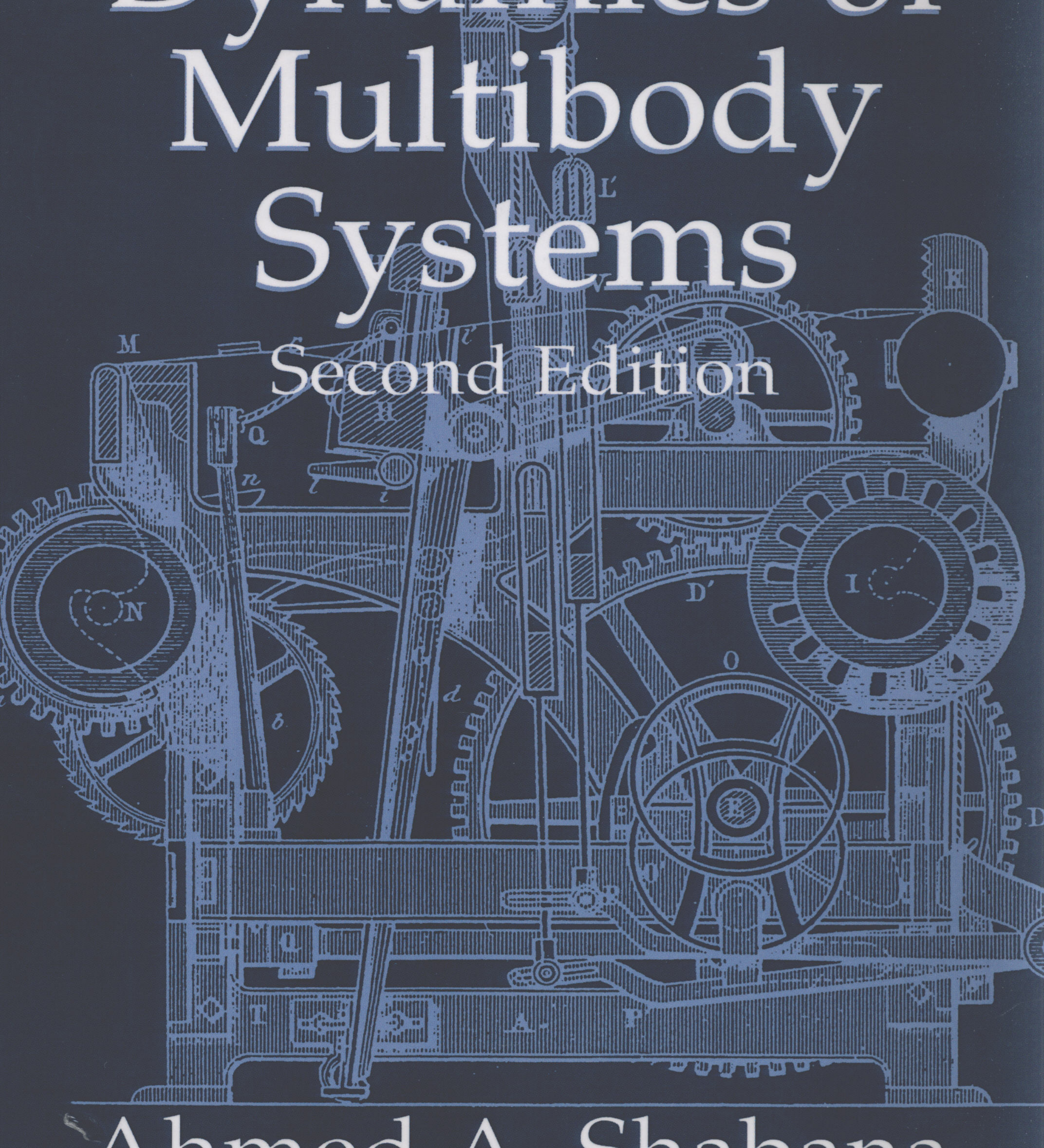
Dynamics of Multibody Systems

Second Edition



Ahmed A. Shabana

DYNAMICS OF MULTIBODY SYSTEMS

Second Edition

Ahmed A. Shabana
University of Illinois at Chicago

CAMBRIDGE
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York NY 10011–4211, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

http://www.cambridge.org

First published 1989 by Wiley & Sons
Published by Cambridge University Press 1998
First paperback edition 2003

Typeset in Times Roman

A catalogue record for this book is available from the British Library

Library of Congress Cataloguing-in-Publication Data

Shabana, Ahmed A., 1951–
Dynamics of multibody systems / Ahmed A. Shabana. – 2nd ed.
p. cm.
Includes bibliographical references (p. 359–366) and index.
ISBN 0 521 59446 4 hardback
1.Dynamics. 2. Kinematics. I. Title.
QA845.S45 1998
531′. 11 – DC21
97-31088
CIP

ISBN 0 521 59446 4 hardback
ISBN 0 521 54411 4 paperback

To my father

and to

the memory of my mother

Preface

The methods for the nonlinear analysis of physical and mechanical systems developed for use on modern digital computers provide means for accurate analysis of large-scale systems under dynamic loading conditions. These methods are based on the concept of replacing the actual system by an equivalent model made up from discrete bodies having known elastic and inertia properties. The actual systems, in fact, form multibody systems consisting of interconnected rigid and deformable bodies, each of which may undergo large translational and rotational displacements. Examples of physical and mechanical systems that can be modeled as multibody systems are machines, mechanisms, vehicles, robotic manipulators, and space structures. Clearly, these systems consist of a set of interconnected bodies which may be rigid or deformable. Furthermore, the bodies may undergo large relative translational and rotational displacements. The dynamic equations that govern the motion of these systems are highly nonlinear which in most cases cannot be solved analytically in a closed form. One must resort to the numerical solution of the resulting dynamic equations.

The aim of this text, which is based on lectures that I have given during the past twelve years, is to provide an introduction to the subject of multibody mechanics in a form suitable for senior undergraduate and graduate students. The initial notes for the text were developed for two first-year graduate courses introduced and offered at the University of Illinois at Chicago. These courses were developed to emphasize both the general methodology of the nonlinear dynamic analysis of multibody systems and its actual implementation on the high-speed digital computer. This was prompted by the necessity to deal with complex problems arising in modern engineering and science. In this text, an attempt has been made to provide the rational development of the methods from their foundations and develop the techniques in clearly understandable stages. By understanding the basis of each step, readers can apply the method to their own problems.

The material covered in this text comprises an introductory chapter on the subjects of kinematics and dynamics of rigid and deformable bodies. In this chapter some background materials and a few fundamental ideas are presented. In Chapter 2, the kinematics of the body reference is discussed and the transformation matrices that define the orientation of this body reference are developed. Alternate forms of the transformation matrix are presented. The material presented in this chapter is essential for understanding the dynamic motion of both rigid and deformable bodies. Analytical techniques for deriving the system differential and algebraic equations of motion of a multibody system consisting of rigid bodies are discussed in Chapter 3. In Chapter 4, an introduction to the theory of elasticity is presented. The material covered in this chapter is essential for understanding the dynamics of deformable bodies that undergo large translational and rotational displacements. In Chapter 5, the equations of motion of deformable multibody systems in which the reference motion and elastic deformation are coupled are derived using classical approximation methods. In Chapters 6 and 7, two finite element formulations are presented. Both formulations lead to exact modeling of the rigid body inertia and lead to zero strains under an arbitrary rigid body motion. The first formulation discussed in Chapter 6, which is based on the concept of the intermediate element coordinate system, uses the definition of the coordinates used in the conventional finite element method. A conceptually different finite element formulation that can be used in the large deformation analysis of multibody systems is presented in Chapter 7. In this chapter, the absolute nodal coordinate formulation in which no infinitesimal or finite rotations are used as element coordinates is introduced.

I am grateful to many teachers, colleagues, and students who have contributed to my education in this field. I owe a particular debt of gratitude to Dr. R.A. Wehage and Dr. M.M. Nigm for their advice, encouragement, and assistance at various stages of my educational career. Their work in the areas of computational mechanics and vibration theory stimulated my early interest in the subject of nonlinear dynamics. Several chapters of this book have been read, corrected, and improved by many of my graduate students. I would like to acknowledge the collaboration with my students Drs. Om Agrawal, E. Mokhtar Bakr, Ipek Basdogan, Michael Brown, Bilin Chang, Che-Wei Chang, Koroosh Changizi, Da-Chih Chen, Jui-Sheng Chen, Jin-Hwan Choi, Hanaa El-Absy, Marian Gofron, Wei-Hsin Gau, Wei-Cheng Hsu, Kuo-Hsing Hwang, Yunn-Lin Hwang, Yehia Khulief, John Kremer, Haichiang Lee, Jalil Rismantab-Sany, and Mohammad Sarwar, and my current students Marcello Berzeri, Marcello Campanelli, Andrew Christensen, Hussien Hussien, and Refaat Yakoub. Their work contributed significantly to the development of the material presented in this book. Special thanks are due to Ms. Denise Burt for the excellent job in typing most of the manuscript. Finally, I thank my family for their patience and encouragement during the time of preparation of this text.

Chicago, Illinois
July 1997

Ahmed Shabana

Contents

1 INTRODUCTION

1.1 MULTIBODY SYSTEMS

The primary purpose of this book is to develop methods for the dynamic analysis of *multibody systems* that consist of interconnected *rigid* and *deformable* components. In that sense, the objective may be considered as a generalization of methods of structural and rigid body analysis. Many mechanical and structural systems such as vehicles, space structures, robotics, mechanisms, and aircraft consist of interconnected components that undergo large translational and rotational displacements. Figure 1 shows examples of such systems that can be modeled as multibody systems. In general, a multibody system is defined to be a collection of subsystems called *bodies*, *components*, or *substructures*. The motion of the subsystems is kinematically constrained because of different types of joints, and each subsystem or component may undergo large translations and rotational displacements.

Basic to any presentation of multibody mechanics is the understanding of the motion of subsystems (bodies or components). The motion of material bodies formed the subject of some of the earliest researches pursued in three different fields, namely, *rigid body mechanics*, *structural mechanics*, and *continuum mechanics*. The term *rigid body* implies that the deformation of the body under consideration is assumed small such that the body deformation has no effect on the gross body motion. Hence, for a rigid body, the distance between any two of its particles remains constant at all times and all configurations. The motion of a rigid body in space can be completely described by using six generalized coordinates. However, the resulting mathematical model in general is highly nonlinear because of the large body rotation. On the other hand, the term *structural mechanics* has come into wide use to denote the branch of study in which the deformation is the main concern. Large body rotations are not allowed, thus resulting in inertia-invariant structures. In many applications, however, a large number of elastic coordinates have to be included in the mathematical model in order to accurately describe the body deformation. From the study of these two

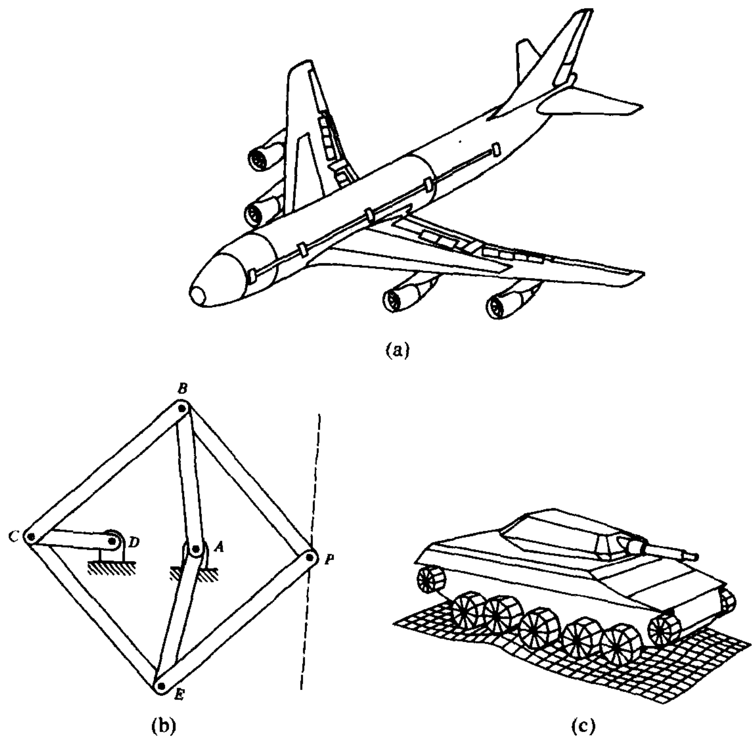


Figure 1.1 Mechanical and structural systems.

subjects, rigid body and structural mechanics, there has evolved the vast field known as *continuum mechanics*, wherein the general body motion is considered, resulting in a mathematical model that has the disadvantages of the previous cases, mainly nonlinearity and large dimensionality. This constitutes many computational problems that will be addressed in subsequent chapters.

The research in the area of multibody dynamics has been motivated by growing interest in the simulation and design of large-scale systems of interconnected bodies that undergo large angular rotations. The analysis and design of such systems require the simultaneous solution of hundreds or thousands of first-order differential equations, a task that could not be accomplished a few decades ago before the development of electronic computers. Most of the work done in this area is based on analyzing rigid multibody systems, and many computer-based techniques that solve complex rigid body systems have been developed.

In recent years, however, greater emphasis has been placed on the design of high-speed, lightweight, precision systems. Generally these systems incorporate various types of driving, sensing, and controlling devices working together to achieve specified performance requirements under different loading conditions. The design and performance analysis of such systems can be greatly enhanced through transient dynamic simulations, provided all significant effects can be incorporated into the mathematical model. The need for a better design, in addition to the fact that many mechanical and structural systems operate in hostile environments, has demanded the

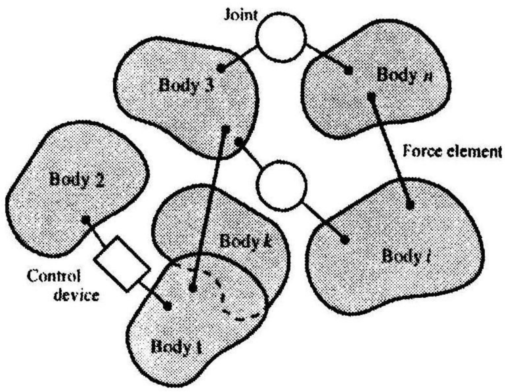


Figure 1.2 Multibody systems.

inclusion of many factors that have been ignored in the past. Systems such as engines, robotics, machine tools, and space structures may operate at high speeds and in very high temperature environments. The neglect of the deformation effect, for example, when these systems are analyzed leads to a mathematical model that poorly represents the actual system.

Consider, for instance, the Peaucellier mechanism shown in Fig. 1(b), which is designed to generate a straight-line path. The geometry of this mechanism is such that $BC = BP = EC = EP$ and $AB = AE$. Points A, C, and P should always lie on a straight line passing through A. The mechanism always satisfies the condition $AC \times AP = c$, where c is a constant called the *inversion constant*. In case $AD = CD$, point C must trace a circular arc and point P should follow an exact straight line. However, this will not be the case when the deformation of the links is considered. If the flexibility of links has to be considered in this specific example, the mechanism can be modeled as a multibody system consisting of interconnected rigid and deformable components, each of which may undergo finite rotations. The connectivity between different components of this mechanism can be described by using revolute joints (turning pairs). This mechanism and other examples shown in Fig. 1, which have different numbers of bodies and different types of mechanical joints, are examples of mechanical and structural systems that can be viewed as a multibody system shown in the abstract drawing in Fig. 2. In this book, computer-based techniques for the dynamic analysis of general multibody systems containing interconnected sets of rigid and deformable bodies will be developed. To this end, methods for the kinematics and dynamics of rigid and deformable bodies that experience large translational and rotational displacements will be presented in the following chapters. In the following sections of this chapter, however, some of the basic concepts that will be subject of detailed analysis in the chapters that follow are briefly discussed.

1.2 REFERENCE FRAMES

The configuration of a multibody system can be described using measurable quantities such as displacements, velocities, and accelerations. These are vector quantities

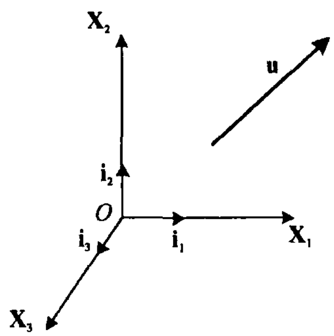


Figure 1.3 Reference frame.

that have to be measured with respect to a proper *frame of reference* or *coordinate system*. In this text, the term *frame of reference*, which can be represented by three orthogonal axes that are rigidly connected at a point called the *origin* of this reference, will be frequently used. Figure 3 shows a frame of reference that consists of the three orthogonal axes $\mathbf{X}_1$, $\mathbf{X}_2$, and $\mathbf{X}_3$. A vector $\mathbf{u}$ in this coordinate system can be defined by three components u_1, u_2, and u_3, along the orthogonal axes $\mathbf{X}_1$, $\mathbf{X}_2$, and $\mathbf{X}_3$, respectively. The vector $\mathbf{u}$ can then be written in terms of its components as

$$\mathbf{u} = [u_1 \quad u_2 \quad u_3]^{\mathrm{T}}$$

or as

$$\mathbf{u} = u_1\mathbf{i}_1 + u_2\mathbf{i}_2 + u_3\mathbf{i}_3$$

where $\mathbf{i}_1$, $\mathbf{i}_2$, and $\mathbf{i}_3$ are unit vectors along the orthogonal axes $\mathbf{X}_1$, $\mathbf{X}_2$, and $\mathbf{X}_3$, respectively.

Generally, in dealing with multibody systems two types of coordinate systems are required. The first is a coordinate system that is fixed in time and represents a unique standard for all bodies in the system. This coordinate system will be referred to as *global*, or *inertial frame* of reference. In addition to this inertial frame of reference, we assign a *body reference* to each component in the system. This body reference translates and rotates with the body; therefore, its location and orientation with respect to the inertial frame change with time. Figure 4 shows a typical body, denoted as body i in the multibody system. The coordinate system $\mathbf{X}_1\mathbf{X}_2\mathbf{X}_3$ is the global inertial frame of reference, and the coordinate system $\mathbf{X}_1^i\mathbf{X}_2^i\mathbf{X}_3^i$ is the body coordinate system. Let $\mathbf{i}_1$, $\mathbf{i}_2$, and $\mathbf{i}_3$ be unit vectors along the axes $\mathbf{X}_1$, $\mathbf{X}_2$, and $\mathbf{X}_3$, respectively, and let $\mathbf{i}_1^i$, $\mathbf{i}_2^i$, and $\mathbf{i}_3^i$ be unit vectors along the body axes $\mathbf{X}_1^i$, $\mathbf{X}_2^i$, and $\mathbf{X}_3^i$, respectively. The unit vectors $\mathbf{i}_1$, $\mathbf{i}_2$, and $\mathbf{i}_3$ are fixed in time; that is, they have constant magnitude and direction, while the unit vectors $\mathbf{i}_1^i$, $\mathbf{i}_2^i$, and $\mathbf{i}_3^i$ have changeable orientations. A vector $\mathbf{u}^i$ defined in the body coordinate system can be written as

$$\mathbf{u}^i = \bar{u}_1^i\mathbf{i}_1^i + \bar{u}_2^i\mathbf{i}_2^i + \bar{u}_3^i\mathbf{i}_3^i$$

where $\bar{u}_1^i$, $\bar{u}_2^i$, and $\bar{u}_3^i$ are the components of the vector $\mathbf{u}^i$ in the local body coordinate system. The same vector $\mathbf{u}^i$ can be expressed in terms of its components in the global

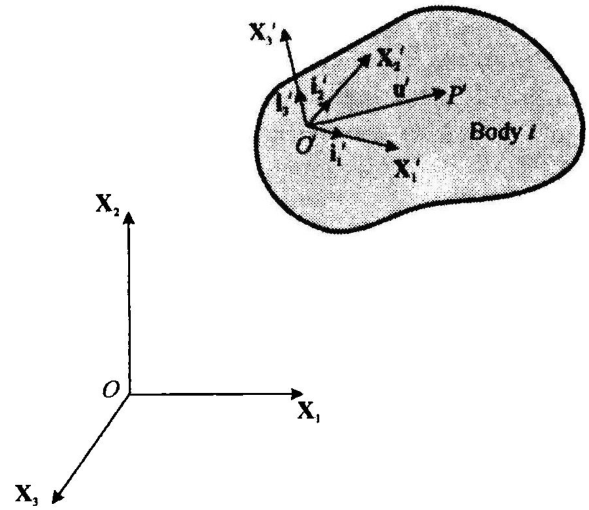


Figure 1.4 Body coordinate system.

coordinate system as

$$\mathbf{u}^i = u_1^i \mathbf{i}_1 + u_2^i \mathbf{i}_2 + u_3^i \mathbf{i}_3$$

where u_1^i, u_2^i, and u_3^i are the components of the vector $\mathbf{u}^i$ in the global coordinate system. We have, therefore, given two different representations for the same vector $\mathbf{u}^i$, one in terms of the body coordinates and the other in terms of global coordinates. Since it is easier to define the vector in terms of the local body coordinates, it is useful to have relationships between the local and global components. Such relationships can be obtained by developing the transformation between the local and global coordinate systems. For instance, consider the *planar motion* of the body shown in Fig. 5. The coordinate system $\mathbf{X}_1\mathbf{X}_2$ represents the inertial frame and $\mathbf{X}_1^i\mathbf{X}_2^i$ is the body coordinate

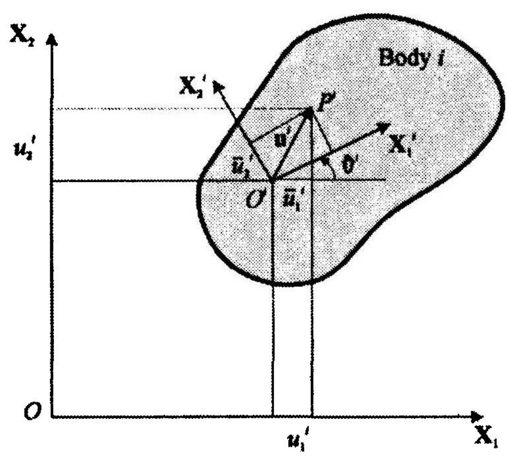


Figure 1.5 Planar motion.

system. Let $\mathbf{i}_1$ and $\mathbf{i}_2$ be unit vectors along the $\mathbf{X}_1$ and $\mathbf{X}_2$ axes, respectively, and let $\mathbf{i}_1^i$ and $\mathbf{i}_2^i$ be unit vectors along the body axes $\mathbf{X}_1^i$ and $\mathbf{X}_2^i$, respectively. The orientation of the body coordinate system with respect to the global frame of reference is defined by the angle θ^i. Since $\mathbf{i}_1^i$ is a unit vector, its component along the $\mathbf{X}_1$ axis is $\cos\theta^i$, while its component along the $\mathbf{X}_2$ axis is $\sin\theta^i$. One can then write the unit vector $\mathbf{i}_1^i$ in the global coordinate system as

$$\mathbf{i}_1^i = \cos\theta^i \mathbf{i}_1 + \sin\theta^i \mathbf{i}_2$$

Similarly, the unit vector $\mathbf{i}_2^i$ is given by

$$\mathbf{i}_2^i = -\sin\theta^i \mathbf{i}_1 + \cos\theta^i \mathbf{i}_2$$

The vector $\mathbf{u}^i$ is defined in the body coordinate system as

$$\mathbf{u}^i = \bar{u}_1^i \mathbf{i}_1^i + \bar{u}_2^i \mathbf{i}_2^i$$

where $\bar{u}_1^i$ and $\bar{u}_2^i$ are the components of the vector $\mathbf{u}^i$ in the body coordinate system. Using the expressions for $\mathbf{i}_1^i$ and $\mathbf{i}_2^i$, one gets

$$\begin{aligned}\mathbf{u}^i &= \bar{u}_1^i(\cos\theta^i \mathbf{i}_1 + \sin\theta^i \mathbf{i}_2) + \bar{u}_2^i(-\sin\theta^i \mathbf{i}_1 + \cos\theta^i \mathbf{i}_2)\\ &= \left(\bar{u}_1^i \cos\theta^i - \bar{u}_2^i \sin\theta^i\right)\mathbf{i}_1 + \left(\bar{u}_1^i \sin\theta^i + \bar{u}_2^i \cos\theta^i\right)\mathbf{i}_2\\ &= u_1^i \mathbf{i}_1 + u_2^i \mathbf{i}_2\end{aligned}$$

where u_1^i and u_2^i are the components of the vector $\mathbf{u}^i$ defined in the global coordinate system and given by

$$\begin{aligned}u_1^i &= \bar{u}_1^i \cos\theta^i - \bar{u}_2^i \sin\theta^i\\ u_2^i &= \bar{u}_1^i \sin\theta^i + \bar{u}_2^i \cos\theta^i\end{aligned}$$

These two equations which provide algebraic relationships between the local and global components in the planar analysis can be expressed in a matrix form as

$$\mathbf{u}^i = \mathbf{A}^i \bar{\mathbf{u}}^i$$

where $\mathbf{u}^i = [u_1^i \quad u_2^i]^{\mathrm{T}}$, $\bar{\mathbf{u}}^i = [\bar{u}_1^i \quad \bar{u}_2^i]^{\mathrm{T}}$, and $\mathbf{A}^i$ is the planar transformation matrix defined as

$$\mathbf{A}^i = \begin{bmatrix}\cos\theta^i & -\sin\theta^i\\ \sin\theta^i & \cos\theta^i\end{bmatrix}$$

In Chapter 2 we will study the spatial kinematics and develop the spatial transformation matrix and study its important properties.

1.3 PARTICLE MECHANICS

Dynamics in general is the science of studying the motion of particles or bodies. The subject of dynamics can be divided into two major branches, *kinematics* and *kinetics*. In kinematic analysis we study the motion regardless of the forces that cause it, while kinetics deals with the motion and forces that produce it. Therefore, in kinematics attention is focused on the geometric aspects of motion. The objective is, then, to determine the positions, velocities, and accelerations of the system under investigation. In order to understand the dynamics of multibody systems containing rigid and deformable bodies, it is important to understand first the body dynamics. We start with a brief discussion on the dynamics of particles that form the rigid and deformable bodies.

Particle Kinematics A *particle* is assumed to have no dimensions and accordingly can be treated as a point in a three-dimensional space. Therefore, in studying the kinematics of particles, we are concerned primarily with the translation of a point with respect to a selected frame of reference. The position of the particle can then be defined using three coordinates. Figure 6 shows a particle p in a three-dimensional space. The position vector of this particle can be written as

$$\mathbf{r} = x_1\mathbf{i}_1 + x_2\mathbf{i}_2 + x_3\mathbf{i}_3 \tag{1.1}$$

where $\mathbf{i}_1$, $\mathbf{i}_2$, and $\mathbf{i}_3$ are unit vectors along the $\mathbf{X}_1$, $\mathbf{X}_2$, and $\mathbf{X}_3$ axes and x_1, x_2, and x_3 are the Cartesian coordinates of the particle.

The velocity of the particle is defined to be the time derivative of the position vector. If we assume that the axes $\mathbf{X}_1$, $\mathbf{X}_2$, and $\mathbf{X}_3$ are fixed in time, the unit vectors $\mathbf{i}_1$, $\mathbf{i}_2$, and $\mathbf{i}_3$ have a constant magnitude and direction. The velocity vector $\mathbf{v}$ of the particle can be written as

$$\mathbf{v} = \dot{\mathbf{r}} = \frac{d}{dt}(\mathbf{r}) = \dot{x}_1\mathbf{i}_1 + \dot{x}_2\mathbf{i}_2 + \dot{x}_3\mathbf{i}_3 \tag{1.2}$$

where $(\dot{\ })$ denotes differentiation with respect to time and $\dot{x}_1, \dot{x}_2$, and $\dot{x}_3$ are the Cartesian components of the velocity vector.

The acceleration of the particle is defined to be the time derivative of the velocity vector, that is,

$$\mathbf{a} = \frac{d}{dt}(\mathbf{v}) = \ddot{x}_1\mathbf{i}_1 + \ddot{x}_2\mathbf{i}_2 + \ddot{x}_3\mathbf{i}_3 \tag{1.3}$$

where $\mathbf{a}$ is the acceleration vector and $\ddot{x}_1, \ddot{x}_2$, and $\ddot{x}_3$ are the Cartesian components of the acceleration vector. Using vector notation, the position vector of the particle in terms of the *Cartesian coordinates* can be written as

$$\mathbf{r} = [x_1 \quad x_2 \quad x_3]^{\mathrm{T}}$$

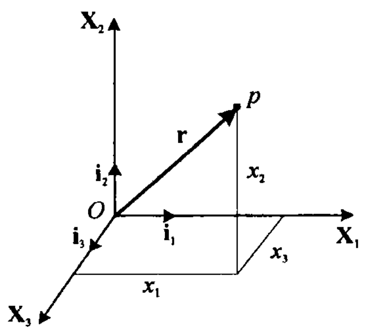


Figure 1.6 Position vector of the particle p.

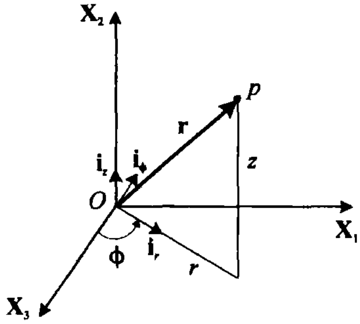


Figure 1.7 Cylindrical coordinates.

while the velocity and acceleration vectors are given by

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \begin{bmatrix} \frac{dx_1}{dt} & \frac{dx_2}{dt} & \frac{dx_3}{dt} \end{bmatrix}^{\mathrm{T}}$$
$$= [\dot{x}_1 \quad \dot{x}_2 \quad \dot{x}_3]^{\mathrm{T}}$$
$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2} = \begin{bmatrix} \frac{d^2x_1}{dt^2} & \frac{d^2x_2}{dt^2} & \frac{d^2x_3}{dt^2} \end{bmatrix}^{\mathrm{T}}$$
$$= [\ddot{x}_1 \quad \ddot{x}_2 \quad \ddot{x}_3]^{\mathrm{T}}$$

The set of coordinates that can be used to define the particle position is not unique. In addition to the Cartesian representation, other sets of coordinates can be used for the same purpose. In Fig. 7, the position of particle p can be defined using the three *cylindrical coordinates*, r, ϕ, and z, while in Fig. 8, the particle position is identified using the *spherical coordinates* r, θ, and ϕ. In many situations, however, it is useful to obtain kinematic relationships between different sets of coordinates. For instance, if we consider the planar motion of a particle p in a circular path as shown in Fig. 9, the position vector of the particle can be written in the fixed coordinate system $\mathbf{X}_1\mathbf{X}_2$ as

$$\mathbf{r} = [x_1 \quad x_2]^{\mathrm{T}} = x_1\mathbf{i}_1 + x_2\mathbf{i}_2$$

where x_1 and x_2 are the coordinates of the particle and $\mathbf{i}_1$ and $\mathbf{i}_2$ are unit vectors along the fixed axes $\mathbf{X}_1$ and $\mathbf{X}_2$, respectively. In terms of the polar coordinates r and θ, the

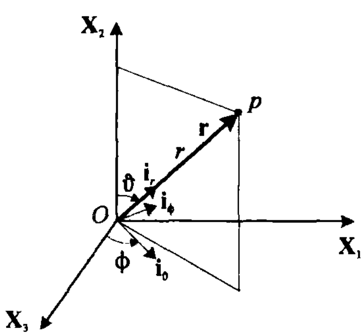


Figure 1.8 Spherical coordinates.