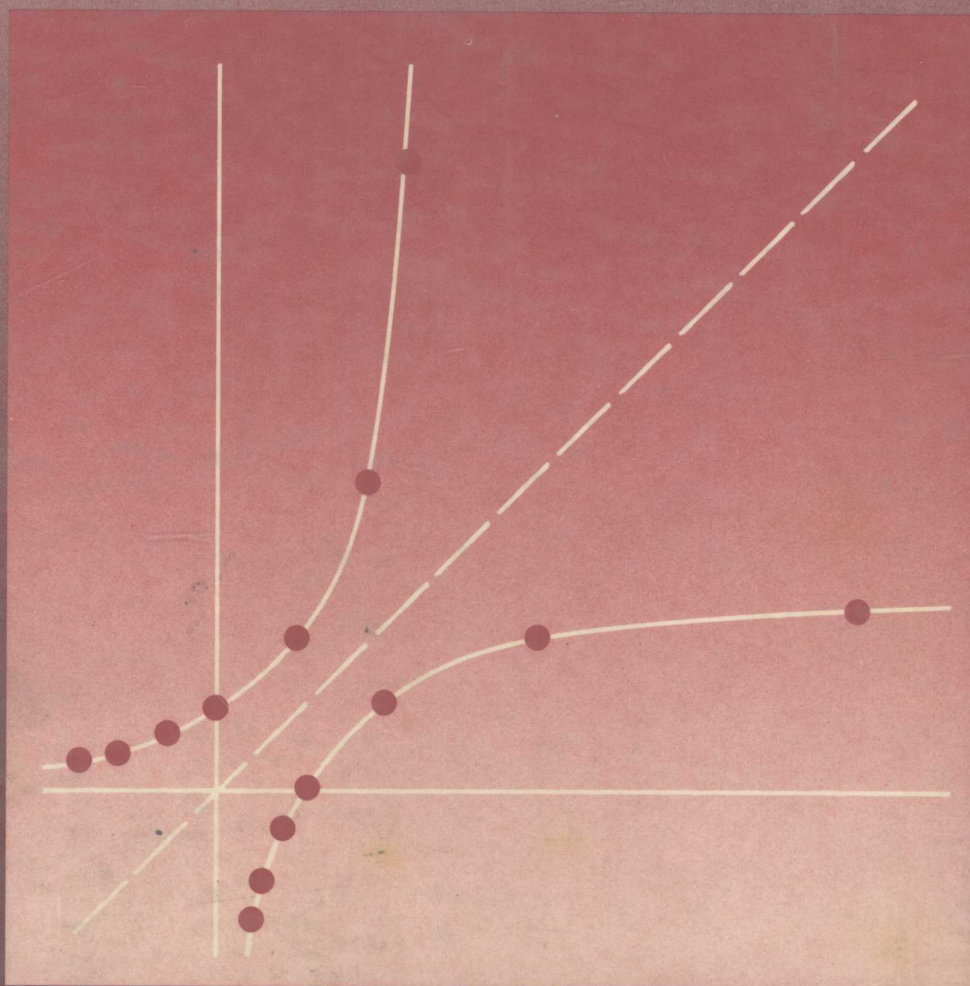


COLLEGE ALGEBRA

FOURTH EDITION

RAYMOND A. BARNETT

MICHAEL R. ZIEGLER



COLLEGE ALGEBRA

FOURTH EDITION

RAYMOND A. BARNETT

Merritt College

MICHAEL R. ZIEGLER

Marquette University

McGRAW-HILL BOOK COMPANY

New York St. Louis San Francisco Auckland Bogotá Caracas
Colorado Springs Hamburg Lisbon London Madrid Mexico Milan
Montreal New Delhi Oklahoma City Panama Paris San Juan
São Paulo Singapore Sydney Tokyo Toronto

COLLEGE ALGEBRA

Copyright © 1989, 1984, 1979, 1974 by McGraw-Hill, Inc. All rights reserved. Printed in the United States of America. Except as permitted under the United States Copyright Act of 1976, no part of this publication may be reproduced or distributed in any form or by any means, or stored in a data base or retrieval system, without the prior written permission of the publisher.

1 2 3 4 5 6 7 8 9 0 HALHAL 8 9 3 2 1 0 9 8

ISBN 0-07-003930-5

This book was set in Janson by Syntax International.
The editors were Robert A. Weinstein and James W. Bradley;
the interior and cover designer was Charles A. Carson;
the cover and chapter illustrator was Joseph Gillians;
the production supervisor was Leroy A. Young.
Project supervision was done by Phyllis Niklas.
Arcata Graphics/Halliday was printer and binder.

LIBRARY OF CONGRESS CATALOGING-IN-PUBLICATION DATA

Barnett, Raymond A.

College algebra/Raymond A. Barnett, Michael R. Ziegler.
4th ed.

p. cm.

Includes index.

ISBN 0-07-003930-5

1. Algebra. I. Ziegler, Michael R. II. Title.

QA154.2.B35 1989

512.9—dc19

88-8201

COLLEGE ALGEBRA

BARNETT & ZIEGLER'S COLLEGE ALGEBRA/PRECALCULUS SERIES

COLLEGE ALGEBRA: Includes less analytic geometry and a more unhurried (two-chapter) intermediate algebra review than in the other two books. Also includes a chapter on probability that is not in PRECALCULUS. No trigonometry is included.

COLLEGE ALGEBRA WITH TRIGONOMETRY: Intermediate algebra review is more concise than in COLLEGE ALGEBRA but less concise than in PRECALCULUS. More analytic geometry topics are included than in COLLEGE ALGEBRA, but less than in PRECALCULUS. Trigonometry starts with angles and right triangles; in PRECALCULUS, trigonometry starts with the unit circle. Also includes a chapter on probability that is not in PRECALCULUS.

PRECALCULUS: FUNCTIONS AND GRAPHS: Involves a higher-level intermediate algebra review and more analytic geometry topics than in either of the other two books. Trigonometry is introduced through the unit circle; in COLLEGE ALGEBRA WITH TRIGONOMETRY, trigonometry starts with angles. There is no probability chapter.

BOOKS BY BARNETT—KEARNS—ZIEGLER

Barnett—Kearns: ELEMENTARY ALGEBRA: STRUCTURE AND USE, 4th edition
Barnett—Kearns: ALGEBRA: AN ELEMENTARY COURSE, 2d edition
Barnett—Kearns: INTERMEDIATE ALGEBRA: STRUCTURE AND USE, 3d edition
Barnett—Kearns: ALGEBRA: AN INTERMEDIATE COURSE, 2d edition
Barnett: ESSENTIALS OF ALGEBRA FOR COLLEGE STUDENTS
Barnett—Ziegler: COLLEGE ALGEBRA, 4th edition
Barnett—Ziegler: COLLEGE ALGEBRA WITH TRIGONOMETRY, 4th edition
Barnett—Ziegler: PRECALCULUS: FUNCTIONS AND GRAPHS, 2d edition

Also Available from McGraw-Hill

SCHAUM'S OUTLINE SERIES IN MATHEMATICS AND STATISTICS

Each outline includes basic theory, definitions, and hundreds of solved problems and supplementary problems with answers.

Current List Includes:

Advanced Calculus	Linear Algebra
Advanced Mathematics	Mathematical Handbook
Analytic Geometry	Matrices
Basic Mathematics	Modern Algebra
Beginning Calculus	Modern Elementary Algebra
Calculus, 2d edition	Modern Introductory Differential Equations
College Algebra	Numerical Analysis
Complex Variables	Partial Differential Equations
Differential Equations	Geometry
Differential Geometry	Probability
Elementary Algebra	Probability and Statistics
Review of Elementary Mathematics	Projective Geometry
Finite Differences and Difference Equations	Real Variables
Finite Mathematics	Set Theory and Related Topics
First Year College Mathematics	Statistics
Fourier Analysis	Technical Mathematics
General Topology	Trigonometry
Laplace Transforms	Vector Analysis

SCHAUM'S SOLVED PROBLEMS BOOKS

Each title in this series is a complete and expert source of solved problems containing thousands of problems with worked-out solutions.

Current List Includes:

3000 Solved Problems in Calculus

AVAILABLE AT YOUR COLLEGE BOOKSTORE

ABOUT THE AUTHORS

Raymond A. Barnett, a native of California, and educated in California, received his B.A. in mathematical statistics from the University of California at Berkeley and his M.A. in mathematics from the University of Southern California. He has been a member of the Merritt College Mathematics Department, and was chairman of the department for four years.

Associated with four different publishers, Raymond Barnett has authored or co-authored eighteen textbooks in mathematics, most of which are still in use. In addition to international English editions, a number of the books have been translated into Spanish. Co-authors include Michael Ziegler, Marquette University; Thomas Kearns, Northern Kentucky University; Charles Burke, City College of San Francisco; and John Fujii, Merritt College.

Michael R. Ziegler received his B.S. from Shippensburg State College and his M.S. and Ph.D. from the University of Delaware. After completing postdoctoral work at the University of Kentucky, he was appointed to the faculty of Marquette University where he currently holds the rank of Professor in the Department of Mathematics, Statistics, and Computer Science.

Dr. Ziegler has published over a dozen research articles in complex analysis and has co-authored ten undergraduate mathematics textbooks with Raymond A. Barnett.

PREFACE

The fourth edition of *College Algebra* is one of three books in a precalculus series. See page ii for a brief comparison of the three books. Most chapters after the fourth are independent of one another and may be rearranged as desired to suit particular preferences or course outlines. See the chapter dependency chart on page xvii.

Improvements contained in this fourth edition of *College Algebra* evolved out of the generous and constructive response from users of the third edition and the careful work of the reviewers of the fourth edition while in manuscript form. Basic to a book's growth and effectiveness are use and feedback. Now in its fourth edition, *College Algebra* has had the benefit of a substantial amount of both.

- **Principal Changes from the Third Edition**

1. **Exposition** has been carefully reviewed by the authors, users in the classroom, and reviewers. Rewriting has been directed toward improving clarity and reducing excess wordiness.
2. **Balance, level, and pace:** The balance between computational skills, applications, and theory and the level and pace of developments have also been carefully adjusted in response to users' and reviewers' comments and suggestions.
3. **Boxed material** has been expanded to highlight important definitions, theorems, results, and step-by-step processes. All conditions for a theorem are now stated within the box containing the theorem.
4. More worked-out **examples** and **matched problems** have been added where appropriate.
5. **Pedagogy**, a continuing concern of the authors, is even stronger in this book. Developments are usually from the concrete to the abstract. There is an increased use of commentary in color small type, "think boxes" to illustrate steps that are usually done mentally, highlighting of common student errors, functional use of a second color, and boxed material for emphasis.
6. **Exercise sets** have been expanded and upgraded throughout. More challenging mechanics have been added to both the B- and C-level exercises and more problems on theory have been included in the C-level exercises.
7. **Applications** have been updated, and many more have been added throughout the book. Also, more worked-out application examples have been included. Applications are normally grouped at the end of an exercise set and are preceded by the title "Applications." The problems are generally of varying degrees of difficulty, with the most difficult problems marked with two stars (★★), the moderately difficult problems with one star (★), and the easier problems not marked.
8. All students should have a scientific calculator, since **calculator use** is assumed in most parts of the book. In some exercise sets there is a set of problems following the C-level problems that especially emphasize the use of a scientific calculator.

These are labeled “Calculator Problems.” Table use is not covered in the text, but tables are included in the back of the book for those who prefer that approach.

9. The **chapter review** sections have been completely reorganized. The first part of the review is composed of a check list of important terms, concepts, symbols, and symbolic forms (with section numbers indicating where each is encountered). A comprehensive review exercise follows. A separate practice test is no longer included in the review section. Instructors will be able to create their own practice tests (as well as quizzes, drill exercises, and tests) using the **computer testing system** available with this text (see Instructor Aids on page xv).
10. **Review topics** from intermediate algebra have been reorganized and rewritten and now are all in Chapter 1.
11. Problems that are especially **calculus-related** have been increased and are now identified with the integral symbol \int . Students not intending to take calculus will still find these problems of use in developing additional computational skills and conceptual insights.
12. **New topics** include: a new treatment of polynomial and rational inequalities (Sections 3-7 and 6-5); an improved treatment of complex numbers (Section 3-5); a completely rewritten chapter on graphs and functions (Chapter 4), which includes an expanded treatment of operations on functions, composition, and inverse functions; a completely rewritten chapter on exponential and logarithmic functions (Chapter 5) with greater emphasis on base e exponential functions and natural logarithmic functions; a new chapter on conic sections (Chapter 10); and a completely rewritten chapter on probability (Chapter 11).

• **Important Features Retained from the Third Edition** _____

1. The text is still **written for student comprehension**. Each concept is illustrated with a worked-out example, followed by a parallel problem with an answer (given at the end of the section) so that a student can immediately check his or her understanding of the concept. These followup problems also encourage active rather than passive reading of the text.
2. An **informal style** is used for exposition, statements of some definitions, and proofs of theorems.
3. The text includes **more than 3,000 carefully selected and graded problems**. The exercises are divided into A, B, and C groupings, with the A problems easy and routine, the B problems more challenging but still emphasizing mechanics, and the C problems a mixture of theoretical and difficult mechanics. In short, the text is designed so that an average or below average student will be able to experience success and a very capable student will be challenged.
4. The subject matter is related to the real world through many carefully selected **realistic applications** from the physical sciences, business and economics, life sciences, and social sciences. Thus, the most skeptical student should be convinced that mathematics is really useful.
5. **Answers** to all chapter review exercises and to all odd-numbered problems from the other exercises are included in the back of the book.

- **Student Aids**

1. **Common student errors** are clearly identified at places where they naturally occur (see Sections 1-5, 3-1, 3-3, 3-6).
2. **Think boxes** (dashed boxes) are used to enclose steps that are usually performed mentally (see Sections 1-2, 1-3, 3-3).
3. **Annotation** of examples and developments is found throughout the text to help students through critical stages (see Sections 1-2, 1-3, 1-4, 3-3).
4. **Functional use of a second color** guides students through critical steps (see Sections 1-3, 1-4, 2-3, 3-3).
5. **Calculator steps** are often included (see Sections 2-1, 2-2).
6. **Calculus-related problems** are often flagged with a special symbol \int (see Sections 2-1, 2-2).
7. **Boxes** are used to highlight important definitions, theorems, results, and step-by-step processes.
8. **Chapter review sections** include a check list of all important terms, concepts, symbols, and symbolic forms keyed to sections in which they occur. Also included is a comprehensive review exercise with all answers given in the back of the book.
9. **Summaries of formulas and symbols** (keyed to the sections in which they are introduced) are found inside the front and back covers of the text for convenient reference.
10. A **solutions manual** is available at a nominal cost through a book store. The manual includes detailed solutions to most chapter review exercises and most other odd-numbered problems.
11. A **computerized study guide** will be available through learning laboratories to students using this text. This tutorial provides additional coverage and support for all sections of the text. Students can work additional problems of many different question types, receiving constructive feedback based on their answers. Virtually no computer training is needed for the student to work with this supplement.

- **Instructor Aids**

1. A unique **computer testing system** is available to adopters without cost. The system provides the instructor with over 2,200 test questions from the text. Several test question types are available, including multiple choice, open-ended, matching, true-false, and vocabulary. The testing system enables the instructor to find these questions by section, topic, question type, difficulty level, and other criteria. In addition, instructors may add their own criteria and edit their own questions.
2. A **printed and bound test bank** is also available. This bank is a hard copy listing of the questions found in the computerized version.
3. An **instructor's answer manual** (that slips inside the back of the book) containing answers to the even-numbered problems not included in the text is available to adopters without charge.
4. A **solutions manual** (see Student Aids) is available to adopters without charge from the publisher.
5. An **instructor's resource manual** provides sample tests, transparency masters, and additional teaching suggestions and assistance.

- **Error Check**

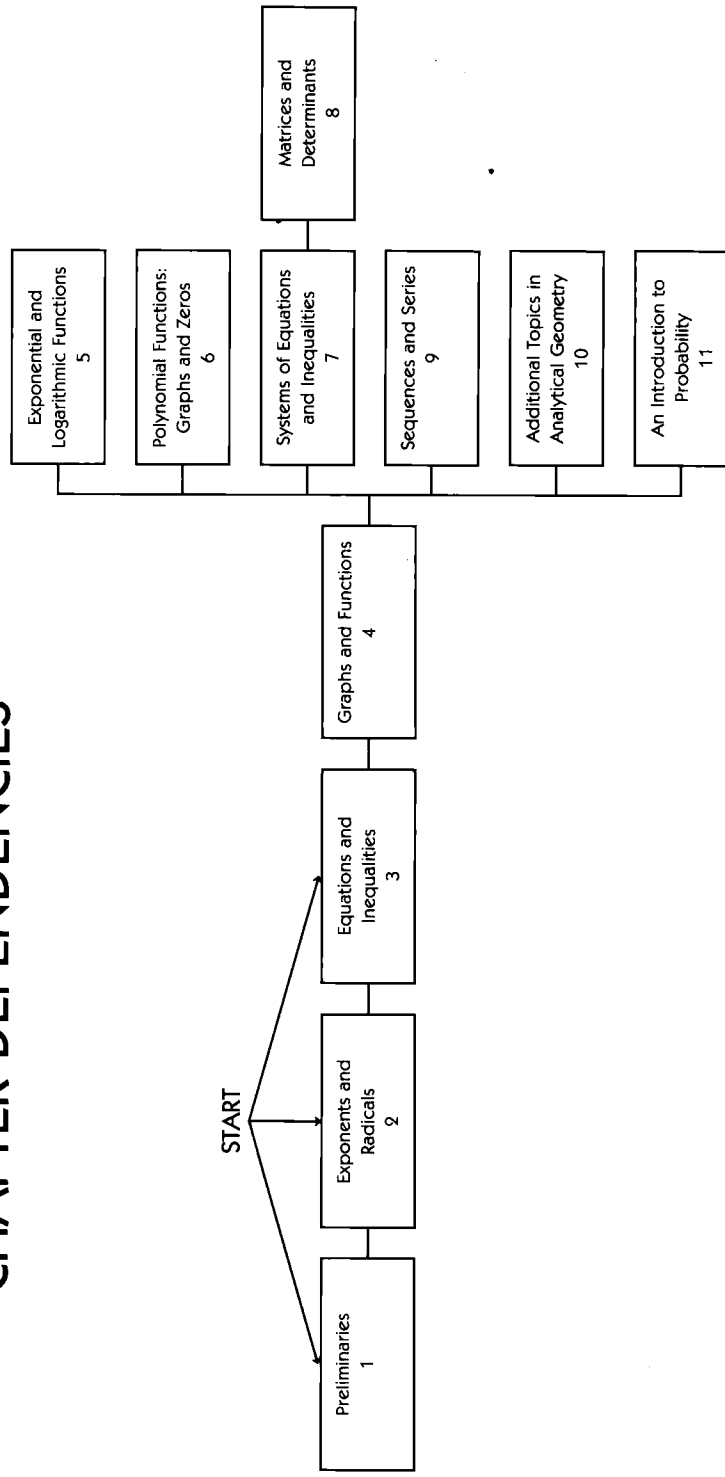
Because of the careful checking and proofing by a number of mathematics instructors (acting independently), the authors and publisher believe this book to be substantially error-free. For any errors remaining, the authors would be grateful if they were sent to: Mathematics Editor, College Division, 27th Floor, McGraw-Hill Book Company, 1221 Avenue of the Americas, New York, NY 10020.

- **Acknowledgments**

The preparation of a book requires the effort and skills of many people in addition to the authors. We would like to extend particular thanks to several very competent people: Phyllis Niklas and Shelly Langman, for their deftness and expertise in guiding the text; Fred Safier of City College of San Francisco, for his careful checking of the exercise sets and his skillful preparation of the solutions manual that accompanies the text; and Stephen J. Merrill and Robert E. Mullins of Marquette University, for their very careful checking of the whole book including the exercise sets.

We wish to thank all the reviewers for their many helpful suggestions and comments. (It is this process of use, feedback, and adjustment that produces an increasingly effective book for both students and instructors.) In particular, we wish to thank Thomas Bengtson, Pennsylvania State University; Robert Calcaterra, University of Wisconsin—Platteville; Ben L. Cornelius, Oregon Institute of Technology; Sami Deek, Richland College; David A. Felland, Ellsworth Community College; Robert Gold, Ohio State University; Mary McCammon, Pennsylvania State University; Julia R. Monte, Daytona Beach Community College; Linda Padilla, Joliet Junior College; Lawrence Runyan, Shoreline Community College; W. Arlene Starwalt—Jeskey, Rose State College; Raymond W. Tebbetts, San Antonio College; and Thomas J. Woods, Central Connecticut State University.

CHAPTER DEPENDENCIES



TO THE STUDENT

The following suggestions are made to help you get the most out of this book and your efforts.

As you study the text we suggest a five-step process. For each section:

- | | | |
|--|---|---|
| <ol style="list-style-type: none">1. Read a mathematical development.2. Work through the illustrative example.3. Work the matched problem.4. Review the main ideas in the section.5. Work the assigned exercise at the end of the section. | } | Repeat the 1-2-3 cycle until the section is finished. |
|--|---|---|

All of this should be done with a scientific calculator and plenty of paper, pencils, and a wastebasket at hand. In fact, no mathematics text should be read without pencil and paper in hand; mathematics is not a spectator sport. Just as you cannot learn to swim by watching someone else swim, you cannot learn mathematics by simply reading worked examples—you must work problems, lots of them.

If you have difficulty with the course, then, in addition to doing the regular assignments, spend more time on the examples and matched problems and work more A exercises, even if they are not assigned. If you find the course too easy, then work more C exercises and applied problems, even if they are not assigned.

Raymond A. Barnett
Michael R. Ziegler

REMARKS ON CALCULATOR USE

In many places in the text calculator steps for new types of calculations are shown (similar to those steps shown here). These are only aids. Try the calculation without the aid; then use the aid only if you get stuck.

Hand calculators are of two basic types relative to their internal logic (the way they compute): algebraic and reverse Polish notation (RPN). Throughout the book we will identify algebraic calculator steps with “A” and reverse Polish notation calculator steps with “P.” Let’s see how each type of calculator would compute

$$\frac{(5)(3)(2) - (7)(6)}{2(11)}$$

PRESS	DISPLAY
A: $\boxed{5} \boxed{\times} \boxed{3} \boxed{\times} \boxed{2} \boxed{-} \boxed{7} \boxed{\times} \boxed{6} \boxed{=} \boxed{\div} \boxed{2} \boxed{\div} \boxed{11} \boxed{=}$	$\boxed{-0.54545455}$
P: $\boxed{5} \boxed{\text{ENTER}} \boxed{3} \boxed{\times} \boxed{2} \boxed{\times} \boxed{7} \boxed{\text{ENTER}} \boxed{6} \boxed{\times} \boxed{-} \boxed{2} \boxed{\div} \boxed{11} \boxed{\div}$	$\boxed{-0.54545455}$

Some people prefer the algebraic logic and others prefer the Polish. Which is better is still being debated. The answer seems to rest with the type of problems encountered and with individual preferences.

In any case, irrespective of the type of calculator you own, it is essential that you read the user’s manual for your own calculator. A large variety of calculators are on the market, and each is slightly different from the others. Therefore, it is important that you take the time to read the manual. Do not try to read and understand everything the calculator can do; this will only tend to confuse you. Read only those sections that pertain to the operations you are or will be using; then return to the manual as necessary when you encounter new operations.

It is important to remember that *a calculator is not a substitute for thinking*. It can save you a great deal of time in certain types of problems, but you still must know how and when to use it.

Raymond A. Barnett
Michael R. Ziegler

COLLEGE ALGEBRA

CONTENTS

PREFACE	xiii
CHAPTER 1 PRELIMINARIES	1
1-1 Algebra and Real Numbers	2
1-2 Polynomials: Basic Operations	12
1-3 Polynomials: Factoring	19
1-4 Rational Expressions: Basic Operations	27
1-5 Chapter Review	35
CHAPTER 2 EXPONENTS AND RADICALS	39
2-1 Integer Exponents	40
2-2 Rational Exponents	47
2-3 Radicals	53
2-4 Chapter Review	61
CHAPTER 3 EQUATIONS AND INEQUALITIES	63
3-1 Linear Equations	64
3-2 Applications Involving Linear Equations	72
3-3 Linear Inequalities	81
3-4 Absolute Value in Equations and Inequalities	89
3-5 Complex Numbers	97
3-6 Quadratic Equations	105
3-7 Polynomial and Rational Inequalities	118
3-8 Equations Reducible to Quadratic Form	126
3-9 Chapter Review	131
CHAPTER 4 GRAPHS AND FUNCTIONS	135
4-1 Basic Tools; Circles	136
4-2 Straight Lines	147
4-3 Functions	158
4-4 Linear and Quadratic Functions	172
4-5 Operations on Functions; Composition	185
4-6 Inverse Functions	194
4-7 Chapter Review	204
CHAPTER 5 EXPONENTIAL AND LOGARITHMIC FUNCTIONS	209
5-1 Exponential Functions	210
5-2 The Exponential Function with Base e	220
5-3 Logarithmic Functions	229

5-4	Common and Natural Logarithms	237	
5-5	Exponential and Logarithmic Equations	245	
5-6	Chapter Review	254	
CHAPTER 6 POLYNOMIAL FUNCTIONS; GRAPHS AND ZEROS			257
6-1	Synthetic Division	260	
6-2	Remainder and Factor Theorems	264	
6-3	Fundamental Theorem of Algebra	270	
6-4	Isolating Real Zeros	275	
6-5	Rational Zero Theorem	281	
6-6	Chapter Review	290	
CHAPTER 7 SYSTEMS OF EQUATIONS AND INEQUALITIES			293
7-1	Systems of Linear Equations in Two and Three Variables		294
7-2	Systems and Augmented Matrices	308	
7-3	Gauss-Jordan Elimination	316	
7-4	Systems Involving Second-Degree Equations	327	
7-5	Systems of Linear Inequalities	333	
7-6	Linear Programming	343	
7-7	Chapter Review	354	
CHAPTER 8 MATRICES AND DETERMINANTS			359
8-1	Matrix Addition; Multiplication by a Number		360
8-2	Matrix Multiplication	366	
8-3	Inverse of a Square Matrix; Matrix Equations		373
8-4	Determinants	388	
8-5	Properties of Determinants	395	
8-6	Cramer's Rule	401	
8-7	Chapter Review	405	
CHAPTER 9 SEQUENCES AND SERIES			409
9-1	Sequences and Series	410	
9-2	Mathematical Induction	417	
9-3	Arithmetic Sequences and Series	424	
9-4	Geometric Sequences and Series	429	
9-5	Additional Applications	436	
9-6	Binomial Formula	437	
9-7	Chapter Review	443	
CHAPTER 10 ADDITIONAL TOPICS IN ANALYTIC GEOMETRY			445
10-1	Conic Sections; Parabola	446	
10-2	Ellipse	454	
10-3	Hyperbola	463	
10-4	Translation of Axes	475	
10-5	Chapter Review	481	