 'invariance' or 'symmetry' of physical phenomena seem to be our most fundamental ones".

Symmetry in this more fundamental sense operates at three levels which may be called the descriptive, the restrictive and the creative. To see this let us first recall with Wigner that there are three ideas of equal importance when discussing any set of physical laws; these are the laws themselves, then the allowed choices of initial conditions, and finally the symmetries of the laws. Again as Wigner says, "The purpose ..... of all equations of physics is to calculate, from the knowledge of the present, the state of affairs that will prevail in the future". To begin with, let us consider such deterministic laws of motion alone. So they tell us, given some observed initial condition of a physical system, how the system evolves and what its condition is at all later times. Thus each solution of the equations determines one possible sequence of states in time, one history, corresponding to one choice of initial condition. In this context, a symmetry is an operation that leads us from one solution of the equations of motion to another generally different one. Such a symmetry is not a property of the condition of a physical system at an initial or any other time; rather it consists in the unchanging relationship at each time between the physical conditions on two different histories or solutions of the equations of motion. As opposed to static symmetry, this is a dynamical concept describing a property of the concerned physical laws and not of this or that state or condition. It is the equations that are preserved under the symmetry operation; this makes it somewhat abstract since the symmetry "cannot be seen by the eye but only by the mind".

In this sense one says that the equations of mechanics of Galileo-Newton are symmetric or invariant under the transformations of the Galilei group. Similarly the Maxwell equations of the Faraday-Maxwell theory of electromagnetism are symmetric under the Lorentz-or better Poincare-transformations. And these are the two prime instances of the descriptive role of symmetry, since it happened in both cases that the relevant equations were discovered well before the complete understanding of their respective symmetries.

DESCRIPTIVE ROLE OF SYMMETRY

GALILEAN-NEWTONIAN MECHANICS	:	GALILEI GROUP AND TRANSFORMATIONS
FARADAY-MAXWELL ELECTROMAGNETISM	:	POINCARE GROUP AND LORENTZ TRANSFORMA TIONS

However from the early years of this century came a shift of emphasis and a change to a new point of view, due principally to Poincare and Einstein. It arose from the realisation that the Lorentz transformations and Lorentz invariance, though first seen in the context of Maxwell's equations, actually described general properties of space, time and measurement and so had a much wider significance. This led to the use of symmetry as a restrictive principle in the construction of new theories. In the words of Bergamann again, speaking of special relativity which governs space-time in the absence of gravitation: "....every physical theory is supposed to conform to the basic relativistic principles, and any concrete physical problem involves a synthesis of relativity and some specific physical theory".

Many striking examples of this restrictive role of symmetry are concerned with special relativity; some are in the framework of classical physics, others in connection with quantum theory and quantum mechanics, and yet others with quantum field theory. It is well worth devoting our attention to quickly recount them.

RESTRICTIVE ROLE OF SYMMETRY

MASS ENERGY EQUIVALENCE $E = mc^2$

TEN CONSERVATION LAWS

DIRAC-LORENTZ EQUATION

SOMMERFELD FINE STRUCTURE FORMULA

PHOTON MOMENTUM $P = E/c$

PLANCK'S $E = h\nu$ TO DE BROGLIE'S $P = \hbar k$

DIRAC ELECTRON EQUATION

WEYL NEUTRINO EQUATION

WIGNER ANALYSIS OF ELEMENTARY SYSTEMS

FERMI WEAK INTERACTION THEORY

PAULI SPIN STATISTICS THEOREM

TOMONAGA FEYNMAN SCHWINGER RENORMALIZATION THEORY

The most famous classical result is perhaps the equivalence of mass and energy, $E = mc^2$; this came from amending the Galilean-Newtonian mechanics of material particles so that it too would share the Lorentz invariance of electromagnetism. Thus the two separate prerelativistic conservation laws of mass and energy were combined into one. More generally, special relativity or Lorentz invariance of a theory (almost) automatically ensures the ten basic conservation laws of energy, momentum, angular momentum and moment of energy . One of the most impressive uses of this was Dirac's 1938 treatment of the classical relativistic point electron; using essentially only the energy-momentum conservation laws he was able to obtain equations of motion, now called the Lorentz-Dirac equations, including the radiation-reaction terms. In the period of the old quantum theory, one can recall the use of special relativity by Sommerfeld in deriving the fine structure of the hydrogen spectrum. To that same period also belongs the association of a momentum to a light quantum with the energy-momentum relation $E = pc$, which requires and can only be understood on the basis of special relativity. Slightly later, special relativity showed de Broglie the way to extend Planck's energy frequency relation $E = h\nu$ to his own momentum wave number relation $p = \hbar k$ for material particles: thus he associated a relativistic wave with a moving particle, the particle properties of energy-momentum being proportional to the wave properties of frequency and wave number through Planck's constant. Turning to quantum mechanics, one has first the amazing discovery of the relativistic wave equation for the electron by Dirac in 1928. It came about by combining three elements - the general structure of quantum mechanics, the requirement of symmetry with respect to special relativity, and the genius of Dirac - and it ended up explaining more things than its discoverer could have hoped for: the spin of the electron, its magnetic moment, the hydrogen fine structure, and the existence of the positron and antimatter. This last was of course a prediction and not an explana tion. After this inauguration of relativistic quantum mechanics, one can mention Weyl's discovery of the wave equation for the massless neutrino; and somewhat later the analysis by Wigner of the quantum mechanical representations of the symmetry group of special relativity, which gave a systematic classification of all possible free relativistic systems. Finally in this recounting of the restrictive role of symmetry we have some instances from quantum field theory and elementary particle physics. Soon after Fermi constructed a theory of the weak interactions in 1934, it was seen that on the basis of special relativity there were five independent forms for this interaction. This was based on the assumption that space reflection was a symmetry of nature. After it was shown by Lee and Yang in 1956 that this was not a valid symmetry for weak processes, the number of forms of interaction allowed by relativity jumped to ten; but it was quickly reduced to one by the discovery in 1957 of the universal V-A interaction by Sudarshan and Marshak.

This incidentally then led to a new symmetry called chirality. In quantum field theory itself the remarkable connection between spin and statistics - the fact for instance that photons obey Bose statistics while electrons obey Fermi statistics - was shown by Pauli to be a consequence of relativity. In fact he concludes his paper on the subject with the words: "....we wish to state, that according to our opinion the connection between spin and statistics is one of the most important applications of the special relativity theory". Later in the 1940's relativistic invariance was one of the crucial guiding principles that enabled Tomonaga, Feynman and Schwinger to develop a consistent way to handle divergences and infinities in quantum field theory calculations, the renormalization theory, and thus to make meaningful predictions that could be compared with experiment.

These illustrative examples of the restrictive function of symmetry show the power and the fruitfulness of the point of view introduced by Poincare and Einstein in the early 1900's. It is by carrying these ideas to one higher level of sophistication - so to speak by pursuing them to their logical conclusion in various contexts - that one arrives at the creative role of symmetry.

CREATIVE ROLE OF SYMMETRY

ABELIAN GAUGE INVARIANCE	----->	ELECTRODYNAMICS
GENERAL COORDINATE TRANSFORMATION INVARIANCE	----->	GENERAL RELATIVITY
NON ABELIAN GAUGE INVARIANCE	----->	YANG-MILLS THEORY