Nonlinear Stochastic Dynamic Engineering Systems

International Union of Theoretical and Applied Mechanics

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IUTAM Symposium Innsbruck/Igls, Austria
June 21-26, 1987

Springer-Verlag
Berlin Heidelberg NewYork
London Paris Tokyo

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ISBN 3-540-18804-5 Springer-Verlag Berlin Heidelberg NewYork
ISBN 0-387-18804-5 Springer-Verlag NewYork Heidelberg Berlin

Offsetprinting: Color-Druck, G. Baucke, Berlin
Bookbinding: Lüderitz & Bauer, Berlin
2161/3020 5 4 3 2 1 0

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Preface

This symposium, held at Innsbruck/Igls on June 21-26, 1987, is the fifth in a series of IUTAM-Symposia on the application of stochastic methods in mechanics. The first two meetings in Warwick (1972) and Southhampton (1976) concentrated on the stability of stochastic dynamical systems and stochastic methods in dynamics, respectively. The third meeting in Frankfurt/Oder (1982) added aspects of reliability, while the fourth symposium in Stockholm (1984) dealt mainly with fatigue and fracture problems. The general theme of the present symposium is devoted to nonlinear stochastic dynamics of engineering systems which is believed of great importance for providing the tools for basic development and progress in various fields of mechanical-, structural- and aeronautical engineering, particularly in the areas of vehicle dynamics, multi-storey structural dynamics, systems identification, offshore structural dynamics, nuclear structures under various stochastic loading conditions (i.e. wind-, earthquake-, parametric excitations, etc.). The contributions collected in this volume cover a wide spectrum of topics ranging from more theoretical, analytical and numerical treatment to practical application in various fields. The truly international character of the meeting is accomplished by 42 contributions and 86 participants from as many as 19 countries and hence, contributed to the original idea of IUTAM, which is to foster international cooperation. It should be recalled, that, for getting this cooperation started again after the First World War, Theodore von Kármán and Tullio Levi-Civita called the world's first international (IUTAM) conference on hydro- and aeromechanics in 1922 in Innsbruck, Austria.

We are indebted to the IUTAM Bureau for the allocation of financial support to participants and would like to express our gratitude to Austrian and German companies and government agencies listed above for their generous financial support to make this meeting possible. Last not least we thank the Scientific Committee who took the burden in helping to select the presentations from a large number of excellent contributions. The major credit, however, goes to the authors who's excellent presentations and papers made the meeting a successful one.

F. Ziegler, Wien
G. I. Schuëller, Innsbruck

Contents

CHAOTIC MOTIONS

EARTHQUAKE AND WIND RELATED PROBLEMS

SIMULATION TECHNIQUES

SPECIAL PROBLEMS

CLOSURE AND STOCHASTIC AVERAGING TECHNIQUES

HYSTERETIC SYSTEMS

RELIABILITY PROBLEMS

Equivalent Linearization and Linearization Techniques

Accuracy and Limitations of the Method of Equivalent Linearization for Hysteretic Multi-Storey Structures

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SUMMARY

A method of solution for determing the response statistics of a nonlinear hysteretic shear building subjected to nonstationary stochastic excitation is suggested by utilizing a stochastic equivalent linearization technique. This paper concentrates on the accuracy of the predicted response for an excitation ranging from low to high intensity and on the computational efficiency of the time step procedure. The nonlinear hysteretic properties of the shear building are modelled by Bouc's [1] model in terms of auxiliary variables following nonlinear differential equations. The auxiliary variables are linearized leading to a set of linear differential equations. They are solved numerically using the state vector formulation and a complex modal analysis time-step procedure in order to consider the time varying linearization coefficients. The procedure is applied to a six storey shear building with no residual linear stiffness to demonstrate the applicability to this special case. For the numerical efficiency of the time step procedure, a generalized Jacobi-iteration is suggested to solve in each time step efficiently the characteristic value problem. The predicted variances are then compared with the variances obtained by applying simulations procedures. The agreement is excellent for the velocity response, but less satisfactory for the displacement response. Finally, probability densities of response quantities are shown (based on 3000 simulated samples).

INTRODUCTION

Due to the severity of the loading conditions caused by earthquakes, sea waves or strong winds, structures may respond nonlinearly when subjected to these natural hazards. In addition, the loading characteristics reveal statistical properties and, consequently, they have to be modeled by stochastic processes. Thus, the stochastic properties of the nonlinear structural response need to be evaluated. In order to be useful for practical application, the analysis has to meet the following requirements:

(a) the procedure should apply to general nonstationary excitation characteristics to be described by evolutionary spectra,

(b) the procedure should be applicable to any type of structure discretized by a MDOF-system.

F. Ziegler, G. I. Schuëller, (Eds.)
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(c) the computational efforts required to obtain the response statistics should be considerable less than those needed for Monte Carlo simulative procedures.

For nonlinear systems, these requirements can only be met partially. The most severe discrepancy between requirement and capabilities of methods presently available holds for item (b). In fact, almost all procedures to evaluate the nonlinear stochastic response, such as those based on Fokker-Planck equation [2] and closure techniques [3] etc., are only applicable if the structure can be idealized by one, or at best, by very few degrees of freedom. Alternative approaches, such as the pertubation method, are restricted to weakly nonlinear structures and are therefore not suitable to treat the extreme range of loading effects which is needed for reliability analyses.

An approach with the highest potential to satisfy all three of necessities as stated above is the method of stochastic equivalent linearization. Hence its application to a hysteretic shear building subjected to nonstationary excitaton is addressed in this paper. Although stochastic equivalent linearization is regarded as most suitable for practical applications, its shortcomings should be pointed out nevertheless. One basic drawback of the approach is, that only second moment properties of the response quantities are obtained. Since the nonlinear response is known to be non-Gaussian, the obtained quantities are insufficient for a complete characterization of the response. Another shortcoming is its restriction to quite simple structures such as the shear beam models. A closer investigation shows that this limitation is not due to the characteristics of the solution technique, but to insufficient knowledge to describe the hysteretic behavior of more complex structures by constitutive laws which are simple enough to be linearized by the stochastic equivalent linearization technique.

Among all methods available within the framework of the equivalent linearization technique, complex modal analysis has been found to be most suitable to calculate the response of systems consisting of a large number of degrees of freedom. A particular advantage of the method when compared with other approaches, such as the use of the Lyapunov equation [4], [5], which for computational reasons, is in practice limited to the order of ten degrees of freedom - is its capability to incorporate evolutionary spectral excitation. This is very important when dealing with earthquake problems, where the low frequency content contributes significantly to the plastic deformation (drift). Another useful property of the approach is - similarly as for linear systems - the possibility of neglecting higher modes. The present investigation is focused on the accuracy of the predicted second moments for a wide range of excitation and on the computational efficiency of the time step procedure. Such an investigation is urgently needed, since most of the procedures available addressing the accuracy apply either for stationary excitation or SDOF-systems only. Only a recent work [6] compares the predicted results for a six storey shear building under nonstationary excitation with simulated results. However, the applied excitation might be classified as weak where the structure reacts basically linearly, and hence the reported good agreement has to be expected. Another important aspect is the computational efficiency of the time step procedure, which becomes more essential with increasing number of degrees-of-freedom.

HYSTERETIC SHEAR BEAM MODEL AND ITS LINEARIZATION

In the study as presented here, a hysteretic multi-storey structure is idealized by a simple coupled N-degree of freedom shear beam model subjected to horizontal ground acceleration a_g as shown in Fig. 1. The i-th restoring force, acting between the masses m_{i-1} and m_i, is represented by

$$q_i = \alpha_i k_i u_i + (1 - \alpha_i) k_i z_i \,; \qquad 1 \leq i \leq N \tag{1}$$

where k_i and u_i represent the i-th stiffness and relative displacement,

$$u_i = d_i - d_{i-1} \,; \qquad 1 \leq i \leq N \tag{2}$$

respectively. The parameter d_i denote the displacements of the i-th mass m_i relative to the ground. The term $(1-\alpha_i)k_i z_i$ models the i-th nonlinear hysteretic restoring force $q_{i,NL}$. The factor α_i, $(0 \leq \alpha_i \leq 1)$, defines the participation of a linear restoring force $k_i u_i$ within the total restoring force q_i. The hysteretic behavior of the nonlinear restoring force $q_{i,NL}$ is described by the auxiliary variable z_i with the dimension of displacements. Using the smooth hysteretic model as proposed first by Bouc [1], and later generalized by Wen [4], the auxiliary variable z_i is governed by the following nonlinear differential equation

$$\dot{z}_{i,NL} = A_i \dot{u}_i - \beta_i |\dot{u}_i| |z_i|^{n_i-1} z_i - \gamma_i \dot{u}_i |z_i|^{n_i} \tag{3}$$

where A_i, β_i, γ_i and n_i are parameters which control the shape of the hysteretic loops. The reader is referred to [4] and [5] for the salient variability of this model, capable to represent a wide class of hysteretic behavior including degradation effects and in close approximation such special cases as elasto-plastic hysteresis and the Coulomb slip model.

The restoring force law in eq. (1) is linear in u_i and z_i. In order to replace the nonlinear restoring force by a stochastic equivalent linear restoring force, only eq. (3) needs to be linearized. Since the right-hand side of eq.(3) is a function of the two variables u_i and z_i only, the linearized version of eq.(3) must be of the form,

$$\dot{z}_{i,L} = c_{e_i}\,\dot{u}_i + k_{e_i}\,z_{i,L} \tag{4}$$

where c_{ei} and k_{ei} depend on the nonstationary stochastic response of the linearized structural system. The linearization coefficients c_{ei} and k_{ei} are found by minimizing the difference function

$$e_i = \dot{z}_{i,NL} - \dot{z}_{i,L} \tag{5}$$

with respect to c_{ei} and k_{ei}, which is equivalent to

$$\frac{\partial\, E[e_i^2]}{\partial c_{e_i}} = \frac{\partial\, E[e_i^2]}{\partial k_{e_i}} = 0 \tag{6}$$

leading to two linear equations with respect to c_{ei} and k_{ei}. Under the assumption that $\dot{u}_i$ and z_i are jointly Gaussian, both with zero mean, Atalik and Utku [7] showed that the linearization coefficients can be derived from the following relation:

$$c_{e_i} = E\,[\,\partial z_{i,NL} / \partial \dot{u}_i\,] \quad \text{and} \quad k_{e_i} = E\,[\,\partial z_{i,NL} / \partial z_i\,] \tag{7}$$

From this, a closed form solution is obtained [4]:

$$\begin{aligned} c_{e_i} &= A_i - \beta_i\,F_{1i} - \gamma_i\,F_{2i} \\ k_{e_i} &= -\,\beta_i\,F_{3i} - \gamma_i\,F_{4i} \end{aligned} \tag{8}$$

where

$$F_{1i} = \frac{\sigma_{z_i}^{n_i}}{\pi}\,\Gamma\left(\frac{n_i+2}{2}\right) 2^{\frac{n_i}{2}}\, I_i\,; \quad F_{2i} = \frac{\sigma_{z_i}^{n_i}}{\sqrt{\pi}}\,\Gamma\left(\frac{n_i+1}{2}\right) 2^{\frac{n_i}{2}}\,; \quad I_i = 2\int_s^{\pi/2} \sin^{n_i}\phi\, d\phi$$

$$F_{3i} = \frac{n_i \sigma_{\dot{u}_i} \sigma_{z_i}^{n_i}}{\pi} \Gamma\left(\frac{n_i + 2}{2}\right) 2^{\frac{n_i}{2}} \left\{ \frac{2}{n_i} \left(1 - \rho_{\dot{u}_i z_i}^2\right)^{\frac{(n_i+1)}{2}} + \rho_{\dot{u}_i z_i} I_i \right\} \tag{9}$$

$$F_{4i} = \frac{n_i}{\sqrt{\pi}} \rho_{\dot{u}_i z_i} \sigma_{\dot{u}_i} \sigma_{z_i}^{n_i - 1} \Gamma\left(\frac{n_i + 1}{2}\right) 2^{\frac{n_i}{2}}; \qquad s = \tan^{-1}\left(\frac{\sqrt{1 - \rho_{\dot{u}_i z_i}^2}}{\rho_{\dot{u}_i z_i}}\right)$$

Considering all forces acting on the i-th mass, the equation of motion is defined by the following relation:

$$m_i \ddot{d}_i + c_i \dot{u}_i - c_{i+1} \dot{u}_{i+1} + q_i - q_{i+1} = -m_i a_g \tag{10}$$

If the relation $d_i = u_1 + u_2 + ... + u_{i-1} + u_i$ is used, eq. (10) can be written in the following matrix form:

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} + [G]\{Z\} = -[Mo]\{I\} a_g(t) \tag{11}$$

The above matrices of dimension N have the following non-zero coefficients [6],

$$\begin{aligned} &M_{ij, j \le i} = m_i; \quad C_{ij, j<i} = \alpha_c m_i; \quad C_{ii} = \alpha_c m_i + \beta_c k_i; \quad C_{i,i+1} = -\beta_c k_{i+1} \\ &K_{ii} = \alpha_i k_i; \quad K_{i,i+1} = -\alpha_{i+1} k_{i+1}; \quad G_{ii} = (1 - \alpha_i) k_i \\ &G_{i,i+1} = -(1 - \alpha_{i+1}) k_{i+1}; \quad Mo_{ii} = m_i \end{aligned} \tag{12}$$

where α_c and β_c are constants approximating the viscous damping and $\{I\}$ is the unit vector, i.e. $\{I\}^T = \{1,1,...,1,1\}^T$.

Finally the linearized restoring force law of eq. (4) is represented in matrix form:

$$\{\dot{Z}\} = [C_e]\{\dot{U}\} + [K_e]\{Z\} \tag{13}$$

where the matrices $[C_e]$ and $[K_e]$ are diagonal matrices with the components

$$C_{e_{ii}} = c_{e_i} \quad \text{and} \quad K_{e_{ii}} = k_{e_i} \tag{14}$$

COMPLEX MODAL ANALYSIS

In this section, the evaluation of the statistical response of a linearized hysteretic system subjected to nonstationary excitation is considered. As pointed out in the following, for several reasons the complex modal analysis is thought to be most suitable to calculate the nonstationary second moments of MDOF-system responses. In particular:

(a) it can be applied to MDOF-systems with a fairly large number N of degrees of freedom. For realistic levels of excitation the second moments can be calculated with sufficient accuracy by considering merely the first N+2p eigenpairs instead of all 3N eigenpairs, where $p < N$.

(b) modal analysis is suitable for colored stochastic excitation. This feature is important, since it is well known that the low frequency content of the excitation contributes significantly to the drift, i.e. the remaining plastic displacements.

(c) modal analysis is applicable to cases of strongly yielding systems with no residual linear stiffness, i.e. $\alpha_i = 0$.

(d) it is suitable for nonstationary excitation as required when dealing with earthquake problems. Nonstationarity is taken into account by a time step procedure, where a degrading hysteresis can be easily adopted.

(e) the time step procedure is computationally efficient. The immediate previous eigenvectors can be used as starting vectors to determine the eigenvectors and eigenvalues for each subsequent discrete time by requiring few iterations only.

Complex modal analysis utilizes the state vector approach to solve eq. (11) and (13) by an equivalent first order differential equation system of the form,

$$[A]\{\dot{X}(t)\} + [B(t)]\{X(t)\} = \{I_0\}\, a_g(t) \tag{15}$$

where the two vectors $\{X(t)\}$ and $\{I_0\}$ are defined as follows,

$$\{X(t)\} = \begin{bmatrix} \{U(t)\} \\ \{\dot{U}(t)\} \\ \{Z(t)\} \end{bmatrix} \quad \text{and} \quad \{I_0\} = \begin{bmatrix} \{0\} \\ -[M_0]\{I\} \\ \{0\} \end{bmatrix} \tag{16}$$

and the matrices [A] and [B(t)] consists of subsequent submatrices already defined in the