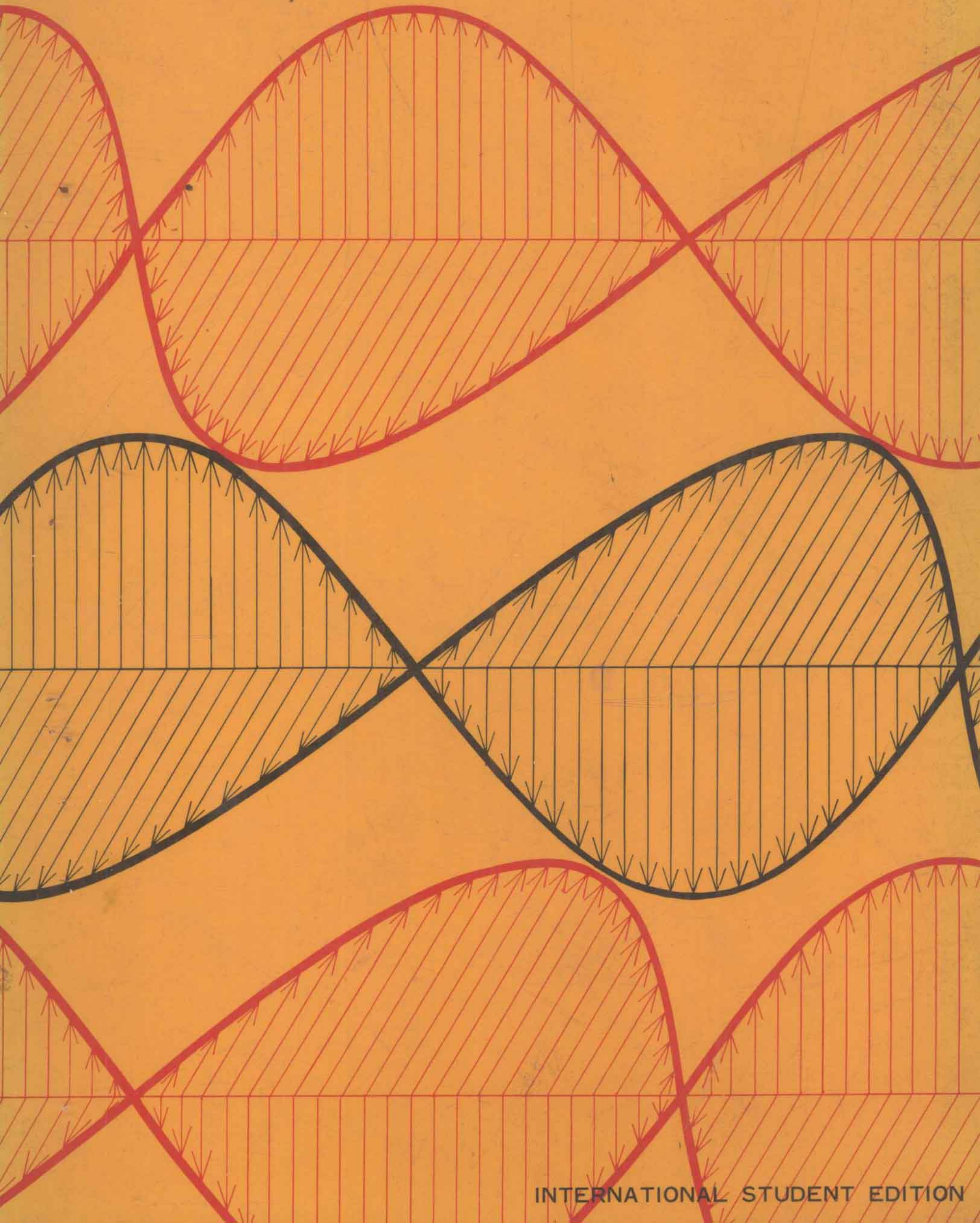
CONTROL SYSTEMS THEORY

Olle I. Elgerd



INTERNATIONAL STUDENT EDITION

CONTROL SYSTEMS THEORY

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International Student Edition

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Library of Congress Catalog Card Number 66-20716

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TOSHO PRINTING CO., LTD., TOKYO, JAPAN

CONTROL SYSTEMS THEORY

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PREFACE

As of this writing (1966), there are over one hundred books and monographs on the general topic of control and servomechanisms in the English language alone. Any new author venturing into this field should therefore be prepared to motivate the need for his particular product and to define its specific *raison d'être*. I believe the time is ripe for a "new look" in control textbooks and offer the following reasons for this assumption:

It is current praxis to distinguish between "classical" and "modern" control, a distinction that not only is arbitrary but also has resulted in an unfortunate and often artificial division between topic areas usually covered in the graduate and undergraduate control programs at our universities.

The vast majority of present texts take the classical view of control. With few exceptions, these texts are direct descendants—first, second, or later generation—of the classic treatises by James, Nichols, and Phillips and Truxal. The outstanding features of the classical approach are:

1. A linear model of the control system is assumed, and the designer then proceeds to describe the system in terms of transfer functions of either the frequency or the Laplace variety, depending upon the type of information he has available.
2. An almost exclusive emphasis is placed upon those specific control systems classified as servomechanisms. In addition, attention is directed in most cases to systems of the single-input–single-output variety.
3. The performance criteria usually specified make it necessary to utilize *indirect* design techniques of the trial-and-error type.

The classical approach, typically relegated to the undergraduate control programs at our universities, has many outstanding and time-tested advantages. No sophisticated mathematics is required beyond the elements of linear differential equations and complex algebra. The present generation of control engineers is thoroughly indoctrinated with classical design techniques, and the majority of the control systems now being manufactured are designed by classical methods.

However, classical design methods have numerous shortcomings. Empiricism, trial-and-error design, and a lack of a *fundamental* theory combine to make conventional control more an art than a science. For the exacting requirements of the more complex automatic-control problems in modern technology, classical methods are, in effect, entirely inadequate. This is where "modern control" enters the picture.

The modern theory approaches the control problem in a more fundamental fashion. For example, it defines the concept of control and establishes test procedures for determining when and under what conditions a system is *controllable*. *Direct* synthesis methods are proposed which lead to *unique* system designs that are *optimal* in some defined sense. Extensive use is made of the modern computer. Theories are developed for systems with learning and adaptive capabilities.

Modern control theory leans heavily on mathematics that typically are not included on the undergraduate menu, e.g., calculus of variation and matrix algebra. It prefers time-domain system description (state variables) to transfer-function methods since it does not limit itself to linear models. The methods of modern control have been developed in many instances by applied and pure mathematicians, with the result that much of the written presentation is very formal and quite inaccessible to most control engineers. At our universities, modern control theory is reserved for the graduate programs and is presented in courses which are usually completely unintegrated with classical viewpoints. Formalism, isolation from the undergraduate program, and unwillingness to merge the modern concepts with existing conventional methods have prevented a wide dissemination of the modern ideas.

This book undertakes the task of bridging the existing gap between the conventional techniques and modern theory. Its objectives are, specifically:

1. To give a basic presentation of the fundamental control problem
2. To integrate modern concepts with conventional design techniques
3. To strip some of the new theories of some of the mathematical clothing in which they are obscured in order to make them understandable to the senior undergraduate engineering student
4. To bring into focus the importance of the modern computer—analog, digital, and hybrid—in design and on-line operation of control systems

The book aims at an audience consisting of the senior undergraduate and the first-year graduate student and the practicing control engineer. Its contents correspond approximately to six semester credit hours, which could be covered entirely in an undergraduate course sequence, or its first half (through Chap. 7) could be offered on the undergraduate level and the second half on the first-year graduate level.

In order to meet the stated objectives, I have chosen an approach which is very different from the usual one. Examples are used prolifically, but instead of starting with a mathematical model divorced from reality, the origin of the model is discussed and every step of the analysis is clearly motivated from there on. The elements of matrix algebra and variational calculus have been included, but it has been assumed that the student has had at least an elementary background in linear differential equations and transform calculus. However, a summary of Laplace and Fourier transform theorems has been included as Appendix B.

On occasion, mathematical shortcuts have been taken to preserve simplicity and continuity in the presentation, and for the same purpose, formalism has been avoided.

It has not been possible to cover componentry to any great extent. It is hoped that the examples chosen are sufficiently practical to give the reader a feeling that he always has "one foot on the ground." I have found that the presentation of the material is greatly enhanced by a parallel laboratory course taking the form of a series of simulation experiments. The examples in the text and also the end-of-chapter exercises are put on the analog computer, and the student is given a comparison between analytical results and computer recordings.

It is, of course, impossible to cover in one volume the entire spectrum of topic areas in a field as wide as control. It has been very difficult to decide where to draw the line, but I finally decided that the most natural grouping is the following one:

1. Deterministic control
2. Stochastic control

Stochastic control encompasses topics such as estimation, filtering, adaptive and learning systems, and also biocontrol and has therefore not been included in the present volume.

The methods and philosophy of teaching control courses as reflected in this book were developed in the graduate and undergraduate programs of the Department of Electrical Engineering, University of Florida. I was given the opportunity to write this text during a year as Visiting Professor at the University of Colorado, and I am particularly indebted to Dr. Max Peters, Dean of the College of Engineering, and Dr. Frank Barnes, Chairman of the Department of Electrical Engineering. Dr. John G. Truxal, Dean of the Polytechnic Institute of Brooklyn, reviewed the entire manuscript, and his comments and suggestions have been of considerable assistance in the preparation of the final manuscript. I also wish to express my special gratitude to Mrs. Barbara Salaman, Mrs. Thyra S. Johnston, and my wife, Margaret, for typing the manuscript.

My former and present students at the Universities of Florida and Colorado supplied the incentive for this project. Their enthusiasm has been truly inspirational, and their questioning minds have provided the necessary feedback. In a real sense, this book represents a joint effort in which the collective contributions of all these students constitute an essential ingredient.

Olle I. Elgerd

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chapter 1

INTRODUCTION

1-1 DEFINITION OF A CONTROL SYSTEM

One of the most attractive aspects of control-systems theory is its general applicability to control problems of the most varying engineering types. In the following, maximum advantage will be taken of this fact in order that the presentation will be fully acceptable to any senior engineering student. Indeed, the book should not prove impenetrable to students of biology, medicine, and business, who quite often are concerned with control problems of great complexity.

At the outset, it should be pointed out that, as the title of the book implies, we shall be concerned with *systems*. Webster defines a "system" as a "collection of objects united by some form of interaction or interdependence." This broad definition will suffice for the present; however, it will be necessary later to somewhat narrow the meaning.

The control engineer invariably will be interested in the *dynamic*, or *live*, characteristics of a system. As a rule, the "objects" making up the system will not be in a state of equilibrium relative to each other and the surrounding world. Under the influence of external stimuli, the *state* of the system will be changing with time in a manner entirely attributable to the character of the stimuli and the bonds of interaction. The reader must be satisfied at this time with his own intuitional concept of system "state." It may perhaps be helpful to make comparisons with expressions like "state of health" and "state of mind." A complete definition will be given in Chap. 3.

In principle, it is possible to change the state of a system in any prescribed fashion by properly choosing the inputs, *at least within reasonable limits.* In other words, one may exert influence on the system state by means of intelligent manipulation of its inputs. This then, in a general sense, constitutes a *controlled system.* The theory of control is concerned with the mathematical formulation of laws for control actions. The control engineer develops the techniques and hardware necessary for the implementation of the control laws to the specific systems in question.

Figure 1-1 depicts the general structure of a control system. The output of the system, or *plant*,† is measured by the p variables $c_1, c_2, \ldots, c_p$, which in some

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† This term is borrowed from the area of process control and has become thoroughly incorporated in the vocabulary of control engineers.

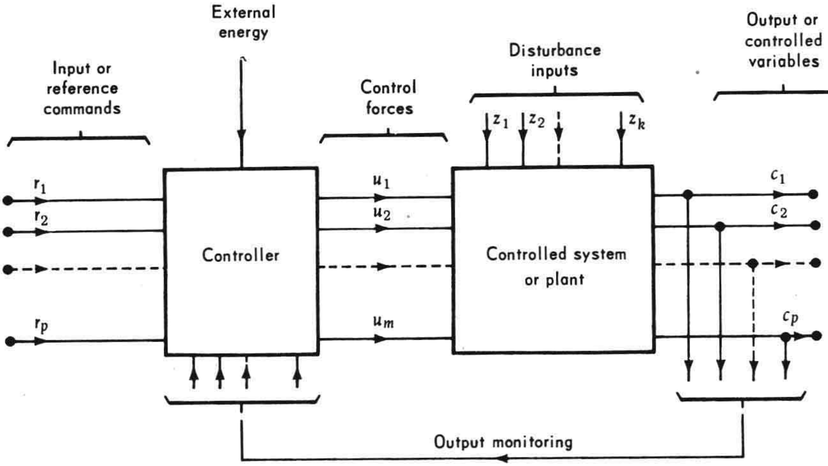

Fig. 1-1 General control-system structure.

way are related to the state of the system. It should be pointed out that these output variables do not necessarily need to tell the whole story about the state of the controlled system. It may be desired to control only part of the system, or it may not be physically possible to measure all the so-called "state variables." In the following, the c variables will be referred to alternately as *outputs* and *controlled variables*.

Direct control of the system is exerted by means of the *m control forces* u_1, $u_2, \ldots, u_m$. These forces are applied by the *controller*, which always constitutes both the brain and the brute-force portion of the overall system.† The controller determines proper control action based upon the *input* or *reference commands* r_1, $r_2, \ldots, r_p$ and information obtained, via output sensors, concerning the actual output. This *constant output monitoring*, made possible through the presence of *feedback channels*, is the distinguishing mark of all high-precision control systems. The feedback results in a *closed-loop* signal flow, and the term *closed-loop control* is often used. *Open-loop* control systems are used in certain applications, e.g., preset-type traffic-control systems. Optimal control systems, to be discussed in the later chapters of this book, are often of the open-loop type.

The general block diagram in Fig. 1-1 would not be complete without the inclusion of the *k disturbance inputs* $z_1, z_2, \ldots, z_k$. In most practical situations, it is necessary to control the plant in spite of the corruptive influence of various effects that we may classify collectively as disturbances. These inputs may be of external origin, or they may emanate from within the system itself. For example, in controlling an aircraft, we must anticipate either external disturbance forces caused by atmospheric turbulence or the corruptive influence due to component failures, which we, of course, must classify as internal disturbances.

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† It should be noted that the physical dimension of a control "force" may not necessarily be that of mechanical force (ML/T^2).

The disturbance forces may be totally *random* in nature, or they may be predictable to various degrees of accuracy. For example, in designing a fin stabilizer for a ship, we must assume complete unpredictability of the wave motions. Consider, on the other hand, the control system for a space booster. Such a booster must penetrate the layer of jetwinds without ill effects. We know a considerable amount about this layer, and it is possible to assume a wind profile that with considerable accuracy represents the true conditions.

The controller in Fig. 1-1 may be a human operator; the system is then *manually* controlled. In an *automatic* control system, a machine has replaced the man. As a general rule, high-speed, high-precision control systems are automatic. The control engineer's job usually is centered around the problem of properly designing a controller that will fit the specific job. He usually is stuck with a plant the physical characteristics of which lie beyond his control. The complexity of the controller that finally will be chosen is a function of the complexity of the plant and the stringency of the control requirements. It may range from a device that simply compares the input command with the actual output, as is often the case in a simple *servomechanism*, to a large digital *control computer* in a complex multidimensional industrial process.

It may be of interest to mention a few examples of control systems which fit the above general description. Consider first an automobile. The vehicle operator continuously and in a closed-loop fashion exerts control over various outputs of this system, e.g., velocity, inside temperature, and (most important) vehicle position in the traffic lane. This is certainly a manually controlled system.

A jet fighter in pursuit of an evasive target is a good example of a semiautomatically controlled system. Certain control functions are in the hands of the pilot, but others, like weapons aiming, are executed automatically.

The temperature-control system of the human body is an example of a high-precision, fully automatic control system. The human body contains a large number of perfectly operating, fully automatic control systems which, in view of the low-level signals they utilize, are marvels of "engineering." Consider, for example, the positioning mechanisms of the eyes. Compare the precision of this system when the person is walking or running with that of a gun-turret positioning servo on a ship heaving in the seas.

Modern control theory is finding increased use in such areas as economics and business administration. An economic system the state of which may be represented by measures such as gross national product and stockmarket indexes may be controlled by manipulation of interest rates and other economic "control forces."

1-2 EVOLUTION OF THE SCIENCE OF CONTROL

Proper functioning of biological systems clearly requires controls of a more or less complicated nature. Control systems have thus been with us for as long as life itself.