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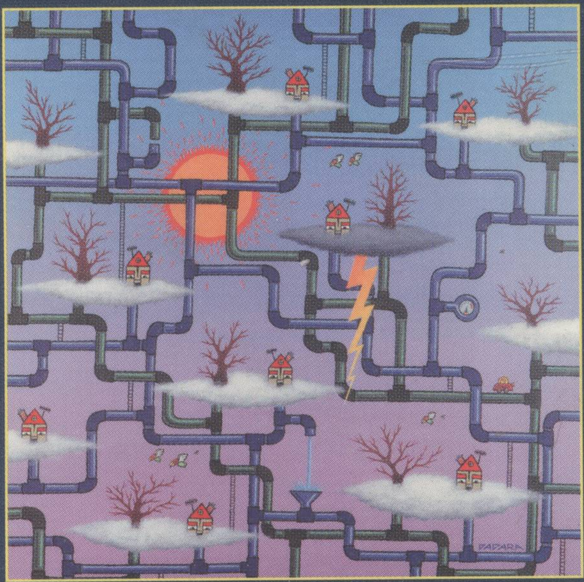
Processes, Terms and Cycles: Steps on the Road to Infinity

Essays Dedicated to Jan Willem Klop on the Occasion of His 60th Birthday



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Preface

This Festschrift is dedicated to Jan Willem Klop on the occasion of his 60th birthday on December 19, 2005. Its focus is on the lambda calculus, term rewriting and process algebra, the fields where Jan Willem has made fundamental contributions. Without attempting to give a balanced account of Jan Willem's scientific achievements, we recall three accomplishments from the early years of his career that especially stand out.

The first is his counterexample showing that the extension of the lambda calculus with surjective pairing lacks the Church–Rosser property, or, in modern terminology, is not confluent [7, 9]. This settled a famous open problem, which had challenged several researchers in the lambda calculus community for years.

The second is his pioneering work in term rewriting. In his PhD thesis [9], Jan Willem gave a systematic study of orthogonal rewriting in the general setting of combinatory reduction systems (CRSs), thereby putting the areas of higher-order rewriting and orthogonality firmly on the map. Some of the ideas in the thesis trace back to the famous Blue Preprint [2], from the period that Jan Willem and some other students were graduating in mathematics and logic, under the supervision of Dirk van Dalen and Henk Barendregt.

The third feat is the creation, together with Jan Bergstra [23], of the algebra of communicating processes (ACP).

With his early work, Jan Willem provided inspiration for many years of fruitful research, continuing to this day. For decades he has been a creative and stimulating force in the areas of term rewriting and process algebra. Some of his recent interests are infinitary rewriting, graph rewriting and the geometry of processes.

Jan Willem's scientific world is inhabited by objects like processes, streams, terms, cycles and many other puzzling and intriguing phenomena, and in this world he paves the way, guided by his extraordinary intuition and his great care in visualizing that intuition. To his colleagues, Jan Willem's adventures and the way he talks about them are a never-ending source of inspiration.

This Festschrift contains scientific papers by close friends and colleagues of Jan Willem, written specifically for this book. The papers are different in nature: some report on new research, others have the character of a survey, and again others are mainly expository. Every contribution has been thoroughly refereed at least twice. In many cases the first round of referee reports led to significant revision of the original paper, which was again reviewed. This introduction includes a list of Jan Willem Klop's publications, for reference, and as an overview of the development of his scientific interests and achievements over the years. Although this bibliography is quite extensive, we do not claim it to be complete.

We thank all authors for their contribution to the Festschrift for Jan Willem Klop and we are grateful to the referees for their constructive cooperation. We

also wish to thank Springer for publishing this book in their LNCS Festschrift series, and for the smooth publishing process.

The Festschrift was presented to Jan Willem on his 60th birthday, during a one-day symposium with the same title as this book: *Processes, terms and cycles: steps on the road to infinity.* The symposium not only celebrated Jan Willem's 60th birthday, but also the 25th anniversary of his connection with the CWI in Amsterdam. His first working day as a post-doc at the Mathematical Centre (MC), the predecessor of the CWI, was December 15, 1980.

We invited Zena Ariola (University of Oregon, USA), Arvind (MIT, USA), Henk Barendregt (Radboud University, The Netherlands), Jan Bergstra (University of Amsterdam and Utrecht University, The Netherlands), Nachum Dershowitz (Tel Aviv University, Israel), Mariangiola Dezani (University of Torino, Italy), Roger Hindley (Swansea University, UK), Jean-Jacques Lévy (INRIA, France), and Ronan Sleep (University of East Anglia, UK) to give a talk at the symposium. We are very pleased that they kindly accepted. All speakers are leading researchers in the fields of term rewriting, lambda calculus and process algebra, and their friendship with Jan Willem dates back many years.

The symposium was organized at and together with the CWI. The collaboration with Susanne van Dam and Jaco van de Pol from the CWI has been a pleasure. We wish to thank them, and more generally the CWI, for all their assistance. We also gratefully acknowledge the generous financial support for the symposium from the Vrije Universiteit, CWI, NWO, and the Radboud University.

Also on behalf of Susanne and Jaco, we would like to conclude by stating that it has been an honour and a pleasure to work on the preparation of this book and the symposium. It gave us the opportunity to experience how much Jan Willem is valued by colleagues all over the world. The willingness to contribute and participate and make the best of the book and symposium has been enormous. For this we are thankful of course, but more importantly, the manifestation of this enthusiasm reflects the esteem that Jan Willem has in the scientific community. Congratulations, Jan Willem! We wish you and all of us a fruitful continuation of our cooperation and sharing of interests for many years to come.

October 2005

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Bibliography of Jan Willem Klop

1. J.W. Klop. On solvability by λI-terms. In C. Böhm, editor, *λ-Calculus and Computer Science Theory*, volume 37 of *Lecture Notes in Computer Science*, pages 342–345. Springer, 1975.
2. H.P. Barendregt, J.A. Bergstra, J.W. Klop, and H. Volken. Degrees, reductions and representability in the lambda calculus. Preprint 22, Department of Mathematics, Utrecht University, 1976. The blue preprint.
3. H.P. Barendregt, J.A. Bergstra, J.W. Klop, and H. Volken. Representability in lambda algebras. *Indagationes Mathematicae*, 38(5):377–387, 1976.
4. J.W. Klop. A counterexample to the Church-Rosser property for λ-calculus + $DMM \to M$. Typed note, Department of Mathematics, Utrecht University, 1976.
5. H.P. Barendregt, J.A. Bergstra, J.W. Klop, and H. Volken. Degrees of sensible lambda theories. *Journal of Symbolic Logic*, 43(1):45–55, 1978.
6. J.A. Bergstra and J.W. Klop. Church-Rosser strategies in the lambda calculus. *Theoretical Computer Science*, 9(1):27–38, 1979.
7. J.W. Klop. A counterexample to the Church-Rosser property for lambda calculus with surjective pairing. In B. Robinet, editor, *Proc. of the École de Printemps d'Informatique Théorique: Lambda Calcul et Sémantique Formelle des Langages de Programmation.* LITP-ENSTA, 1979. Also published as Preprint 102, Department of Mathematics, Utrecht University.
8. J.A. Bergstra and J.W. Klop. Invertible terms in the lambda calculus. *Theoretical Computer Science*, 11(1):19–37, 1980.
9. J.W. Klop. *Combinatory Reduction Systems.* PhD thesis, Utrecht University, 1980. Also published as volume 127 of *Mathematical Centre Tracts*, Mathematisch Centrum, Amsterdam, 1980.
10. J.W. Klop. Reduction cycles in combinatory logic. In J.R. Hindley and J.P. Seldin, editors, *To H.B. Curry: Essays on Combinatory Logic, Lambda Calculus and Formalism*, pages 193–214. Academic Press, 1980.
11. J.W. de Bakker, J.W. Klop, and J.-J.Ch. Meyer. Correctness of programs with function procedures. In D. Kozen, editor, *Proc. of the 1981 Workshop on Logic of Programs*, volume 131 of *Lecture Notes in Computer Science*, pages 94–113. Springer, 1981.
12. J.A. Bergstra and J.W. Klop. Algebraic specifications for parametrized data types with minimal parameter and target algebras. In M. Nielsen and E.M. Schmidt, editors, *Proc. of the 9th International Colloquium on Automata, Languages and Programming (ICALP'82)*, volume 140 of *Lecture Notes in Computer Science*, pages 23–34. Springer, 1982.
13. J.A. Bergstra and J.W. Klop. Fixed point semantics in process algebras. Technical Report IW 206/82, Mathematical Centre, Amsterdam, 1982. Revised version: A convergence theorem in process algebra, CWI Report CS-R8733, Amsterdam, 1987.
14. J.A. Bergstra and J.W. Klop. A formalized proof system for total correctness of while programs. In *Proc. of the 5th International Symposium on Programming*, volume 137 of *Lecture Notes in Computer Science*, pages 26–36. Springer, 1982.
15. J.A. Bergstra and J.W. Klop. Strong normalization and perpetual reductions in the lambda calculus. *Elektronische Informationsverarbeitung und Kybernetik*, 18(7/8):403–417, 1982.
16. J.W. Klop. Extending partial combinatory algebras. *Bulletin of the European Association for Theoretical Computer Science*, 16:472–482, 1982.

17. J.W. de Bakker, J.A. Bergstra, J.W. Klop, and J.-J.Ch. Meyer. Linear time and branching time semantics for recursion with merge. In *Proc. of the 10th International Colloquium on Automata, Languages and Programming (ICALP'83)*, pages 39–51, 1983. Extended version appeared as [21].
18. J.A. Bergstra and J.W. Klop. Formal proof systems for program equivalence. In D. Bjørner, editor, *Proc. of the 2nd IFIP WG 2.2 Conference on Formal Description of Programming Concepts*, pages 289–303. North-Holland, 1983.
19. J.A. Bergstra and J.W. Klop. Initial algebra specifications for parametrized data types. *Elektronische Informationsverarbeitung und Kybernetik*, 19(1/2):17–31, 1983.
20. J.A. Bergstra and J.W. Klop. A proof rule for restoring logic circuits. *Integration, the VLSI Journal*, 1:161–178, 1983.
21. J.W. de Bakker, J.A. Bergstra, J.W. Klop, and J.-J.Ch. Meyer. Linear time and branching time semantics for recursion with merge. *Theoretical Computer Science*, 34(1/2):135–156, 1984. Extended version of [17].
22. J.A. Bergstra and J.W. Klop. The algebra of recursively defined processes and the algebra of regular processes. In J. Paredaens, editor, *Proc. of the 11th International Colloquium on Automata, Languages and Programming (ICALP'84)*, volume 172 of *Lecture Notes in Computer Science*, pages 82–95. Springer, 1984.
23. J.A. Bergstra and J.W. Klop. Process algebra for synchronous communication. *Information and Control*, 60(1-3):109–137, 1984.
24. J.A. Bergstra and J.W. Klop. Proving program inclusion using Hoare's logic. *Theoretical Computer Science*, 30(1):1–48, 1984.
25. J.A. Bergstra, J.W. Klop, and J.V. Tucker. Algebraic tools for system construction. In E. Clarke and D. Kozen, editors, *Proc. of the 1983 Workshop on Logic of Programs*, volume 164 of *Lecture Notes in Computer Science*, pages 34–44. Springer, 1984.
26. J.A. Bergstra and J.W. Klop. Algebra of communicating processes with abstraction. *Theoretical Computer Science*, 37(1):77–121, 1985.
27. J.A. Bergstra and J.W. Klop. Verification of an alternating bit protocol by means of process algebra. In W. Bibel and K.P. Jantke, editors, *Proc. of the Spring School on Mathematical Methods of Specification and Synthesis of Software Systems*, volume 215 of *Lecture Notes in Computer Science*, pages 9–23. Springer, 1985.
28. J.A. Bergstra, J.W. Klop, and J.V. Tucker. Process algebra with asynchronous communication mechanisms. In S.D. Brookes, A.W. Roscoe, and G. Winskel, editors, *Seminar on Concurrency*, volume 197 of *Lecture Notes in Computer Science*, pages 76–95. Springer, 1985.
29. J.W. Klop. Term rewriting systems. Unpublished lecture notes for the seminar on reduction machines, Ustica, 1985.
30. J.C.M. Baeten, J.A. Bergstra, and J.W. Klop. Syntax and defining equations for an interrupt mechanism in process algebra. *Fundamenta Informaticae*, 9(2):127–167, 1986.
31. J.A. Bergstra and J.W. Klop. Algebra of communicating processes. In J.W. de Bakker, M. Hazewinkel, and J.K. Lenstra, editors, *Mathematics and Computer Science*, CWI Monograph 1, pages 89–138. North-Holland, Amsterdam, 1986.
32. J.A. Bergstra and J.W. Klop. Conditional rewrite rules: Confluence and termination. *Journal of Computer and System Sciences*, 32(3):323–362, 1986.
33. J.A. Bergstra and J.W. Klop. Process algebra: Specification and verification in bisimulation semantics. In M. Hazewinkel, J.K. Lenstra, and L.G.L.T. Meertens, editors, *Mathematics and Computer Science II*, CWI Monograph 4, pages 61–94. North-Holland, Amsterdam, 1986.

34. J.C.M. Baeten, J.A. Bergstra, and J.W. Klop. Conditional axioms and α/β calculus in process algebra. In M. Wirsing, editor, *Proc. of the 3rd IFIP WG 2.2 Conference on Formal Description of Programming Concepts*, pages 53–75. North-Holland, 1987.
35. J.C.M. Baeten, J.A. Bergstra, and J.W. Klop. Decidability of bisimulation equivalence for processes generating context-free languages. In J.W. de Bakker, A.J. Nijman, and P.C. Treleaven, editors, *Proc. of the 1st International Conference on Parallel Architectures and Languages Europe (PARLE'87), Vol. I: Parallel Languages*, volume 259 of *Lecture Notes in Computer Science*, pages 94–111. Springer, 1987. Extended version appeared as [64].
36. J.C.M. Baeten, J.A. Bergstra, and J.W. Klop. On the consistency of Koomen's fair abstraction rule. *Theoretical Computer Science*, 51(1&2):129–176, 1987.
37. J.C.M. Baeten, J.A. Bergstra, and J.W. Klop. Ready-trace semantics for concrete process algebra with the priority operator. *The Computer Journal*, 30(6):498–506, 1987.
38. J.C.M. Baeten, J.A. Bergstra, and J.W. Klop. Term rewriting systems with priorities. In P. Lescanne, editor, *Proc. of the 2nd International Conference on Rewriting Techniques and Applications (RTA'87)*, volume 256 of *Lecture Notes in Computer Science*, pages 83–94. Springer, 1987. Extended version appeared as [47].
39. H.P. Barendregt, J.R. Kennaway, J.W. Klop, and M.R. Sleep. Needed reduction and spine strategies for the lambda calculus. *Information and Computation*, 75(3):191–231, 1987.
40. J.A. Bergstra and J.W. Klop. ACP_τ: A universal axiom system for process specification. In M. Wirsing and J.A. Bergstra, editors, *Proc. of the Workshop on Algebraic Methods: Theory, Tools and Applications*, volume 394 of *Lecture Notes in Computer Science*, pages 447–463. Springer, 1987.
41. J.A. Bergstra and J.W. Klop. *Semi-complete termherschrijfsystemen.* Programmatuurkunde. Kluwer, 1987. In Dutch.
42. J.A. Bergstra, J.W. Klop, and E.-R. Olderog. Failures without chaos: a new process semantics for fair abstraction. In M. Wirsing, editor, *Proc. of the 3rd IFIP WG 2.2 Conference on Formal Description of Programming Concepts*, pages 77–103. North-Holland, 1987.
43. J.W. Klop. Term rewriting systems: a tutorial. *Bulletin of the European Association for Theoretical Computer Science*, 32:143–183, 1987.
44. J.W. Klop and E. Kranakis. Lower bounds for a class of Kostka numbers. *Ars Combinatoria*, 23:121–129, 1987.
45. J.A. Bergstra and J.W. Klop. A complete inference system for regular processes with silent moves. In F.R. Drake and J.K. Truss, editors, *Proc. of the 1986 Logic Colloquium*, pages 21–81. North-Holland, 1988.
46. J.A. Bergstra, J.W. Klop, and E.-R. Olderog. Readies and failures in the algebra of communicating processes. *SIAM Journal on Computing*, 17(6):1134–1177, 1988.
47. J.C.M. Baeten, J.A. Bergstra, J.W. Klop, and W.P. Weijland. Term-rewriting systems with rule priorities. *Theoretical Computer Science*, 67(2&3):283–301, 1989. Extended version of [38].
48. J.A. Bergstra and J.W. Klop. Process theory based on bisimulation semantics. In J.W. de Bakker, W.-P. de Roever, and G. Rozenberg, editors, *Proc. of the REX Workshop on Linear Time, Branching Time and Partial Order in Logics and Models for Concurrency*, volume 354 of *Lecture Notes in Computer Science*, pages 50–122. Springer, 1989.
49. J.A. Bergstra, J.W. Klop, and A. Middeldorp. *Termherschrijfsystemen.* Programmatuurkunde. Kluwer, 1989. In Dutch.

50. R.N. Bol, K.R. Apt, and J.W. Klop. On the safe termination of PROLOG programs. In G. Levi and M. Martelli, editors, *Proc. of the 6th International Conference on Logic Programming*, pages 353–368. The MIT Press, 1989.
51. J.W. Klop and R.C. de Vrijer. Unique normal forms for lambda calculus with surjective pairing. *Information and Computation*, 80(2):97–113, 1989.
52. Y. Toyama, J.W. Klop, and H.P. Barendregt. Termination for the direct sum of left-linear term rewriting systems – preliminary draft. In N. Dershowitz, editor, *Proc. of the 3rd International Conference on Rewriting Techniques and Applications (RTA'89)*, volume 355 of *Lecture Notes in Computer Science*, pages 477–491. Springer, 1989. Extended version appeared as [79].
53. J.C.M. Baeten and J.W. Klop, editors. *Proc. of the 1st International Conference on Theories of Concurrency: Unification and Extension (CONCUR'90)*, volume 458 of *Lecture Notes in Computer Science*. Springer, 1990.
54. J.A. Bergstra and J.W. Klop. An introduction to process algebra. In J.C.M. Baeten, editor, *Applications of Process Algebra*, volume 17 of *Cambridge Tracts in Theoretical Computer Science*, pages 1–21. Cambridge University Press, 1990.
55. R.N. Bol, K.R. Apt, and J.W. Klop. On the power of subsumption and context checks. In A. Miola, editor, *Proc. of the International Symposium on Design and Implementation of Symbolic Computation Systems (DISCO'90)*, volume 429 of *Lecture Notes in Computer Science*, pages 131–140. Springer, 1990.
56. J.W. Klop. Term rewriting systems: From Church-Rosser to Knuth-Bendix and beyond. In M.S. Paterson, editor, *Proc. of the 17th International Colloquium on Automata, Languages and Programming (ICALP'90)*, volume 443 of *Lecture Notes in Computer Science*, pages 350–369. Springer, 1990.
57. R.N. Bol, K.R. Apt, and J.W. Klop. An analysis of loop checking mechanisms for logic programs. *Theoretical Computer Science*, 86(1):35–79, 1991.
58. N. Dershowitz, J.-P. Jouannaud, and J.W. Klop. Open problems in rewriting. In Ronald V. Book, editor, *Proc. of the 4th International Conference on Rewriting Techniques and Applications (RTA'91)*, volume 488 of *Lecture Notes in Computer Science*, pages 445–456. Springer, 1991.
59. J.R. Kennaway, J.W. Klop, M.R. Sleep, and F.J. de Vries. Transfinite reductions in orthogonal term rewriting systems (extended abstract). In Ronald V. Book, editor, *Proc. of the 4th International Conference on Rewriting Techniques and Applications (RTA'91)*, volume 488 of *Lecture Notes in Computer Science*, pages 1–12. Springer, 1991. Extended version appeared as [78].
60. J.W. Klop and A. Middeldorp. Sequentiality in orthogonal term rewriting systems. *Journal of Symbolic Computation*, 12:161–195, 1991.
61. J.W. Klop and R.C. de Vrijer. Extended term rewriting systems. In S. Kaplan and M. Okada, editors, *Proc. of the 2nd International Workshop on Conditional and Typed Rewriting Systems (CTRS'90)*, volume 516 of *Lecture Notes in Computer Science*, pages 26–50. Springer, 1991.
62. F.S. de Boer, J.W. Klop, and C. Palamidessi. Asynchronous communication in process algebra. In *Proc. of the 7th Annual Symposium on Logic in Computer Science (LICS'92)*, pages 137–147. IEEE Computer Society Press, 1992.
63. J.W. Klop. Term rewriting systems. In S. Abramsky, D.M. Gabbay, and T.S.E. Maibaum, editors, *Handbook of Logic in Computer Science*, volume 2, pages 1–116. Oxford University Press, 1992.
64. J.C.M. Baeten, J.A. Bergstra, and J.W. Klop. Decidability of bisimulation equivalence for processes generating context-free languages. *Journal of the ACM*, 40(3):653–682, 1993. Extended version of [35].

65. N. Dershowitz, J.-P. Jouannaud, and J.W. Klop. More problems in rewriting. In Claude Kirchner, editor, *Proc. of the 5th International Conference on Rewriting Techniques and Applications (RTA'93)*, volume 690 of *Lecture Notes in Computer Science*, pages 468–487. Springer, 1993.
66. J.R. Kennaway, J.W. Klop, M.R. Sleep, and F.J. de Vries. The adequacy of term graph rewriting for simulating term rewriting. In M.R. Sleep, M.J. Plasmeijer, and M.C.J.D. van Eekelen, editors, *Term Graph Rewriting – Theory and Practice*, pages 157–169. John Wiley & Sons, 1993.
67. J.R. Kennaway, J.W. Klop, M.R. Sleep, and F.J. de Vries. Event structures and orthogonal term graph rewriting. In M.R. Sleep, M.J. Plasmeijer, and M.C.J.D. van Eekelen, editors, *Term Graph Rewriting – Theory and Practice*, pages 141–155. John Wiley & Sons, 1993.
68. J.R. Kennaway, J.W. Klop, M.R. Sleep, and F.J. de Vries. An infinitary Church-Rosser property for non-collapsing orthogonal term rewriting systems. In M.R. Sleep, M.J. Plasmeijer, and M.C.J.D. van Eekelen, editors, *Term Graph Rewriting – Theory and Practice*, pages 47–59. John Wiley & Sons, 1993.
69. J.R. Kennaway, J.W. Klop, M.R. Sleep, and F.J. de Vries. An introduction to term graph rewriting. In M.R. Sleep, M.J. Plasmeijer, and M.C.J.D. van Eekelen, editors, *Term Graph Rewriting - Theory and Practice*, pages 1–12. John Wiley & Sons, 1993.
70. J.W. Klop, V. van Oostrom, and F. van Raamsdonk. Combinatory Reduction Systems: Introduction and survey. *Theoretical Computer Science*, 121(1&2):279–308, 1993. Special issue in honour of Corrado Böhm.
71. Z.M. Ariola, J.R. Kennaway, J.W. Klop, M.R. Sleep, and F.J. de Vries. Syntactic definitions of undefined: on defining the undefined. In M. Hagiya and J.C. Mitchell, editors, *Proc. of the 1st International Conference on Theoretical Aspects of Computer Software (TACS'94)*, volume 789 of *Lecture Notes in Computer Science*, pages 543–554. Springer, 1994.
72. Z.M. Ariola and J.W. Klop. Cyclic lambda graph rewriting. In *Proc. of the 9th Annual Symposium on Logic in Computer Science (LICS'94)*, pages 416–425. IEEE Computer Society Press, 1994.
73. J.R. Kennaway, J.W. Klop, M.R. Sleep, and F.J. de Vries. On the adequacy of graph rewriting for simulating term rewriting. *ACM Transactions on Programming Languages and Systems*, 16(3):493–523, 1994.
74. J.W. Klop, A. Middeldorp, Y. Toyama, and R.C. de Vrijer. Modularity of confluence: a simplified proof. *Information Processing Letters*, 49:101–109, 1994.
75. J.A. Bergstra and J.W. Klop. The algebra of recursively defined processes and the algebra of regular processes. In *Algebra of Communicating Processes*, Workshops in Computing, pages 1–25. Springer, 1995.
76. N. Dershowitz, J.-P. Jouannaud, and J.W. Klop. Problems in rewriting III. In Jieh Hsiang, editor, *Proc. of the 6th International Conference on Rewriting Techniques and Applications (RTA'95)*, volume 914 of *Lecture Notes in Computer Science*, pages 457–471. Springer, 1995.
77. J.R. Kennaway, J.W. Klop, M.R. Sleep, and F.J. de Vries. Infinitary lambda calculus and Böhm models. In Jieh Hsiang, editor, *Proc. of the 6th International Conference on Rewriting Techniques and Applications (RTA'95)*, volume 914 of *Lecture Notes in Computer Science*, pages 257–270. Springer, 1995. Extended version appeared as [87].
78. J.R. Kennaway, J.W. Klop, M.R. Sleep, and F.J. de Vries. Transfinite reductions in orthogonal term rewriting systems. *Information and Computation*, 119(1):18–38, 1995. Extended version of [59].

79. Y. Toyama, J.W. Klop, and H.P. Barendregt. Termination for direct sums of left-linear complete term rewriting systems. *Journal of the ACM*, 42(6):1275–1304, 1995. Extended version of [52].
80. Z.M. Ariola and J.W. Klop. Equational term graph rewriting. *Fundamenta Informaticae*, 26(3/4):207–240, 1996.
81. I. Bethke and J.W. Klop. Collapsing partial combinatory algebras. In G. Dowek, J. Heering, K. Meinke, and B. Möller, editors, *Proc. of the 2nd International Workshop on Higher-Order Algebra, Logic, and Term Rewriting (HOA'95)*, volume 1074 of *Lecture Notes in Computer Science*, pages 57–73. Springer, 1996.
82. I. Bethke, J.W. Klop, and R.C. de Vrijer. Completing partial combinatory algebras with unique head-normal forms. In *Proc. of the 11th Annual Symposium on Logic in Computer Science (LICS'96)*, pages 448–454. IEEE Computer Society Press, 1996.
83. J.R. Kennaway, J.W. Klop, M.R. Sleep, and F.J. de Vries. Comparing curried and uncurried rewriting. *Journal of Symbolic Computation*, 21(1):15–39, 1996.
84. J.W. Klop. Term graph rewriting. In G. Dowek, J. Heering, K. Meinke, and B. Möller, editors, *Proc. of the 2nd International Workshop on Higher-Order Algebra, Logic, and Term Rewriting (HOA'95)*, volume 1074 of *Lecture Notes in Computer Science*, pages 1–16. Springer, 1996.
85. Z.M. Ariola and J.W. Klop. Lambda calculus with explicit recursion. *Information and Computation*, 139(2):154–233, 1997.
86. Z.M. Ariola, J.W. Klop, and D. Plump. Confluent rewriting of bisimilar term graphs. *Electronic Notes in Theoretical Computer Science*, 7, 1997.
87. J.R. Kennaway, J.W. Klop, M.R. Sleep, and F.J. de Vries. Infinitary lambda calculus. *Theoretical Computer Science*, 175(1):93–125, 1997. Extended version of [77].
88. M. Bezem, J.W. Klop, and V. van Oostrom. Diagram techniques for confluence. *Information and Computation*, 141(2):172–204, 1998.
89. I. Bethke, J.W. Klop, and R.C. de Vrijer. Extending partial combinatory algebras. *Mathematical Structures in Computer Science*, 9(4):483–505, 1999.
90. Z.M. Ariola, J.W. Klop, and D. Plump. Bisimilarity in term graph rewriting. *Information and Computation*, 156:2–24, 2000.
91. I. Bethke, J.W. Klop, and R.C. de Vrijer. Descendants and origins in term rewriting. *Information and Computation*, 159:59–124, 2000.
92. J.W. Klop, V. van Oostrom, and R.C. de Vrijer. A geometric proof of confluence by decreasing diagrams. *Journal of Logic and Computation*, 10(3):437–460, 2000.
93. Terese. *Term Rewriting Systems*, volume 55 of *Cambridge Tracts in Theoretical Computer Science*. Cambridge University Press, 2003.
94. J. Ketema, J.W. Klop, and V. van Oostrom. Vicious circles in orthogonal term rewriting systems. In S. Antoy and Y. Toyama, editors, *Proc. of the 4th International Workshop on Reduction Strategies in Rewriting and Programming (WRS'04)*, volume 124(2) of *Electronic Notes in Theoretical Computer Science*, pages 65–77. Elsevier Science, 2005.

Table of Contents

The Spectra of Words

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Abstract. The k-spectrum of a word is the multiset of its non-contiguous subwords of length k. For given k, how small can n be for a pair of different words of length n to exist, with equal k- spectra? From the Thue-Morse word we find that n is at most 2^k. The construction of this paper decreases this upper bound to θ^k, where $\theta \simeq 1.6$ is the golden ratio; the construction was found, though not published, over thirty years ago. Recently the bound has been further reduced, but remains considerably greater than the greatest known lower bound.

Jan Willem Klop is renowned for his contributions to process algebra, term rewriting, and graphical models of computation; also for his exquisite diagrammatic presentations. Alongside these interests he finds delight in phenomena involving complex illustrations and counterexamples; for example, a single lambda term that takes several pages. He enjoys long words.

One long – indeed infinite – word is the Thue-Morse word. This word, and its finite prefixes of length 2^k, have a rich variety of properties. This paper is about one property that they almost, but not quite, enjoy. There is a sense in which these finite prefixes are not optimal, but appear to be nearly so.

I discovered this phenomenon in 1972, together with a counterexample to the optimality, but did not to publish it. In the last decade or so, the results have been repeated and indeed improved; in my concluding remarks I give a brief survey of some of this more recent work, enough for interested readers to discover the current state of this special problem area. But this festschrift for Jan Willem gives an opportunity to publish the phenomenon as it first appeared to me, in a form which I hope is readily digestible by many who, like me, are not expert in combinatorics.

We are concerned with words a over an alphabet A; for simplicity, we henceforth assume $A = \{0, 1\}$. A word s is a *subword* of a if the members of s occur in a in the correct sequence, not necessarily contiguously. A subword may occur often; for example, 01 occurs once in 010 but five times in 01011. (It is worth noting that there is not universal agreement of terminology; some authors use 'subword' to mean a contiguous occurrence, and in that case 01 appears only twice in 01011.) Denote by $\#(s, a)$ the number of non-contiguous occurrences of s in (i.e. as a subword of) a. Remarkably, the quantities $\#(s, a)$ share many properties with the binomial coefficients; indeed, the latter correspond to the case in which the alphabet A is a singleton. An elegant theory of these quantities $\#(s, a)$ is presented by Sakarovitch and Simon [8]. What little we need of that theory is included here, to make the paper self-contained.

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For any natural number k, the *k-spectrum* of a word a is a function giving, for each word s of length k, the value of $\#(s,a)$. For example, the 2-spectrum of 0110 is

s	$\#(s, 0110)$
00	1
01	2
10	2
11	1

We ask the question: if we know the k-spectrum of a, does this fix a? Certainly not, if a has length at least 2^k. The finite prefixes of the Thue-Morse word provide a counterexample. The prefix a_k of length 2^k is given by

$$a_0, b_0 \stackrel{\text{def}}{=} 0, 1$$
$$a_{k+1}, b_{k+1} \stackrel{\text{def}}{=} a_k\, b_k, b_k\, a_k \ .$$

For example $a_3, b_3 = 01101001, 10010110$. An easy inductive proof shows that a_k and b_k have the same k-spectrum.

One way of presenting this proof depends on a Lemma that will come in useful later. It involves the notion of a *rewriting rule* $u \longrightarrow v$, which may be used to replace a (contiguous) occurrence of u by v in a larger string. When the rule is understood we shall write $a \longrightarrow b$, or sometimes $b \longleftarrow a$, to mean that a single use of the rule can transform a into b.

Lemma. *Let u, v have the same k-spectrum. Suppose a can be rewritten into b by applying the rewrite rule $u \longrightarrow v$ once and then applying $v \longrightarrow u$ once, i.e. $a = cuc'$, $cvc' = dvd'$ and $dud' = b$. Then a, b have the same $k{+}1$-spectrum.*

To see that this provides an inductive proof that the Thue-Morse pair a_k, b_k have the same k-spectrum, assume the property for k, and consider a_{k+1}, b_{k+1}. In the Lemma take a, b to be this pair, and u, v to be a_k, b_k; then the result follows by taking $c = d' = \varepsilon$ (the empty word) and $c' = d = b_k$.

Let us look more closely at this when $k = 3$. The rewriting rule is $0110 \longrightarrow 1001$, so if we underline the occurrences to be rewritten we have

$$a_3 = \underline{0110}1001 \longrightarrow 1001\underline{1001} \longrightarrow 10010110 = b_3 \ .$$

Note that the two rewrites are non-overlapping. Now, consider a sequence of pairs a_k, b_k of unequal strings of equal length ℓ_k, where ℓ_k grow *more slowly* than 2^k; a similar inductive proof that a_k and b_k have the same k-spectrum would require that a_{k+1} can be transformed into b_{k+1} by an application of $a_k \longrightarrow b_k$ followed by an application of $b_k \longrightarrow a_k$ in which the second rewrite *overlaps the first*! It seemed unlikely that such pairs a_k, b_k could be defined. And it is not easy to think of constructing a sequence of unequal pairs with equal k-spectra other than inductively, applying something like the Lemma to yield the proof.

Therefore it was reasonable to hope to prove that each Thue-Morse pair a_k, b_k is *optimal*, in the sense that no shorter unequal pair has equal k-spectra.

But this claim is false; we now give a counter-example. Considering the recurrence relation for the Thue-Morse pairs, we naturally attempt to define pairs based instead on a Fibonacci sequence, whose length will grow asymptotically as θ^k, where $\theta = (1+\sqrt{5})/2 \doteq 1.6$ is the golden ratio. The details require care. By analogy with the Thue-Morse pairs, given two initial pairs a_0, c_0 and a_1, c_1 we define for $k \geq 0$:

$$a_{k+2},\ c_{k+2} \stackrel{\text{def}}{=} \begin{cases} c_k\, a_{k+1},\ a_{k+1}\, c_k & (k \text{ even}) \\ a_{k+1}\, c_k,\ c_k\, a_{k+1} & (k \text{ odd}). \end{cases}$$

Then, for all $k \geq 0$ define

$$b_{k+2} \stackrel{\text{def}}{=} \begin{cases} a_{k+1}\, a_k & (k \text{ even}) \\ a_k\, a_{k+1} & (k \text{ odd}). \end{cases}$$

Now let us choose $a_0, c_0 = 010, 100$ and $a_1, c_1 = 01, 10$ (we discuss this choice later). Then the following table shows the first few pairs a_k, b_k:

k	a_k	b_k	c_k	ℓ_k
0	010	–	100	3
1	01	–	10	2
2	10001	01010	01100	5
3	1000110	0110001	1010001	7
4	011001000110	100011010001	100011001100	12
5	0110010001101010001	1000110011001000110	⋯	19

Theorem 1. *Choose $a_0, c_0 = 010, 100$ and $a_1, c_1 = 01, 10$ and define a_k, b_k, c_k ($k \geq 2$) as above. Then for all $k \geq 2$ the strings a_k, b_k are unequal, with equal lengths and equal k-spectra. Also the lengths ℓ_k of the members of the pairs form a Fibonacci sequence, and $\ell_k < 2\theta^k$ for all $k \geq 2$.*

Proof. It is obvious that the strings a_k, b_k have equal length. To show that they differ, one can prove by induction that (i) their first letters always differ, and (ii) the first letters of b_k and c_k differ iff k is odd.

Now, a simple calculation yields that for $k \geq 0$

$$b_{k+2} = \begin{cases} a_k\, c_{k+1} & (k \text{ even}) \\ c_{k+1}\, a_k & (k \text{ odd}). \end{cases}$$

Using this, we give an inductive proof that, for $k \geq 2$, the strings a_k, b_k have equal k-spectra. For $k = 2$, i.e. for the pair 10001 and 01010, this is easily checked. For the inductive step, we assume the property for k and prove it for $k+1$. Let us use $\rightarrow$ for a rewrite by $a_k \rightarrow b_k$, and $\leftarrow$ for the inverse rewrite. Then for odd $k \geq 2$ we have

$$\begin{aligned} a_{k+1} &= c_{k-1}\, a_k && \text{by the recurrence} \\ &\rightarrow c_{k-1}\, b_k && \text{by the rewrite} \\ &= c_{k-1}\, a_{k-2}\, a_{k-1} && \text{by definition} \\ &= b_k\, a_{k-1} && \text{by the above expression} \end{aligned}$$