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Richard S. Sanford

Physical Networks

Richard S. Sanford

Associate Professor of Electrical Engineering
Clarkson College

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Preface

This book was designed to present a complete one or two term circuits and systems course suitable for all sophomore or junior engineers, while still giving sufficient detail and background for electrical engineering majors who might go on to more advanced courses in these areas. It gives an introductory analysis of linear, one-dimensional, constant-parameter, physical systems using the network approach as developed extensively in the "circuit theory" of electrical engineering, and which has recently been applied to many other types of lumped-parameter linear systems. Electrical, mechanical, hydraulic, acoustical, and thermal systems are treated in a unified manner as special cases of general network-analysis techniques using "through" and "across" variables and a general definition of impedance as a ratio of an "across" variable to a "through" variable. In this general definition, impedance is considered as a d-c impedance in d-c circuit analysis, an integral-differential impedance in classical differential-equation solutions, a transformed impedance in Laplace-transform analysis, or a complex a-c impedance in steady-state sinusoidal analysis. Thus, in Chapter 1 and 2, all of the basic techniques of network analysis and networks theorems are developed in general terms, using d-c resistive networks as specific examples. In subsequent chapters these concepts are extended to complete transient and steady-state analysis of electrical, mechanical, hydraulic, acoustical, and thermal systems. Signal-flow graph concepts and transfer functions are also introduced in Chapter 2.

Chapter 3 develops equivalent networks and integral-differential equations for reciprocal electrical and mechanical systems of all types, including those with mutual inductance and mutual inertia, and ideal transformers, levers, and gears. These networks are solved for initial conditions and initial and final values, although complete solution of the network differential equations is deferred until Chapter 5. Basic physical concepts of inductance, capacitance, masses, inertias, springs, etc., are developed briefly.

Chapter 4 presents the "background mathematics" necessary to solve the differential equations of Chapter 3. Topics include complex numbers, the complex plane, zeros and poles, sinusoidal functions, pulse functions, partial-fraction expansion, and Laplace transforms. These concepts are presented as briefly as possible as "mathematical tools" necessary for the solution of physical networks, with rigorous derivations, and so forth, left for concurrent mathematics courses.

In Chapter 5 the differential equations developed in Chapter 3 are solved by Laplace-transform methods for transient and steady-state solutions. Sinusoidal (a-c) solutions are presented as a special case of the general steady-state (particular-integral) solutions. The concepts of zeros and poles, frequency response, and resonance are introduced and applied to simple electrical and mechanical systems.

Chapter 6 presents a classical treatment of a-c circuits, including phasor diagrams, power relationships, and multi-phase systems. The latter part of this chapter covers the basic concepts of a-c machines and transformers in a manner most suitable for non-electrical engineering majors, since electrical engineers would get a much more thorough treatment in energy-conversion or machines courses.

Chapter 7 derives equivalent networks for hydraulic, acoustic, and thermal systems, as well as for combined systems, such as electro-mechanical systems and mechanical-hydraulic systems, which involve various types of signal transducers. Concepts of matrix analysis and two-port parameters are introduced and applied to various reciprocal and non-reciprocal systems, including simple vacuum-tube and transistor circuits.

Chapter 8 gives a brief introduction to general systems analysis involving closed-loop systems. Included are: stability analysis; and analysis and design by frequency-response (Bode) plots, by Nyquist plots, and by root-locus techniques. Design of passive one-port networks and of simple filters and equalizers, both d-c and a-c, is covered briefly, as are the basic concepts of carrier systems and modulation. Signal-flow graphs are treated fairly extensively and applied to the programming of analog computers in the simulation of various physical systems.

Three brief appendices cover use of determinants, calculation of limits, and give an introduction to Fourier series and the Fourier Integral and Fourier Transform.

This book thus gives a unified treatment to most of the topics covered in the typical electrical engineering sophomore-junior sequence of two or three one-term courses in the areas of d-c, a-c, and transient electrical circuit analysis, elementary network theory, and elementary systems analysis and control systems. In addition, many topics in linear mechanical, hydraulic, acoustic, and thermal systems not normally covered in these courses are treated in some detail.

To further the dual purpose of presenting a complete treatment of circuits and systems for non-electrical engineers, while still giving a good foundation in these areas for electrical engineers, a number of topics in the first half of the book are marked (OPTIONAL) and can be omitted or delayed without loss of continuity. Thus, most of chapters 1, 2, 3, 5, and 6, and whatever parts of 4 are necessary depending on prior and concurrent mathematics courses, can constitute a basic one-term course in circuits suitable for all engineers. Parts of chapters 7 and 8, possibly supplemented by some work on transistor and electron-tube devices, round out a second term on circuits and systems for non-electrical engineers. The optional portions of the first six chapters, along with a thorough coverage of chapters 7 and 8, constitute a good second term, and possibly much of a third term for electrical engineering majors. To keep the book topical for different engineering disciplines, many examples and problems from the different areas of engineering are given, including a number of problems typical of the professional engineering exams required of civil engineers in particular.

Prerequisites for this book include an introduction to the basic concepts of electricity and magnetism as taught in the usual freshman or sophomore physics courses and some knowledge of differentials, integrals, and mathematical functions as covered in the usual first-term sophomore mathematics course. Actual solution of differential equations is not assumed, although such a study in concurrent mathematics courses would be highly desirable.

I wish to thank the many people at Clarkson, both students and staff, (in particular our former E.E. Department Chairman, Whit Reed) whose suggestions, encouragements, and contributions over the past few years have been invaluable in preparing this book. Many valuable concepts and ideas acquired in the past three Summer Institutes in Elecrtical Engineering at Worcester Polytechnic Institute, directed by Dr. Glen Richardson, are also reflected in these pages. I also give thanks to Margaret, Ellen, Lorri, and especially Angie, for their help in typing, with a special word of appreciation to Eric Peper of Prentice-Hall for his assistance and suggestions in preparing the manuscript.

RICHARD SANFORD

Potsdam, N.Y.

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Chapter One

Characteristics and Classifications of Physical Systems

1.1 Introduction

This book is an introduction to the analysis of linear, constant-parameter, one-dimensional physical systems which can be represented by equivalent networks composed of ideal linear two-terminal elements. This chapter explains that statement and the terms in it, and gives an introduction to the characteristics and definitions of physical systems and the ways in which these systems may be classified. In short, this first chapter defines the area of our study and its relationship to other types of physical problems.

These physical systems embrace, from both a design and an analysis viewpoint, a large portion of the problems with which engineers deal. They include problems in electronics and communications, mechanical control and positioning systems, system engineering, computers and data processing systems, missile and guidance systems, and a host of other fields. While it is true that many, if not most, of these problems are too complex to be included in our limited category of "linear constant-parameter one-dimensional systems," still many of them can be approximated by such systems or include such systems as component parts of the complete problem. Moreover, the basic network analysis techniques developed here can be extended to handle many of these more complex problems, and are also stepping stones to other powerful and useful methods and techniques which you may study later.

Actually, engineering in its most useful form is more concerned with the design or *synthesis* of physical systems to do a specific job, than with the analysis of systems already in existence. However, a thorough understanding of methods for analyzing existing systems is obviously a prerequisite for being able to synthesize a new system or improve on the design of an existing one. While there are a number of ways of analyzing various types of physical systems, the so-called *network approach*, which we shall use, combines relative simplicity with considerable generality when applied to the linear one-dimensional systems with which we shall be concerned. This network approach consists, in its simplest form, of representing a system by an *equivalent network* composed of idealized linear two-terminal elements related to the parameters or constants of the actual system. The equivalent network represents one form of a "linear graph" of the equations of the original physical system, and these equations can be written in a straightforward manner by inspection of the network. The solution of these equations then gives the reaction, or *response*, of the original system to a given set of conditions and inputs. This then, in brief, is the "network approach": (1) derive an appropriate equivalent network; (2) write network equations; (3) solve equations; (4) apply this network solution with care to the actual system. An important advantage of the network approach over some other methods of analysis is that it is a systematic method, the steps of which follow a straightforward, easy-to-learn pattern, readily adaptable to many types of problems.

The network approach to the solution of physical problems was originally developed for analyzing electrical circuits, for which it is particularly well suited. However, the generality of this method is now well recognized, and it is also applied to many different types of physical systems, such as mechanical problems of translational and rotational motion, hydraulic or "fluid-flow" problems, thermal or "heat-flow" problems, acoustical problems, etc. We shall, therefore, study network analysis techniques from a very general point of view which recognizes this basic underlying unity among problems in apparently unrelated fields. Examples will be taken from all these areas although, to begin with, most of the examples will be from d-c electrical circuits because of their relative simplicity and your presumed familiarity with them.

1.2 Response of Physical Systems

1.2.1. ***Response to an input.*** Physical systems will respond in a predictable manner to a given *input* or *driving function* or stimulus; and it is the calculation of this *response* or *output* with which we shall be concerned. For example,

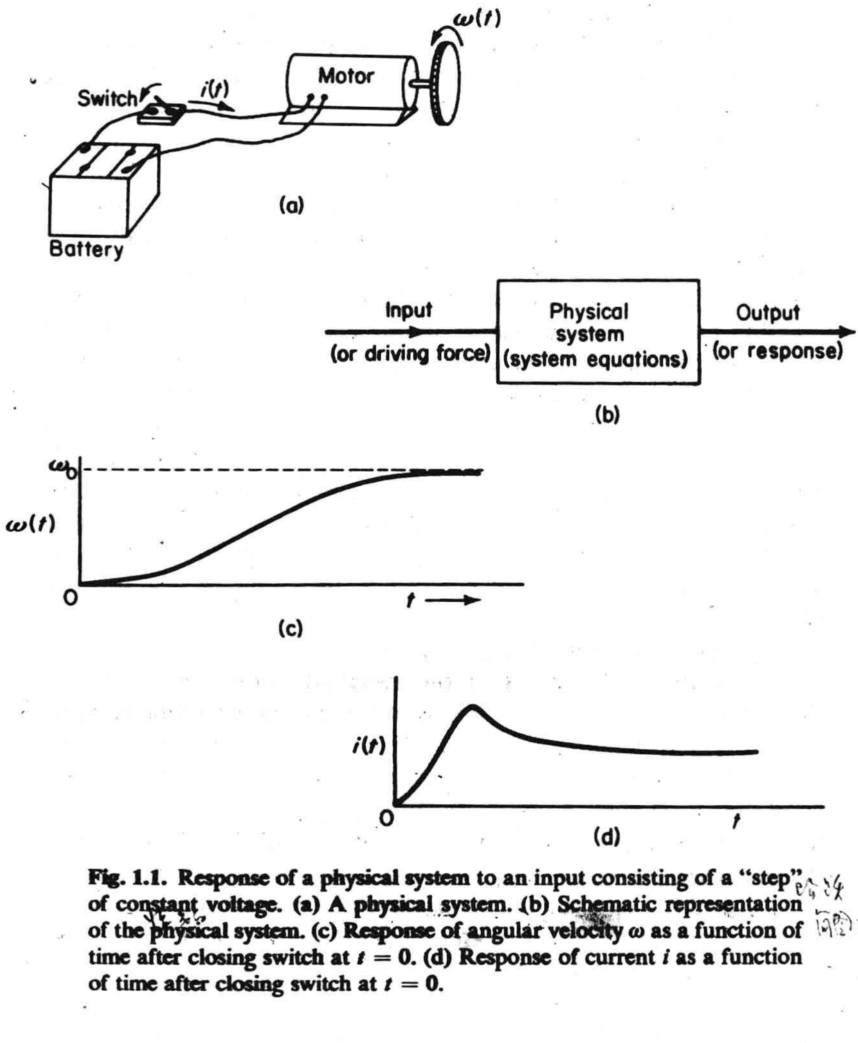

Fig. 1.1. Response of a physical system to an input consisting of a "step" of constant voltage. (a) A physical system. (b) Schematic representation of the physical system. (c) Response of angular velocity ω as a function of time after closing switch at $t = 0$. (d) Response of current i as a function of time after closing switch at $t = 0$.

if a motor is connected to a voltage source by closing a switch, as shown in Fig. 1.1, current flows through the motor and the motor comes up to some final speed ω_0 as shown. The response of the system to the applied voltage (the driving function) is thus either the current $i(t)$ or the angular velocity $\omega(t)$. It could also be the shaft angle $\theta(t)$ or any other observable or measurable dependent variable.

The relationships or equations that describe the system responses are usually some form of differential equations, and we shall thus be concerned with setting up and solving these equations. We will see that physical systems can be classified according to the type of these *system equations* which describe the system responses to a given input. The coefficients of these system equations are directly related to the elements and parameters of the physical system. The *passive elements* of a physical system are related to the uniquely identifiable physical properties of the system. Each passive element consists of only one physical quantity and has a specific equation relating the dependent variables associated with it. Thus, mass M, electrical resistance R, electrical capacitance C, rotational moment of inertia J, and so on, are examples of passive elements of physical systems. The *active elements* of a system are related to the energy sources of the system and include ideal amplifiers, voltage sources, and so on. The elements of a physical system determine the physical characteristics or properties of the system, and also the coefficients and form of the differential equations which describe the system (that is, the system equations). The *parameters* of a system are constants, either dimensionless or having the dimensions of some power of time (sec, 1/sec, sec^2, etc.) and which are directly related to the passive system elements (and some types of active elements) and also to the coefficients of the system equations. These terms will be clarified later with specific examples.

A system will also respond to changes in the magnitude of a passive system element, caused by either an abrupt switching process—as when a resistor is suddenly "shorted" in or out of an electric circuit—or by a time-varying change induced by some outside force. The former case can be treated by considering the effect of the switched element in terms of an equivalent input or driving force acting on a constant-parameter system. However, a system containing a time-varying passive element (where the element magnitude varies as some continuous function of time and not just in discrete steps) is a *non-constant-parameter system* and, in general, leads to system equations with non-constant coefficients which may be quite difficult to solve. We shall, therefore, not study responses resulting from *parametric forcing*, as the effect of a time-varying passive element is often called, but will consider only *constant-parameter systems* for which all the passive elements (and thus all the system parameters) are constant.

1.2.2. Types of systems. We have stated that we shall be concerned with finding the response of physical systems, so let us see if we can define such a system more fully, and also identify other possible types of systems.

By a *physical system* we mean any cohesive collection of physical components and elements which, because they are all related by one set of system equations, can reasonably be taken as a complete "package," or as a component part of some larger system. For example, consider a radio transmitter and a receiver tuned to it. We could reasonably consider the transmitter itself as a system, or sub-system, the receiver as another system, and the entire transmitter-radio waves-antenna-receiver as still another more comprehensive system; or each component could be broken down still finer