CLASSICAL TOPOLOGY AND QUANTUM STATES

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A. P. Balachandran
Physics Department
Syracuse University
Syracuse, N.Y. 13244-1130
USA

G. Marmo
Department of Physical Sciences
Naples University and INFN, Naples Section
I-80125 Naples
Italy

B. S. Skagerstam
Chalmers University of Technology
University of Göteborg
S-41296, Göteborg
Sweden

A. Stern
Department of Physics and Astronomy
University of Alabama
Tuscaloosa, AL 35487-0324
USA

World Scientific
Singapore • New Jersey • London • Hong Kong

Published by

World Scientific Publishing Co. Pte. Ltd.
P O Box 128, Farrer Road, Singapore 9128
USA office: 687 Hartwell Street, Teaneck, NJ 07666
UK office: 73 Lynton Mead, Totteridge, London N20 8DH

Library of Congress Cataloging-in-Publication Data

Balachandran, A. P., 1938-
Classical topology and quantum states/A.P. Balachandran ... [et al.].
p. cm.
Includes bibliographical references and index.
ISBN 9810203292. -- ISBN 9810203306 (pbk.)
1. Topology. 2. Quantum field theory. 3. Nuclear forces
(Physics) I. Title.
QC20.7.T65B35 1991
514--dc20 91-10473
CIP

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Printed in Singapore by Loi Printing Pte Ltd.

We dedicate this book to

Appa, Indra and Vinod;
Patrizia;
Helene, Emilie and Cecilia;
and
Lore, Max and Phoebe.

PREFACE

The topology of the configuration space for many classical systems of physical importance, such as the free particle and the harmonic oscillator in $\mathbf{R}^N$, happens to be very simple. The conventional exposition of classical and quantum mechanics generally concentrates on such systems, explains their quantization and goes on to show that there is a representation of wave functions as functions on the configuration space Q. However, already in 1931, Dirac understood in his work on magnetic monopoles that there are problems of interest where it is not natural to regard wave functions as functions on Q. Subsequent developments, especially in recent years, have made it clear that there are numerous important physical theories, all distinguished by a nontrivial topology of Q, where too it is not appropriate to regard wave functions as functions on Q. Rather they are to be thought of as special kinds of functions on a principal fibre bundle over Q or as sections of an associated vector bundle.

The nature of wave functions is only one aspect of the properties of a physical system subject to the influence of the configuration space topology. It is now appreciated that there are several other attributes which may show the effects of this topology. In particular, for appropriate topologies of Q, already the classical theory can predict the existence of novel sorts of stable configurations, such as line and point defects in condensed matter systems, solitons and monopoles. It happens that the nature of wave functions of the quantal version of many of these configurations is in turn influenced by the topology of Q. The latter can thus influence quantum dynamics both by leading to the existence of new states such as those describing solitons and by affecting their qualitative properties.

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This book is an introduction to the role of topology in the quantization of classical systems. It is also an introduction to topological solitons with special emphasis on Skyrmions. As regards the first aspect, several issues of current interest are dealt with at a reasonably elementary level. Examples of such topics we cover are principal fibre bundles and their role in quantum physics, the possibility of spinorial quantum states in a Lagrangian theory based on tensorial variables, and multiply connected configuration spaces and associated quantum phenomena like the QCD θ angle and exotic statistics. The ideas are also illustrated by simple examples such as the spinning particle, the charge-monopole system and strings in $3+1$ dimensions. The application of these ideas to quantum gravity is another subject treated at an introductory level. In the field of topological solitons, our main interest has been in the exposition of Skyrmion physics. For this reason, we have limited ourselves to a comparatively brief treatment of the general theory of solitons, adequate to follow the subsequent chapters on Skyrmion physics. Some Skyrmion phenomenology is also discussed although it is far from being exhaustive. A chapter on electroweak Skyrmions has also been included. Although these are not topological solitons, they do resemble Skyrmions in many ways so that such a chapter seemed appropriate. There is also another class of solitons called topological geons to which this book may serve as an introduction. They were discovered by Friedman and Sorkin and possess many remarkable properties because of the rich topological complexities to be found in gravitational models.

An attempt has been made in this book to introduce the reader to the significance of topology for many distinct physical systems such as spinning particles, the charge-monopole system, strings, Skyrmions, QCD and gravity. It is our hope that it will contribute to a wider appreciation of the profound role of topology in classical and quantum dynamics.

There are several important aspects of the role of topology in quantization and soliton physics that we have not dealt with in this book. A major omission has been the subject of anomalies. As indicated in the text, this is a topic which can be naturally approached using the concepts that we develop. Limitations on our time have forced us into this omission and we are obliged to refer the interested reader to the several excellent reviews which exist today. Another interesting aspect of quantization which we shall not discuss is the relation between our approach to quantization with its emphasis on topology and the one which concentrates instead on domain and extension problems of operators. As alluded to previously, our treatment of general soliton theory and Skyrmion phenomenology has also been rather brief. In Skyrmion physics, there exists in particular a substantial body of research which considers the applications of Skyrme's model and its variants to low energy physics and which we have not attempted to cover adequately because of our limitations. Fortunately detailed reviews of these developments are also available to which the interested reader can refer.

With regard to references, no attempt has been made to give an exhaustive bibliography. We shall list essentially only those publications which we have frequently used during the preparation of this book. We shall also list a few representative review articles which cover material not treated by us here. We apologize to those authors whose work we have overlooked and to those who feel that their work should have been referred to.

The book is an outgrowth of lectures given by the authors at various institutions and conferences. We thank the audience at these lectures as well as our collegues at Syracuse and elsewhere for their suggestions and criticism. We are especially grateful to Rafael Sorkin for numerous discussions about the material treated in this book and for collaboration on its title. We thank Ted Allen for carefully proofreading the manuscript and several useful suggestions, and David Dallman and Kumar Gupta for their generous help in collecting references. The typing of the several versions of the manuscript of this book was done by Jane Boyd at Tuscaloosa, by Guido Celentano at Naples and by Annika Hofling at Göteborg. We also gratefully record our appreciation of their patience and accurate work here. Finally, the TH-division at CERN is acknowledged for hospitality while this book was completed.

A.P.B. was supported by the U.S. Department of Energy under contract number DE-FG02-85ER40231 and B.S.S. was supported by the Swedish National Science Research Council under contract number 8244-103 during the preparation of this work. A.S. was supported in part by the U.S. Department of Energy under contract number DE-FG05-84ER40141.

Göteborg, Naples, Syracuse, Tuscaloosa
September, 1990

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Chapter 1

INTRODUCTION

The dynamics of a system in classical mechanics can be described by equations of motion on a configuration space Q. These equations are generally of second order in time. Thus if the position $q(t_0)$ of the system in Q and its velocity $\dot{q}(t_0)$ are known at some time t_0, then the equations of motion uniquely determine the trajectory $q(t)$ for all time t.

When the classical system is quantized, the state of a system at time t_0 is not specified by a position in Q and a velocity. Rather, it is described by a wave function ψ which in elementary quantum mechanics is a (normalized) function on Q. The correspondence between the quantum states and wave functions however is not one to one since two wave functions which differ by a phase describe the same state. The quantum state of a system is thus an equivalence class $\{e^{i\alpha}\psi | \alpha \text{ real}\}$ of normalized wave functions. The physical reason for this circumstance is that experimental observables correspond to functions like $\psi^*\psi$ which are insensitive to this phase.

In discussing the transformation properties of wave functions, it is often convenient to enlarge the domain of definition of wave functions in elementary quantum mechanics in such a way as to naturally describe all the wave functions of an equivalence class. Thus instead of considering wave functions as functions on Q, we can regard them as functions on a larger space $\hat{Q} = Q \times S^1 \equiv \{(q, e^{i\alpha})\}$. The space $\hat{Q}$ is obtained by associating circles S^1

to each point of Q and is said to be a $U(1)$ bundle on Q. Wave functions on $\hat{Q}$ are not completely general functions on $\hat{Q}$, rather they are functions with the property $\psi(q, e^{i(\alpha+\theta)}) = \psi(q, e^{i\alpha})e^{i\theta}$. [Here we can also replace $e^{i\theta}$ by $e^{ni\theta}$ where n is a fixed integer]. Because of this property, experimental observables like $\psi^*\psi$ are independent of the extra phase and are functions on Q as they should be. The standard elementary treatment which deals with functions on Q is recovered by restricting the wave functions to a surface $\{(q, e^{i\alpha_0})|q \in Q\}$ in $\hat{Q}$ where α_0 has a fixed value. Such a choice α_0 of α corresponds to a phase convention in the elementary approach.

When the topology of Q is nontrivial, it is often possible to associate circles S^1 to each point of Q so that the resultant space $\hat{Q} = \{\hat{q}\}$ is not $Q \times S^1$ although there is still an action of $U(1)$ on $\hat{Q}$. We shall indicate this action by $\hat{q} \to \hat{q}e^{i\theta}$. It is the analogue of the transformation $(q, e^{i\alpha}) \to (q, e^{i\alpha}e^{i\theta})$ we considered earlier. We shall require this action to be free, which means that $\hat{q}e^{i\theta} = \hat{q}$ if and only if $e^{i\theta}$ is the identity of $U(1)$. When $\hat{Q} \neq Q \times S^1$, the $U(1)$ bundle $\hat{Q}$ over Q is said to be twisted. It is possible to contemplate wave functions which are functions on $\hat{Q}$ even when this bundle is twisted provided they satisfy the constraint $\psi(\hat{q}e^{i\theta}) = \psi(\hat{q})e^{ni\theta}$ for some fixed integer n. If this constraint is satisfied, experimental observables being invariant under the $U(1)$ action are functions on Q as we require. However, when the bundle is twisted, it does not admit globally valid coordinates of the form $(q, e^{i\alpha})$ so that it is not possible (modulo certain technical qualifications) to make a global phase choice, as we did ealier. In other words, it is not possible to regard wave functions as functions on Q when $\hat{Q}$ is twisted.

The classical Lagrangian L often contains complete information on the nature of the bundle $\hat{Q}$. We can regard the classical Lagrangian as a function on the tangent bundle $T\hat{Q}$ of $\hat{Q}$. The space $T\hat{Q}$ is the space of positions in $\hat{Q}$ and the associated velocities. When $\hat{Q}$ is trivial, it is possible to reduce any such Lagrangian to a Lagrangian on the space TQ of positions and velocities associated with Q thereby obtaining the familiar description. On the other hand, when $\hat{Q}$ is twisted, such a reduction is in general impossible. Since the equations of motion deal with trajectories on Q and not on $\hat{Q}$, it is necessary that there is some principle which renders the additional $U(1)$ degrees of freedom in such a Lagrangian nondynamical. This principle is the principle of gauge invariance for the gauged group $U(1)$. Thus under the gauge transformation $\hat{q}(t) \rightarrow \hat{q}(t)e^{i\theta(t)}$, these Lagrangians change by constant times $d\theta/dt$, where t is the time. Since the equations of motion therefore involve only gauge invariant quantities which can be regarded as functions of positions and velocities associated with Q, these equations describe dynamics on Q. The Lagrangians we deal with in this book split into two terms L_0 and L_{WZ}, where L_0 is gauge invariant while L_{WZ} changes as indicated above. This term L_{WZ} has a geometrical interpretation. It is the one which is associated with the nature of the bundle $\hat{Q}$.

In particle physics, such a topological term was first discovered by Wess and Zumino [1] in their investigation of nonabelian anomalies in gauge theories. The importance and remarkable properties of such "Wess-Zumino terms" have been forcefully brought to the attention of particle physicists in recent years because of the realization that they play a critical role in creating fermionic states in a theory with bosonic fields and in determining the anomaly structure of effective field theories.

In point particle mechanics, the existence and significance of Wess-Zumino terms have long been understood. For example, such terms play an essential role in the program of geometric quantization [2] and related investigations which study the Hamiltonian or Lagrangian description of particles of fixed spin [3-18]. A similar term occurs in the description of the charge-monopole system [19-24] and has also been discussed in the literature. Recently such terms have been found in dual string models as well [25,26].

The Wess-Zumino term affects the equations of motion and has significant dynamical consequences already at the classical level. Its impact however is most dramatic in quantum theory where as was indicated above it affects the structure of the state space. For example, in the $SU(3)$ chiral model it is this term which is responsible for the fermionic nature of the Skyrmion [27].

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