INTRODUCTION TO SPECIAL RELATIVITY

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SECOND EDITION

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CLARENDON PRESS · OXFORD

1991

Oxford University Press, Walton Street, Oxford OX2 6DP
Oxford New York Toronto
Delhi Bombay Calcutta Madras Karachi
Petaling Jaya Singapore Hong Kong Tokyo
Nairobi Dar es Salaam Cape Town
and associated companies in
Berlin Ibadan

Oxford is a trade mark of Oxford University Press

Published in the United States
by Oxford University Press, New York

First published 1982 (Reprinted four times)
Second edition 1991

British Library Cataloguing in Publication Data
Rindler, Wolfgang 1924–
Introduction to special relativity—2nd ed.
1. Physics. Special theory of relativity
I. Title
530.11
ISBN 0-19-853953-3
ISBN 0-19-853952-5 pbk

Library of Congress Cataloging in Publication Data
Rindler, Wolfgang. 1924–
Introduction to special relativity / Wolfgang Rindler.—2nd ed.
p. cm.
1. Special relativity (Physics) I. Title.
QC173.65.R56 1991 530.1'1—dc20 90-48748
ISBN 0-19-853953-3
ISBN 0-19-853952-5 pbk

Typeset by Integral Typesetting, Gorleston, Norfolk NR31 6RG
Printed in Great Britain by Courier International Ltd., Tiptree, Essex

PREFACE TO THE SECOND EDITION

The aim of this second edition is the refinement and improvement of the text, as well as the incorporation of new data and ideas. The most radical changes are to be found in a novel and particularly careful derivation of the Lorentz transformation, and in a somewhat different logic for the development of continuum mechanics. In connection with the former I have returned to the primacy of Einstein's Relativity Principle as opposed to Dixon's 'Principle of Uniformity' for inertial frames. Lesser changes have been made on almost every page, and many paragraphs and even whole sections have been rewritten. Over 20 new exercises have been added and a few old ones deleted. Also, I am introducing here a new notation, $\mathbf{ds}^2$, for the metric that is traditionally written as ds^2. It is to be regarded as the square of the infinitesimal displacement vector $\mathbf{ds} = (c\,dt, dx, dy, dz)$, so that its possible negativity does not conflict with the reality and non-negativity of the arc ds, here regarded as the magnitude of $\mathbf{ds}$

Much of the work for this second edition was done during the Easter Term at Churchill College, Cambridge. I would like to thank the Fellows of the College and, in particular, the Master, Sir Hermann Bondi, for their stimulating hospitality.

Cambridge W.R.
July 1990

PREFACE TO THE FIRST EDITION

Apart from being a vehicle for communicating my joy in the subject, this book is intended to serve as a text for an introductory course on special relativity, which is rather more conceptually and mathematically than experimentally oriented. In this context it should be suitable from the upper undergraduate level onwards. But the book might well be used autodidactively by a somewhat more advanced reader. It assumes no prior knowledge of relativity. Thus it elaborates the underlying logic, dwells on the subtleties and apparent paradoxes, and also contains a large collection of problems which should just about cover all the basic modes of thinking and calculating in special relativity. Much emphasis has been laid on developing the student's intuition for space-time geometry and four-tensor calculus. But the approach is not so dogmatically four-dimensional that three-dimensional methods are rejected out of hand when they yield a result more directly. Such methods, too, belong to the basic arsenal even of experts.

In fact, the viewpoint in the first three chapters is purely three-dimensional. Here the reader will find a simple introduction to such topics as the relativity of simultaneity, length contraction, time dilation, the twin paradox, and the appearance of moving objects. But beginning with Chapter 4 (on spacetime) the strongest possible use is made of four-dimensional techniques. Pure tensor theory as such is relegated to an appendix, in the belief that it should really be part of a physicist's general education. Still, this appendix will serve as Chapter '$3\frac{1}{2}$' for readers unfamiliar with that theory. In Chapters 5 and 6—on mechanics and electromagnetism—a purely synthetic four-tensor approach is adopted. Not only is this simpler and more transparent than the historical approach, and a good example of four-dimensional reasoning, but it also brings the student face to face with the 'man-made' aspect of physical laws. In the last chapter (on the mechanics of continua), the synthetic approach is somewhat softened by the well-known analogy with electromagnetism.

In the discussion of electromagnetism I have reluctantly adopted the SI units now so widely used in spite of their awkwardness for the theoretician. But I have indicated how the equations can easily be translated into their Gaussian (c.g.s.) forms in terms of which most relativists think. A commitment to follow a consistent notation (capital letters for four-dimensional and lowercase for three-dimensional tensors) resulted im some other awkwardnesses, such as **e** and **b** for the electric and magnetic field vectors and **w** for the vector potential (since **a** was already used for the acceleration). I can only hope that the reader will give these symbols a try and not automatically transcribe them.

I should perhaps say a word on the genesis of this book. It has a predecessor after which it is loosely structured, namely my *Special Relativity* (Oliver & Boyd, 1960), which went out of print in 1975. That little book seems to have won some faithful friends and there have been frequent requests for a new edition. But when I finally attempted such an edition I realized how much my ideas—and perhaps the subject itself—had changed and how impossible it was simply to revise the old text. So I found myself much more pleasantly engaged in writing a new book, this book, though a few of the old arguments and problems have been taken over and, I hope, some of the old spirit as well. There are also ties to my *Essential Relativity* (Second Edition, Springer-Verlag, 1977). In a number of contexts I became uneasily aware that I could neither improve upon, nor omit, nor usefully paraphrase what I have already written there. So eventually (with the publisher's kind permission) I decided simply to borrow the relevant passages *verbatim*; these may account for a total of about ten pages of the present book. My conscience was somewhat eased by the fact that, in its time, *Essential Relativity* had similarly borrowed from the older *Special Relativity*.

I clearly owe much to many authors, some by now forgotten. But I would like to acknowledge the special influence on this book of W. G. Dixon, J. Ehlers, Z. Papapetrou, R. Penrose, I. Robinson, D. W. Sciama, R. Sexl, J. L. Synge, H. Weyl, and N. Woodhouse. I also owe a considerable debt to my students. As just one example I like to recall the innocent class question 'but what if . . .' which, many years ago, precipated the 'length contraction paradox'— herein included.

Dallas W.R.
November 1981

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I

THE FOUNDATIONS OF SPECIAL RELATIVITY

1. Introduction

One of the greatest triumphs of Maxwell's electromagnetic theory (*c*. 1864) was the explanation of light as an electromagnetic wave phenomenon. But waves in what? In conformity with the mechanistic view of nature then prevailing, it seemed imperative to postulate the existence of a medium—the *ether*—which would serve as a carrier for these waves (and for electromagnetic 'stress' in general). This led to the most urgent physical problem of the time: the detection of the earth's motion through the ether.

Of the many experiments devised for this purpose, we shall mention just three. Michelson and Morley (1887, see Sec. 2), looked for a directional variation in the velocity of light on earth. Fizeau (1860), Mascart (1872), and later Lord Rayleigh (1902), looked for an expected effect of the earth's motion on the refractive index of certain dielectrics. And Trouton and Noble [1903, see Ex. VI(11)] tried to detect an expected tendency of a charged plate condenser to face the 'ether drift'. All failed. The facile explanation that the earth might drag the ether along with it only led to other difficulties with the observed aberration of starlight, and could not resolve the problem.

In order to explain nature's apparent conspiracy to hide the ether drift, Lorentz between 1892 and 1909 developed a theory of the ether that was eventually based on two *ad hoc* hypotheses: the longitudinal contraction of rigid bodies[1] and the slowing down of clocks ('time-dilation')[2] when moving through the ether at a speed v, both by a factor $(1 - v^2/c^2)^{1/2}$, where c is the speed of light. This would so affect every apparatus designed to measure the ether drift as to neutralize all expected effects.

In 1905, in the middle of this development, Einstein proposed the *principle of relativity* which is now justly associated with his name. Actually Poincaré had discussed essentially the same principle during the previous year, but it was Einstein who first recognized its full significance and put it to brilliant use. In it, he elevated the complete equivalence of all inertial reference frames to the status of an axiom or principle, for which no proof or explanation is to be sought. On the contrary, *it* explains the failure of all the ether-drift experiments, much as the principle of energy conservation explains *a priori* (i.e. without the need for a detailed examination of the mechanism) the failure of all attempts to construct a perpetual motion machine.

[1] Proposed independently by Fitzgerald as early as 1889.
[2] Based directly on a feature of Einstein's special relativity of 1905.

At first sight Einstein's relativity principle seems to be no more than a whole-hearted acceptance of the null results of all the ether-drift experiments. But by ceasing to look for special explanations of those results, and using them rather as empirical evidence for a new principle of nature, Einstein had turned the tables: predictions could be made. The situation can be compared to that obtaining in astronomy at the time when Ptolemy's intricate geocentric system (corresponding to Lorentz's 'etherocentric' theory) gave way to the ideas of Copernicus, Galileo, and Newton. In both cases the liberation from a venerable but inconvenient reference frame ushered in a revolutionary clarification of physical thought, and consequently led to the discovery of a host of new and unexpected results.

Soon a whole theory based on Einstein's relativity principle (and on a 'second axiom' asserting the invariance of the speed of light) was in existence, and this theory is called *special relativity*. Its programme was to modify all the laws of physics, where necessary, so as to make them equally valid in all inertial frames. For Einstein's principle is really a *metaprinciple*: it puts constraints on *all* the laws of physics. The modifications suggested by the theory (especially in mechanics), though highly significant in many modern applications, have negligible effect in most classical problems, which is of course why they were not discovered earlier. However, they were not exactly needed empirically in 1905 either. This is a beautiful example of the power of pure thought to leap ahead of the empirical frontier—a feature of all good physical theories, though rarely on such a heroic scale.

Today, over eighty years later, the enormous success of special relativity theory has made it impossible to doubt the wide validity of its basic premises. It has led, among other things, to a new theory of space and time, and in particular to the relativity of simultaneity and the existence of a maximum speed for all particles and signals, to a new mechanics in which mass increases with speed, to the formula $E = mc^2$, and to de Broglie's association of waves with particles. One of the ironies of these developments is that Newton's theory, which had always been known to satisfy a relativity principle within the classical framework of space and time, now turned out to be in need of modification, whereas Maxwell's theory, with its apparent conceptual dependence on a preferred ether frame, came through with its formalism intact—in itself a powerful recommendation for special relativity.

Apart from leading to new laws, special relativity leads to a useful technique of problem-solving, namely the possibility of switching reference frames. This often simplifies a problem. For although the totality of laws is always the same, the configuration of the problem may be simpler, its symmetry enhanced, its unknowns fewer, and the relevant subset of physical laws more convenient, in a judiciously chosen inertial frame.

Our main concern in this chapter will be to set Einstein's principle in its proper perspective and to derive from it the so-called Lorentz transformation equations, which are the mathematical core of the special theory of relativity. With their help we can subject the various branches of classical physics to the test of Einstein's principle, and with their help, too, find the necessary modifications where the principle is not satisfied.

2. Schematic account of the Michelson–Morley experiment

Certainly the most famous of all the experiments designed to measure the ether drift was that due to Michelson and Morley, first performed in 1887 and repeated many times thereafter. Its essential principle was to split a beam of light and then to send the two half-beams along orthogonal arms of equal length, at whose ends mirrors reflected the beams back to the starting point where they were made to interfere. Then the entire apparatus was rotated in the plane of the arms. If this causes a differential change in the to-and-fro light travel times along the two arms, the interference pattern should change. Suppose originally one of the arms, marked L_1 in Fig. 1, lies in the direction of an ether drift of velocity v. Figure 1 should make it clear that the respective to-and-fro light travel times along the two arms would then be expected to be

$$T_1 = \frac{L_1}{c+v} + \frac{L_1}{c-v} = \frac{2L_1}{c(1-v^2/c^2)},$$

$$T_2 = \frac{2L_2}{(c^2-v^2)^{1/2}} = \frac{2L_2}{c(1-v^2/c^2)^{1/2}},$$

where L_1 and L_2 are the purportedly equal lengths of the two arms. Since $T_1 \neq T_2$, a rotation of the experiment through 90° should produce a shift in the interference fringes. None was ever observed, which seems to imply $v = 0$. Yet at some point in its orbit around the sun the earth must move through the ether with a speed of at least 18 miles per second (its orbital velocity) and this should have been easily detected by the apparatus. Of course, in Einstein's theory, this null result is to be expected *a priori*: Light propagates

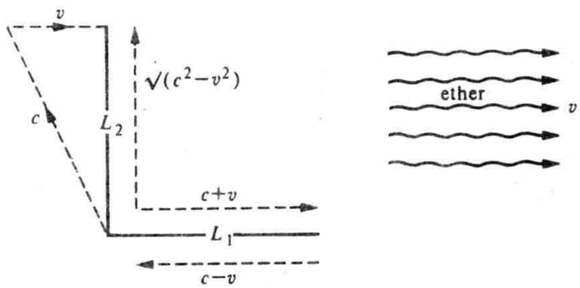

Fig. 1

in all inertial frames, and thus also in a laboratory speeding through space, just as it would in 'still ether'. It is not for us to ask how!

In the Lorentz theory the null result of the Michelson–Morley experiment was explained by the contraction of the arm that moves longitudinally through the ether, so that the *actual* lengths of the arms are related by $L_1 = L_2(1 - v^2/c^2)^{1/2}$, which yields the observed equality $T_1 = T_2$. (It can be shown that the contraction hypothesis ensures $T_1 = T_2$ for *all* positions of the arms.) That there is also need of a second hypothesis—time dilation—in the Lorentz theory can be appreciated by considering a simple thought experiment. Suppose we could measure the original to-and-fro time T_2 directly with a clock, and suppose we could then move the arm L_2 along with the ether so that v becomes zero. Then the to-and-fro time should be $T_3 = 2L_2/c = T_2(1 - v^2/c^2)^{1/2}$. But if nature's conspiracy to hide the ether is complete, we would instead measure $T_3 = T_2$. This could be accounted for by the hypothesis that a clock moving with speed v through the ether goes slow by a factor $(1 - v^2/c^2)^{1/2}$. For then the *measured* time in the original position is less by that factor than the actual time T_2, and is thus equal to T_3.

A recent laser version[1] of the Michelson–Morley experiment has demonstrated the isotropy of the to-and-fro speed of light to an accuracy of $\sim 10^{-15}$. And, as has been stressed by Sexl, modern equivalents of the Michelson–Morley experiment are being performed daily. For example, the synchrony of the various atomic clocks around the earth that serve to define 'International Atomic Time' is continually being tested by an exchange of radio signals. Any interference with these signals by a variable ether wind of the expected magnitude could be detected by the clocks. Needless to say, none has been detected: day or night, summer or winter, the signals from one clock to another always arrive with the same time delay. Again, the incredible accuracy of some modern radio navigational systems hinges crucially on the independence of the speed of radio signals of any ether wind.

3. Inertial frames in special relativity

A frame of reference is a conventional standard of rest relative to which measurements can be made and experiments described. For example, if we choose a frame rigidly attached to the earth, the various points of the earth remain at rest in this frame while the 'fixed' stars all trace out vast circles in the course of each day; if, on the other hand, we choose a rigid frame attached to the fixed stars then these remain at rest while points on the earth, other than those on its axis, trace out approximate circles in the course of each day, and the earth itself traces out an ellipse in the course of each year; and so on. Among all possible reference frames there is one class which

[1] Brillet, A. and Hall, J. L. (1979) *Phys. Rev. Letters* **42**, 549.

plays a special role in classical mechanics, namely the class of *inertial frames.* These frames play an even more fundamental role in the special theory of relativity and we shall therefore define and discuss them carefully:

An inertial frame is one in which spatial relations, as determined by rigid scales at rest in the frame, are Euclidean and in which there exists a universal time in terms of which free particles remain at rest or continue to move with constant speed along straight lines (i.e. in terms of which free particles obey Newton's first law).

Free particles placed without velocity at fixed points in an inertial frame will remain at those points, by definition. We can therefore picture an inertial frame as an aggregate of actual or virtual free test-particles mutually at rest, as determined by rigid scales. The distances between these 'defining' particles satisfy the Euclidean axioms—an important stipulation in view of later developments. Straight lines in such a frame can be defined as geodesics (lines of minimum length) and free particles not belonging to the defining aggregate move along such lines. We can further picture the defining particles as carrying clocks that indicate the universal time throughout the frame.

We shall consider two inertial frames equal if they have the same defining particles. The choice of coordinates within such a frame is still free; once it is made, one should logically use a different term for the frame-plus-its-coordinates, such as *inertial (coordinate-) system.* In practice, however, we shall usually be less precise and let the context define which of the above we mean by 'inertial frame'.

Now let us see the relevance of all this to special relativity. We shall adopt the modern view (largely due to Einstein) that a physical theory is an abstract mathematical model (much like Euclidean geometry) whose applications to the real world consist of correspondences between a subset of it and a subset of the real world. In line with this view, *special relativity is the theory of an ideal physics referred to an ideal set of infinitely extended gravity-free inertial frames*, such as we described above.[1] Why 'gravity-free'? Classically, gravity was regarded as an overlay which did not affect the rest of physics. So it was logical for Newton to treat the frame of the fixed stars as inertial, in the sense that *but for gravity* free particles would move uniformly relative to it. But Einstein, in his *general relativity* (the details of which are beyond the scope of this book) taught us that gravity is *curvature* (of space and time) and so affects *all* the rest of physics, which has no choice but to play on a stage of space and time. Particles not subject to forces *except* gravity move

[1] On a more sophisticated level, the arena of special relativity will eventually (in Chapter IV) be seen to be Minkowski's four-dimensional 'spacetime'. This is an abstraction from the set of all inertial frames. At first, however, we shall rely on the set of inertial frames itself to formulate the theory.

'as straight as possible' in curved 'spacetime'. In E. T. Whittaker's phrase, gravity ceased to be one of the players and became part of the stage. Thus, extended inertial frames cannot be realized in nature, because gravity destroys Euclidicity. But this does not affect in any way the logic of special relativity as an abstract theory (just as it does not invalidate Euclidean geometry). It does, however, put limitations on its correspondence with the real world. These are spelled out by Einstein's *equivalence principle* of 1907 (on which he eventually based his general theory of relativity): *the reference frames in the real world that correspond to (portion of) the ideal inertial frames discussed in special relativity are the freely falling nonrotating local frames.* At any given place and time in the real world there is a family of such frames, each member of which can be realized by an aggregate of test-particles momentarily at rest relative to each other and falling freely under gravity. Certainly in Newton's theory such a local frame is equivalent to an inertial frame from which gravity has been eliminated, for in a gravitational field all particles suffer the same acceleration. Most of us have at least vicariously experienced such freely falling local frames: we need only recall the televized pictures of space capsules in which astronauts are weightless and, if unrestrained, move according to Newton's first law. Such capsules, evidently of limited extent, are the primary reference frames in the real world relative to which the laws of special-relativistic physics would be expected to apply most accurately.[1]

In this book *all* reference frames used (unless otherwise stated) will be ideal infinitely extended gravity-free inertial frames, and all observers will be considered to use such frames ('*inertial observers*'). Sometimes the term 'inertial' may be omitted, but it will always be understood.

It will turn out that, just as in Newtonian mechanics, the ideal inertial frames of special relativity are all in uniform translatory motion relative to each other, and, conversely, that any frame having such motion relative to an inertial frame is itself inertial.

It will also turn out, as a direct consequence of the relativity principle, that all inertial frames are spatially homogeneous and isotropic, not only in their assumed Euclidean geometry but for the performance of all physical experiments. By this we mean that the outcome of an experiment is the same whenever its initial conditions differ only by a translation (homogeneity) and rotation (isotropy) in some inertial frame.

It may be noted that, whereas our definition of inertial frame already determines the *rate* of time in each inertial frame, up to an overall constant factor, as that in which free particles move uniformly, isotropy determines

[1] There is a close analogy between plane Euclidean geometry and its applications to 'small' portions of curved surfaces (like the surface of the earth), on the one hand, and special relativity and its applications to 'small' portions of the real world curved by gravity, on the other hand.

the clock *zero-point settings* up to an overall additive constant. For suppose isotropy holds in an inertial frame referred to Cartesian coordinates x, y, z and we define a new time $t' = t + kx$ (k = constant > 0). Then Newton's first law will still hold. But any given rifle will now shoot bullets faster in the negative x-direction than in the positive x-direction (i.e. with greater *coordinate* velocity).

Again, as a consequence of the relativity principle, it will presently turn out that inertial frames are temporally homogeneous, i.e. that identical experiments (relative to a given inertial frame) performed at different times yield identical results. In particular, this implies that all methods of time keeping based on repetitive processes are equivalent, and it denies such possibilities (envisaged by E. A. Milne) as that inertial time—relative to which free particles move uniformly—falls out of step over the centuries with atomic time, e.g. that indicated by a caesium clock.

4. Einstein's two axioms for special relativity

As we have seen, Einstein's reaction to the failure of all attempts to detect the ether frame was radical. He advanced as an axiom the following *principle of relativity*:

The laws of physics are identical in all inertial frames, or, equivalently, *the outcome of any physical experiment is the same when performed with identical initial conditions relative to any inertial frame.*

Strictly speaking we should read 'inertial coordinate system' for 'inertial frame' in the above statement. Since orthogonal axes can be set up with origin at any point, and with axes in any direction, and since the zero point of time can be chosen arbitrarily, the relativity principle as applied to various coordinate systems within a single inertial frame immediately leads to the spatial homogeneity and isotropy and to the temporal homogeneity of each inertial frame, for the performance of any physical experiment.

Note that Einstein's principle is a generalization to the whole of physics of a relativity principle long known to be satisfied by Newtonian mechanics. Such a generalization is strongly supported by the essential unity of physics. For it would be very disturbing if, for example, the electromagnetic laws governing the behaviour of matter on the atomic scale did not share in this very profound and remarkable invariance property of the laws of mechanics; which govern the behaviour of matter on the macroscopic scale. And, indeed, Einstein cited instances of manifest relativity from electromagnetism. For example, the current induced in a conductor by a magnet is the same whether the conductor is at rest and the magnet moving, or vice versa. But, of course. the chief recommendation for this as for any other axiom is the success of the theory resulting from it.

The acceptance of the relativity principle—Einstein's *first axiom*—seems harmless enough until we come to his *second axiom: There exists an inertial frame in which light signals in vacuum always travel rectilinearly at constant speed c, in all directions, independently of the motion of the source.* (The value of c is $2.997\,9245\ldots \times 10^8$ m s^{-1}, but $c = 3 \times 10^8$ m s^{-1} is good enough for many applications.)

By itself, this axiom is also perfectly reasonable. Even Einstein's contemporaries, familiar with Maxwell's electromagnetic theory of light, did not expect the speed of light to depend on the speed of the source, and they had empirical evidence for this axiom in their pseudo-inertial terrestrial frame of reference. In particular, the direction-independence had been very accurately tested by the Michelson–Morley experiment. But when combined with the first axiom, the second leads to the following apparently absurd state of affairs, which we shall call *Einstein's law of light propagation*:

Light signals in vacuum are propagated rectilinearly, with the same speed c, at all times, in all directions, in all *inertial frames.*

Thus if a light signal recedes from me and I transfer myself to ever faster-moving inertial frames in pursuit of it, I shall not alter the velocity of that light signal relative to me by one iota. This is totally irreconcilable with our classical concepts of space and time. But it was a mark of Einstein's genius to realize that those concepts were dispensable, and could be replaced by others. The final form of those others is due to the mathematician Minkowski, and consists in a certain blend of space and time into a four-dimensional 'spacetime' (1908), as we shall see in due course.

A first logical consequence of Einstein's two axioms was the elimination of the ether concept from physics. Each inertial frame now has the properties with which the ether frame had been credited, and so it makes no sense to single out one inertial frame arbitrarily and call it the ether frame. It is true that Lorentz's theory—gentler to the classical prejudices than Einstein's, and observationally equivalent to it—kept the ether idea alive a few years longer. But soon Einstein's far more elegant and powerful ideas prevailed, and Lorentz's theory, together with the ether concept, fell into oblivion.

Finally, in spite of its historical and practical importance, we must de-emphasize the *logical* role of the law of light propagation as a pillar of special relativity. As we shall see in Section 7(x), *a* second axiom is needed *only* to determine the value of an invariant velocity c that occurs naturally in the theory. But this could come from any number of branches of physics—we need only think of the energy formula $E = mc^2$, or de Broglie's velocity relation $uv = c^2$. Special relativity would exist even if light and electromagnetism were somehow eliminated from nature. It is primarily a new theory of space and time, and only secondarily a theory of the physics in that new space and time, with no preferred relation to any one branch.

5. Coordinates. The relativity of time

An *event* is an instantaneous point-occurrence, like the collision of two particles or the flash of a flash bulb. It will therefore be specified by *four* coordinates, one of time and three of position, e.g. (t, x, y, z). In special relativity events play a central role and we must be clear how to assign coordinates to them, at least conceptually.

The standard spatial coordinates for inertial frames are orthonormal Cartesian coordinates x, y, z. To assign these to events, the 'presiding' observer at the origin of an inertial frame needs to be equipped only with a standard clock (e.g. one based on the vibrations of the caesium atom), a theodolite, and a means of emitting and receiving light signals. He will also need an agreed standard of length, e.g. a metre stick or the wavelength of a specified atomic emission line, at least in order to assign a numerical value to the speed of light once and for all. In accordance with the law of light propagation, he can then measure the distance of any particle (at which an event may be occurring) by the radar method of bouncing at light-echo off that particle and multiplying the elapsed time by $\frac{1}{2}c$. Angle measurements with the theodolite on the returning light signal will serve to determine the relevant (x, y, z) once a set of coordinate directions has been chosen. The same signal can be used to determine the time t of the reflection event at the particle as the average of the time of emission and the time of reception.

But conceptually it is preferable to *pre*coordinatize the frame and to read off the coordinates of all events *locally*. For this purpose we imagine standard clocks placed at rest at the vertices $(m\varepsilon, n\varepsilon, p\varepsilon)$ of an arbitrarily fine lattice, where m, n, p run over the integers and ε is arbitrarily small. The spatial coordinates of these clocks can be determined once and for all by the origin-observer and then engraved upon them. To synchronize the clocks it is sufficient to emit a single light signal from the origin, say at time t_0: each lattice clock is set to read $t_0 + r/c$ as the signal passes it, where r is its distance from the origin. An event is then coordinatized by noting the time and space coordinates (t, x, y, z) on the clock nearest to it.

In view of our remarks at the end of the last section about the dispensability of the law of light propagation as an axiom, it will be well to point out that identical coordinates can be assigned *without* the use of light signals—though perhaps less conveniently. For example, the basic lattice could be laid out with rigid scales of equal length ε. And the vertex clocks could be synchronized by a *sound* signal from the origin if the frame were filled with still air, or by rifle bullets of known velocity shot from the origin in all directions at time t_0. It is clear that *if* there exists a time in terms of