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International Workshops of COST Action 274, TARSKI, 2002-2005
Selected Revised Papers

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Preface

This book is a follow-up of LNCS volume 2929 with the same title, and presents the major results of COST action 274 (2002-2005), TARSKI: Theory and Applications of Relational Structures as Knowledge Instruments.

Relational structures abound in the daily environment: relational databases, data-mining, scaling procedures, preference relations, etc. Reasoning about, and with, relations has a long-standing European tradition, which may be divided into three broad areas:

1. *Algebraic Logic*: algebras of relations, relational semantics, and algebras and logics derived from information systems.
2. *Computational Aspects* of Automated Relational Reasoning: decidability and complexity of algorithms, network satisfaction.
3. *Applications*: social choice, AI, linguistics, psychology, economics, etc.

The main objective of the first TARSKI book (LNCS 2929) was to advance the understanding of relational structures and the use of relational methods in applicable object domains. There were the following sub-objectives:

1. To study the semantical and syntactical aspects of relational structures arising from 'real world' situations
2. To investigate automated inference for relational systems, and, where possible or feasible, develop deductive systems which can be implemented into industrial applications, such as diagnostic systems
3. To develop non-invasive scaling methods for predicting relational data
4. To make software for dealing with relational systems commonly available

We are confident that the present book will further the understanding of interdisciplinary issues involving relational reasoning. This book consists of papers which give a clear and self-contained overview of the results obtained by the TARSKI action, typically obtained by different persons from different work areas. The study and possible integration of different approaches to the same problem, which may have arisen at different locations, will be of practical value to the developers of information systems.

The first three papers concern *applications*. In the first paper a fair procedure for coalition formation is given. The software tool MacBeth for multi-criteria decision making is used to determine the utilities of the different alternatives to parties and the RelView tool is used to compute the stable governments and to visualize the results. If there is no stable government, graph-theoretical results are used to find a government as stable as possible and if there are several stable governments negotiations or consensus reaching may be used to choose one.

In computer science, scenarios with interacting agents are often developed using modal logic. The second paper shows how to interpret modal logic of knowledge in relation algebra. This allows the use of the RelView tool for the purpose of investigating finite models and for visualizing certain properties. This approach is illustrated with the well-known 'muddy children' puzzle using modal logic of knowledge.

The authors of the third paper use a regional health care perspective on maintenance and analysis of data, information and knowledge. Examples are drawn from cardiac diseases. Analysis and development are viewed from the by-pass surgery point of view. Association rules are used for analysis, and they show how these rules take logical forms so as to prepare for development of guidelines.

Computational aspects are treated in the next four papers. The fourth paper gives a generalization of the Hoede–Bakker index, which is a measure for the power of players in a network, taking into account the mutual influences between the players.

The fifth paper gives a relational presentation of nonclassical logics, providing a general scheme for automatic translation. The translation process is supported by a flexible Prolog tool.

The sixth paper provides a translation of the multimodal logic of qualitative order-of-magnitude reasoning into relational logics and presents a sound and complete proof system for the relational version of the language.

Logics of binary relations are presented in the seventh paper, together with the proof systems in the style of dual tableaux. Applications of these logics to reasoning in nonclassical logics are mentioned.

The remaining papers may be classified in the field of *algebraic logic*.

Papers 8 till 11 deal with different aspects of fuzzy preference relations. Fuzzy information relations and operators are studied in paper 8, where an algebraic approach is given based on residuated lattices. The authors of paper 9 give an overview of results on the aggregation of fuzzy relations and the related property of dominance of aggregation operators. The authors of the next paper, paper 10, address the added value that is provided by using distance-based fuzzy relations in flexible query answering. The last paper in this group gives a state-of-the-art overview of general representation results for fuzzy weak orders.

The next four papers deal with *lattices*. Relational representation theorems for lattices endowed with various negation operations are presented in a uniform framework in paper 12. The next paper gives relational representation theorems for classes of algebras which may be viewed as weak relation algebras, where a Boolean part is replaced by a not necessarily distributive lattice. Paper 14 treats aspects of lattice and generalized pre-lattice effect algebras. And the last paper in this group presents a decision procedure for the quantifier-free satisfiability problem of the language BLmf of bounded lattices with monotone unary functions.

Paper 16 addresses the relation of dominance on the class of continuous t-norms with a particular focus on continuous ordinal sum t-norms. Geometrical insight is provided into dominance relationships involving prototypical Archimedean t-norms, the Łukasiewicz t-norm and the product t-norm.

The last paper in this volume addresses the problem of extending aggregation operators typically defined on [0,1] to the symmetric interval [-1,1], where the '0' value plays a particular role (neutral value).

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Table of Contents

TARSKI: Theory and Applications of Relational Structures as Knowledge Instruments II

Social Software for Coalition Formation*

Agnieszka Rusinowska[1,2], Rudolf Berghammer[3], Patrik Eklund[4], Jan-Willem van der Rijt[5], Marc Roubens[6,7], and Harrie de Swart[8]

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Abstract. This paper concerns an interdisciplinary approach to coalition formation. We apply the MacBeth software, relational algebra, the RelView tool, graph theory, bargaining theory, social choice theory, and consensus reaching to a model of coalition formation. A feasible government is a pair consisting of a coalition of parties and a policy supported by this coalition. A feasible government is stable if it is not dominated by any other feasible government. Each party evaluates each government with respect to certain criteria. MacBeth helps to quantify the importance of the criteria and the attractiveness and repulsiveness of governments to parties with respect to the given criteria. Feasibility, dominance, and stability are formulated in relation-algebraic terms. The RelView tool is used to compute the dominance relation and the set of all stable governments. In case there is no stable government, i.e., in case the dominance relation is cyclic, we apply graph-theoretical techniques for breaking the cycles. If the solution is not unique, we select

* Co-operation for this paper was supported by European COST Action 274 "Theory and Applications of Relational Structures as Knowledge Instruments" (TARSKI). We thank Gunther Schmidt for his most valuable contributions to this paper.

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the final government by applying bargaining or appropriate social choice rules. We describe how a coalition may form a government by reaching consensus about a policy.

Keywords: stable government, MacBeth, relational algebra, RelView, graph theory, bargaining, social choice rule, consensus.

1 Introduction

This paper presents an overview of the results on coalition formation obtained from cooperation within the European COST Action 274: TARSKI (Theory and Applications of Relational Structures as Knowledge Instruments). The authors were connected to two different Work Areas of the COST Action, namely Work Area WA2 (Mechanization and Relational Reasoning) and Work Area WA3 (Relational Scaling and Preferences). This cooperation, which was not foreseen but gradually evolved over the years, resulted in an interdisciplinary approach to coalition formation. The MacBeth technique, relational algebra, the RelView tool, graph theory, bargaining theory, social choice theory, and consensus reaching were applied to the basic model of coalition formation described in Rusinowska et al. [44].

Coalition formation is one of the more interesting and at the same time more popular topics, and consequently a lot of work has already been done in this field. There are several ways to distinguish different coalition formation theories: one may talk, for instance, about power-oriented versus policy-oriented theories, one-dimensional versus multi-dimensional models, or actor-oriented versus non-actor oriented theories. The power-oriented theories, where the motivation for political parties to join a coalition is based only on their personal gains, are the earliest theories of coalition formation. One may mention here the theory of minimal winning coalitions (von Neuman and Morgenstern [55]), the minimum size theory (Riker [40]), and the bargaining proposition (Leiserson [35]). In policy-oriented theories, the process of coalition formation is determined by both policy and power motivations. Some of the most important early policy-oriented theories were the minimal range theory (Leiserson [34]), conflict of interest theory (Axelrod [2]), and the policy distance theory (de Swaan [21]). Actor-oriented theories, like the dominant player theory (Peleg [38], [39]) and the center player theory (van Deemen [53]), select an actor that has a more powerful position in the process of coalition formation. Also a lot of work has been done on spatial coalition formation theories, especially with respect to multi-dimensional policy-oriented theories. A main assumption in such models is that policy positions of parties are very important in the coalition formation process. One must mention here the political heart solution (Schofield [48], [49], [50]), the protocoalition formation (Grofman [29]), the winset theory (Laver and Shepsle [32], [33]), and the competitive solution (McKelvey, Ordeshook and Winer [36]). Many authors also considered institutional theories of coalition formation. One of the first theorists who acknowledged the important role of institutions was Shepsle [52], followed, in particular, by Austen-Smith and Banks [1], Laver and Schofield [31], and

Baron [6]. For an overview of coalition formation models we also like to refer to van Deemen [54], de Vries [24], Kahan and Rapoport [30].

The point of departure in this paper is a multi-dimensional model of coalition formation (see Rusinowska et al. [44]) in which the notion of stable government is central. In the model, the approach we use to represent party preferences allows us to include both rent-seeking and idealistic (policy-seeking) motivations. Moreover, a policy space does not have to be a Euclidean space, as is assumed frequently in coalition formation models, but may be any kind of space. The policy space is assumed to be multi-dimensional, which allows us to consider many political issues at the same time.

A government is defined as a pair consisting of a coalition and a policy supported by that coalition. It has a value (utility) to each party with respect to every given issue. In order to determine these values in practice, we propose to use the MacBeth approach; see also Roubens et al. [41]. MacBeth, which stands for *Measuring Attractiveness by a Categorical Based Evaluation Technique*, is an interactive approach to quantify the attractiveness of each alternative, such that the measurement scale constructed is an interval scale. For an overview and some applications of the software, we refer to the web site (www.m-macbeth.com), Bana e Costa and Vansnick [3]; Bana e Costa et al. [5]. The notion of absolute judgement has also been used in Saaty's Analytical Hierarchy Process (AHP); see Saaty [45], [46]. In the MacBeth technique, the absolute judgements concern differences of attractiveness, while in Saaty's method they concern ratios of priority, or of importance. One of the advantages of using the MacBeth approach is related to ensuring consistency. In case of any inconsistency of the initial evaluations, the MacBeth software indicates to the user what is the cause of the inconsistency and how to reach consistency. For a critical analysis of the AHP, see Bana e Costa and Vansnick [4].

Another application to the coalition formation model we propose here concerns Relational Algebra and the RelView tool which helps us to calculate stable governments; see also Berghammer et al. [11]. The RelView system, which has been developed at Kiel University, is a computer system for the visualization and manipulation of relations and for relational prototyping and programming. The tool is written in the C programming language, uses reduced ordered binary decision diagrams for implementing relations, and makes full use of the X-windows graphical user interface. For details and applications see, for instance, Berghammer et al. [14], Behnke et al. [7], Berghammer et al. [10], and Berghammer et al. [13].

In this paper, we also present an application of Graph Theory to the model of coalition formation in question; see Berghammer et al. [12]. We present a graph-theoretical procedure for choosing a government in case there is no stable government. If, on the other hand, more than one stable government exists, we may apply Social Choice Theory to choose one government. For an overview and comparison of social choice rules see, for instance, Brams and Fishburn [16], and de Swart et al. [23]. Another natural application is based on Bargaining Theory. We use a strategic approach to bargaining; see Rubinstein [42], Fishburn and

Rubinstein [27], Osborne and Rubinstein [37]. We formulate several bargaining games in which parties bargain over the choice of one stable government, and next we look for refinements of Nash equilibria called subgame perfect equilibria (Selten [51]) of these games; see also Rusinowska and de Swart [43].

We describe a procedure for a coalition to choose a policy in order to propose a government, based on consensus reaching, by combining some ideas from Carlsson et al. [18] and Rusinowska et al. [44]. It has been first proposed in Eklund et al. [25], where the authors consider consensus reaching in a committee, and next in Eklund et al. [26], where a more complicated model, i.e., consensus reaching in coalition formation, is presented.

The paper is structured as follows. Section 2 introduces the model of coalition formation. In Section 4, the basic notions of relational algebra are presented. In Sections 3 and 5, we present applications of the MacBeth and RelView tools, respectively, to the model in question. Section 6 concerns applications of Social Choice Theory and Bargaining Theory to the model, in order to choose a stable government in the case there exists more than one. Next, an application of Graph Theory to the model of coalition formation is proposed in Section 7, in order to choose a 'rather stable' government in the case that there exists no stable one. Section 8 describes how a coalition may reach consensus about a policy in order to propose a government. In Section 9, we present our conclusions.

2 The Model of Coalition Formation

In this section we recapitulate a model of coalition formation, first introduced in Rusinowska et al. [44], and further refined, in particular, in Eklund et al. [26].

2.1 Description of the Model

Let $N = \{1, \ldots, n\}$ be the set of political parties in a parliament, and let w_i denote the number of seats received by party $i \in N$. Moreover, let W denote the *set of all winning coalitions.* The model concerns the creation of a government by a winning coalition. It is assumed that there are some independent policy issues on which a government has to decide. Let P denote the *set of all policies.*

A *government* is defined as a pair $g = (S, p)$, where S is a winning coalition and p is a policy. Hence, the *set G of all governments* is defined as

$$G := \{(S, p) \mid S \in W \ \wedge \ p \in P\}. \tag{1}$$

Each party has preferences concerning all policies and all (winning) coalitions. A coalition is called *feasible* if it is acceptable to all its members. A policy is *feasible for a given coalition* if it is acceptable to all members of that coalition. A government (S, p) is *feasible* if both, S and p, are acceptable to each party belonging to S. By G^* we denote the *set of all feasible governments*, and by G^*_i the *set of all feasible governments containing party i*, i.e., for each $i \in N$,

$$G^*_i := \{(S, p) \in G^* \mid i \in S\}. \tag{2}$$

A *decision maker* is a party involved in at least one feasible government, i.e., the *set DM of all decision makers* is equal to

$$DM := \{i \in N \mid G_i^* \neq \emptyset\}. \tag{3}$$

Moreover, let the subset W^* of W be defined as

$$W^* := \{S \in W \mid \exists p \in P : (S,p) \in G^*\}. \tag{4}$$

A feasible government is evaluated by each decision maker with respect to the given policy issues and with respect to the issue concerning the coalition. Let C^* be the finite *set of criteria.* The criteria do not have to be equally important to a party, and consequently, each decision maker evaluates the importance of the criteria. Formally, for each $i \in DM$, we assume a function $\alpha_i : C^* \to [0,1]$, such that the following property holds:

$$\forall i \in DM : \sum_{c \in C^*} \alpha_i(c) = 1. \tag{5}$$

The number $\alpha_i(c)$ is i's evaluation of criterion c. Moreover, each decision maker evaluates each feasible government with respect to all the criteria. Hence, for each $i \in DM$, we assume $u_i : C^* \times G^* \to \mathbb{R}$ where the real number $u_i(c,g)$ is called the *value of government* $g \in G^*$ *to party* $i \in DM$ *with respect to criterion* $c \in C^*$. Moreover, for each $i \in DM$, we define $U_i : G^* \to \mathbb{R}$ such that

$$(U_i(g))_{g \in G^*} = (\alpha_i(c))_{c \in C^*} \cdot (u_i(c,g))_{c \in C^*, g \in G^*}, \tag{6}$$

where $(\alpha_i(c))_{c \in C^*}$ is the $1 \times |C^*|$ matrix representing the evaluation (comparison) of the criteria by party i, $(u_i(c,g))_{c \in C^*, g \in G^*}$ is the $|C^*| \times |G^*|$ matrix containing party i's evaluation of all governments in G^* with respect to each criterion in C^*, and $(U_i(g))_{g \in G^*}$ is the $1 \times |G^*|$ matrix containing party i's evaluation of each government in G^*.

In order to determine in practice the values of $\alpha_i(c)$ and $u_i(c,g)$ for all parties $i \in DM$, criteria $c \in C^*$ and governments $g \in G^*$, we can use the MacBeth technique. We do so in Section 3.

The central notion of the model introduced in Rusinowska et al. [44] is the notion of *stability.* A feasible government $h = (S,p) \in G^*$ *dominates* a feasible government $g \in G^*$ (denoted as $h \succ g$) if the property

$$(\forall i \in S : U_i(h) \geq U_i(g)) \wedge (\exists i \in S : U_i(h) > U_i(g)) \tag{7}$$

holds. A feasible government is said to be *stable* if it is dominated by no feasible government. By

$$SG^* := \{g \in G^* \mid \neg \exists h \in G^* : h \succ g\} \tag{8}$$

we denote the *set of all (feasible) stable governments.* In Rusinowska et al. [44], necessary and sufficient conditions for the existence and the uniqueness of a stable government are investigated. Moreover, the authors introduce some alternative definitions of 'stability', and establish the relations between the new notions of 'stability' and the chosen one. In the present paper, we decide for the definition of a stable government given by (8), which we find the most natural definition of stability.

2.2 A Running Example

Let us consider a very small parliament consisting of only three parties. We assume each coalition consisting of at least two parties is winning and there are only two policy issues and four policies, i.e., we have

$$N = \{A, B, C\}, \quad W = \{AB, AC, BC, ABC\}, \quad P = \{p_1, p_2, p_3, p_4\}.$$

As a consequence, we have 16 governments. Assume that the grand coalition is not feasible, but all two-party coalitions are feasible. Further, assume both policies p_1 and p_2 are acceptable to all three parties, policy p_3 is not acceptable to party C, while policy p_4 is not acceptable to party B. Hence, policies p_1 and p_2 are feasible for coalitions AB, AC, and BC, policy p_3 is feasible for coalition AB, and p_4 is feasible for coalition AC.

Consequently, there are eight feasible governments, i.e.,

$$G^* = \{g_1, g_2, g_3, g_4, g_5, g_6, g_7, g_8\},$$

which are given as

$$g_1 = (AB, p_1), \quad g_2 = (AC, p_1), \quad g_3 = (BC, p_1), \quad g_4 = (AB, p_2),$$
$$g_5 = (AC, p_2), \quad g_6 = (BC, p_2), \quad g_7 = (AB, p_3), \quad g_8 = (AC, p_4)$$

and therefore obtain the governments containing the parties as

$$G^*_A = \{g_1, g_2, g_4, g_5, g_7, g_8\},$$
$$G^*_B = \{g_1, g_3, g_4, g_6, g_7\},$$
$$G^*_C = \{g_2, g_3, g_5, g_6, g_8\}.$$

Moreover, we have

$$DM = N, \quad W^* = \{AB, AC, BC\}, \quad C^* = \{1, 2, 3\},$$

where the criteria 1 and 2 refer to the first and the second policy issue, while criterion 3 concerns the (attractiveness of the) 'coalition'. In order to determine $\alpha_i(c)$ and $u_i(c, g)$ for each $i \in DM$, $c \in C^*$, and $g \in G^*$, we will use the MacBeth technique in the next section.

3 Applying MacBeth to Coalition Formation

When applying the coalition formation model described in Section 2 in practice, the question arises how to determine the $\alpha_i(c)$ and the $u_i(c, g)$ for $i \in DM$. The answer to this question will be given in this section, where we propose to use the MacBeth software to determine these values. In Subsection 3.1, we show how the utilities of governments to parties may be calculated using the MacBeth technique (see also [41]), while in Subsection 3.2 the application is illustrated by an example. It is assumed here that each party judges only a finite number of governments differently, even if there is an infinite number of possible governments.