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# Theory and Analysis of Phased Array Antennas

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**WILEY-INTERSCIENCE, a Division of John Wiley & Sons, Inc.**  
New York • London • Sydney • Toronto

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Library of Congress Catalogue Card Number: 70-174768

ISBN 0-471-02553-4

Printed in the United States of America.

10 9 8 7 6 5 4 3 2 1

# **Theory and Analysis of Phased Array Antennas**

*To my wife Joan Amitay,*

להורי חיים ובלהה אמת

*To my darling wife and daughters, Adele, Stephanie Dee, and  
Maria Louise Galindo—my devoted Wandering Jews of America*

*In memory of my parents, Tsu Chai and Li Hsieh Wu*

# Preface

Modern radar, communications, and radio astronomy place increasing demands for large-aperture antennas which are capable of high-speed scanning and simultaneous operation of multiple functions. An array of radiating elements with independent phase control, that is, a phased array, can provide these capabilities. Most of the methods that have been used in the design of phased arrays have relied on classical array theory, which neglects mutual coupling, approximate analytical methods, experimental techniques, and combinations thereof. In modern array radar antennas which require wide angle scanning, it has been found that the mutual coupling between antenna elements plays a most significant role in determining the array performance. The effects of mutual coupling are not properly accounted for in the approximate analytical methods. The design of large phased arrays, often using many thousands of antenna elements, by experimental methods is a time consuming and very costly procedure. A need exists, therefore, for a general understanding of the performance of phased array antennas as well as for efficient design methods in which the effects of mutual coupling are incorporated.

The purpose of this book is to present a mathematical approach for the analysis and design of a broad class of phased array antennas and to examine their electromagnetic properties. It is hoped that this book will provide not only a basic understanding of the properties of phased array antennas, but also the analysis and synthesis techniques needed to design these expensive antennas efficiently before large funds are committed for construction.

Emphasis is placed on the analysis of uniformly and closely spaced planar arrays of open ended waveguides. The boundary value problem associated with these arrays is rigorously formulated and expressed by one of several types of integral equations. The integral equations can be solved numerically (and analytically in special cases) with a very high degree of accuracy. The solution provides a complete characterization of the electromagnetic field, from which pertinent array properties, such as the reflection and radiation (including polarization) characteristics and mutual coupling coefficients, are derived. The formulation and numerical methods can be extended to

dipoles and other related types of array elements. Since dipole arrays have been treated elsewhere, we shall not dwell upon them here. The pattern synthesis and system aspects of phased array antennas have also been extensively discussed in the literature and therefore will not be considered herein.

In Chapter 1 we introduce the reader to the terminology and concepts of phased arrays, and review various approaches to array analysis. The dependence of the array properties on mutual coupling is demonstrated. In Chapter 2 we discuss Floquet's theorem and utilize it in the formulation of the boundary value problems associated with infinite and periodic arrays. Various integral equations and variational expressions are derived. Methods of solving these equations are presented in Chapter 3. Emphasis is placed on numerical methods which can be handled efficiently by high speed and large storage digital computers. A general discussion of various techniques for ascertaining the validity and accuracy of a solution is presented.

In Chapter 4 we derive an analytical solution to the infinite parallel-plate phased array with thin walls. Certain basic array relationships are presented, and examples of performance data are also given. The characteristics of planar arrays of rectangular waveguides are discussed in Chapter 5. The existence of forced surface wave resonances which cause blindness in phased arrays is demonstrated. For the purpose of illustration, some of the techniques developed in Chapters 2 and 3 are rederived for parallel-plate arrays. In Chapter 6 we examine the effects of dielectric loading on the basic properties of parallel-plate arrays. The improvement of impedance match and the appearance of forced surface wave resonances are considered in detail.

The properties of infinite planar arrays of circular waveguides with and without dielectric loading are investigated in Chapter 7. Again, forced surface wave resonances are observed and their salient features are discussed. The extension of the integral equation formulation to skewed periodic and planar grid arrays is derived. In Chapter 8 we extend the integral equation formulation to uniformly and nonuniformly spaced finite arrays and a certain class of infinite aperiodic arrays. Dispersion relations for periodically modulated surfaces are derived. The relationship between the forced surface wave resonances in infinite arrays and these dispersion relations is demonstrated. The relation between the array element characteristics and the proximity of an element to the edge of a finite array is examined. In the final chapter, we present systematic methods to optimize the array match over a prescribed scan region and frequency band. These methods are quite general and are applicable to any type of array element. Experimental methods to characterize the array terminal relationships are also discussed.

Basically, this book is a research monograph. Although the material presented is drawn from many technical publications, the book is based

mainly on work performed at Bell Telephone Laboratories, where a considerable effort has been devoted to phased array research and development.

A bibliography is provided to assist the reader in finding additional information that could not be included in a book of this scope. However, there has been no attempt to make the bibliography complete.

This book is addressed to engineers, physicists, and students who are active or interested in phased arrays and antennas. It is written on an intermediate level. Familiarity with Maxwell's equations, guided wave theory, and elementary antenna theory is assumed. The book may serve as a supplementary text for graduate courses in antennas and applied electromagnetic theory and applications.

We are indebted to our colleagues at Bell Telephone Laboratories for their cooperation and advice. Special thanks are due to Messrs. J. S. Cook and R. Lowell Dr. E. R. Nagelberg for their constructive comments and encouragement. The support of Bell Telephone Laboratories and TRW systems in the preparation of the manuscript is gratefully acknowledged.

We wish also to thank the following organizations for permission to use parts of certain articles which appeared in their various publications: The Institute of Electrical and Electronic Engineers, Inc., the American Telephone and Telegraph Co., and the American Geophysical Union.

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*August 1971*



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# 1. Introduction to Array Theory

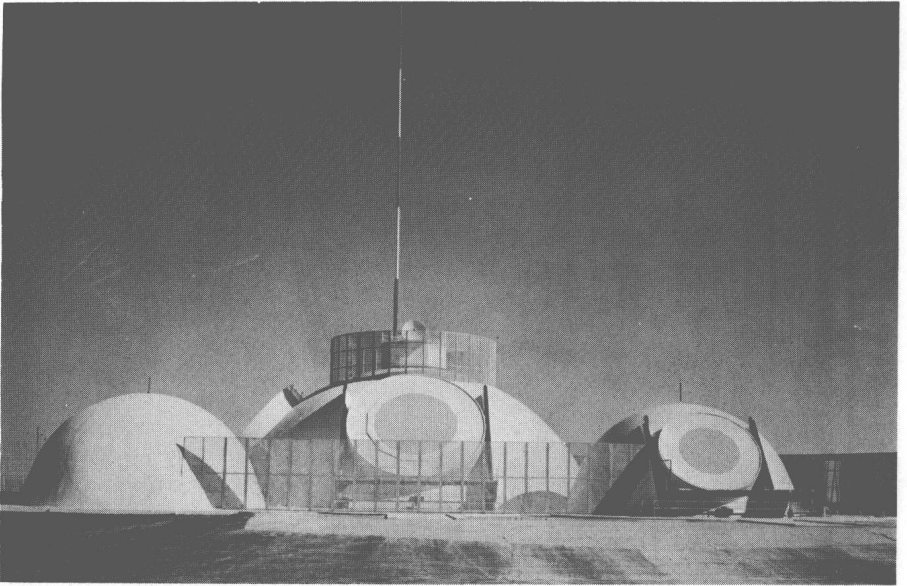
## 1. INTRODUCTION

The ever-increasing demands of the space and missile age upon modern radar and communication systems have propelled phased array antennas into the limelight. The need for specialized multifunction operation (i.e., simultaneous surveillance, discrimination, tracking, etc.), coupled with high power, high data rates, and the ability to withstand adverse environmental conditions, has stimulated considerable activity in the research, design, and deployment of phased array radars and antennas. Figures 1.1–1.4 show some of the phased array antennas that have already been designed and constructed. Phased array antenna systems have been deployed on the ground and aboard ships. It is anticipated that phased arrays with miniaturized components will eventually be deployed in space and aboard aircraft as well.

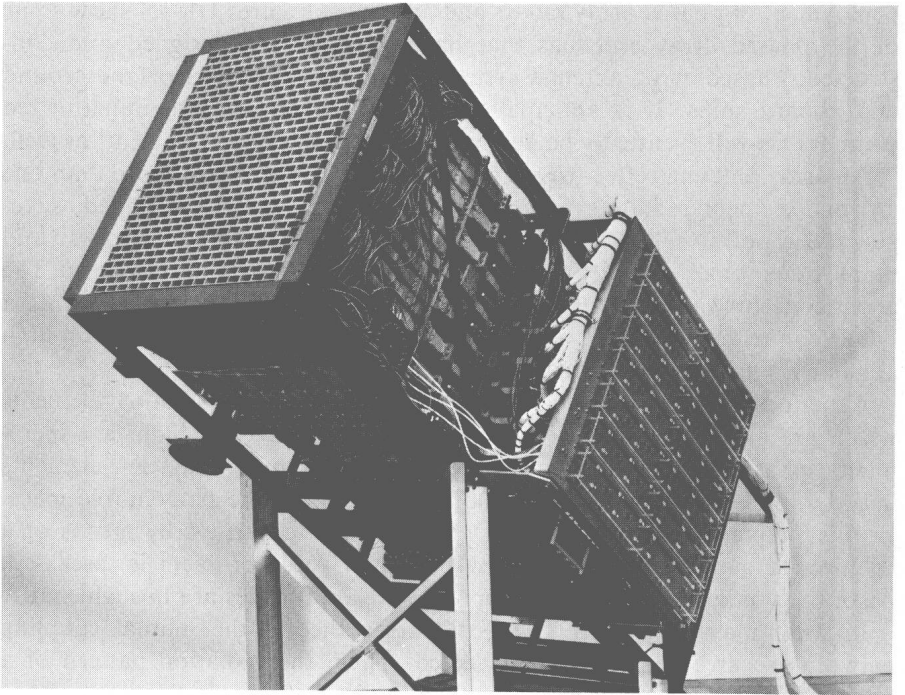
An array antenna refers to an antenna system consisting of more than one antenna element radiating in phase coherence. Horns (open ended waveguides), dipoles, helices, spiral antennas, polyrods, parabolic dishes, and many other types of antennas may constitute the radiating elements. The array concept was originally introduced mainly as a means for achieving a better control of the antenna aperture illumination. In order to obtain high angular resolution, large planar arrays are being utilized.

A periodic planar phased array consists of identical radiating elements which are arranged in a planar and doubly periodic grid. Constant incremental phase shifts (steering phases) are applied between adjacent elements in order to electronically scan the radiation pattern of the array in free space. In this chapter, the significance of analyzing phased arrays by means of a solution to a rigorously formulated boundary value problem is discussed. Also, the weaknesses of various approximate approaches are brought out.

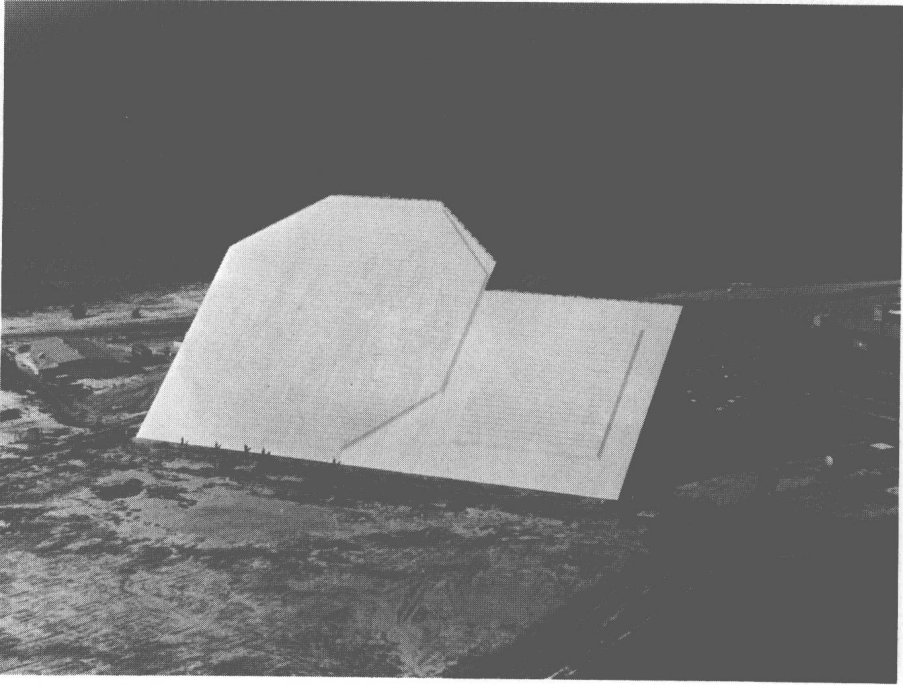
In what may be termed “classical” array theory, the mutual coupling between the array elements is neglected. Thus, the radiation pattern of a finite sized phased array is expressed as a product of the array factor and the radiation pattern of the array element. The array factor determines the beam



**Fig. 1.1.** MAR/WSMR: Multifunction array radar installed at White Sands Missile Range (courtesy Bell Telephone Laboratories).



**Fig. 1.2.** Mobile surface to air missile radar (courtesy Hughes Aircraft Company).



**Fig. 1.3.** AN/FPS-85: Air Force space track radar (courtesy Bendix Communication Division).

shape and sidelobe level of the array radiation pattern and also relates the array geometry and steering phases to the direction cosines of the beam pointing directions. The special functional relationship between the array factor, the steering phases, and the beam directional cosines lends itself to a convenient graphical diagram, known as the grating lobe diagram, which is a useful concept often employed in the discussion of array properties and the design of phased array antennas. This diagram will be discussed in more detail later.

Experimental and theoretical investigations have clearly indicated that mutual coupling cannot be ignored, especially in the case of closely spaced array elements (i.e., spacing of the order of half a wavelength). The mutual coupling strongly affects the radiation and reflection characteristics (as a function of the beam pointing direction) of phased array antennas. Furthermore, in a finite sized array, the radiation patterns and the reflection characteristics of the various elements may depend on their relative positions in the array. They are strongly influenced by the proximity of an element to the



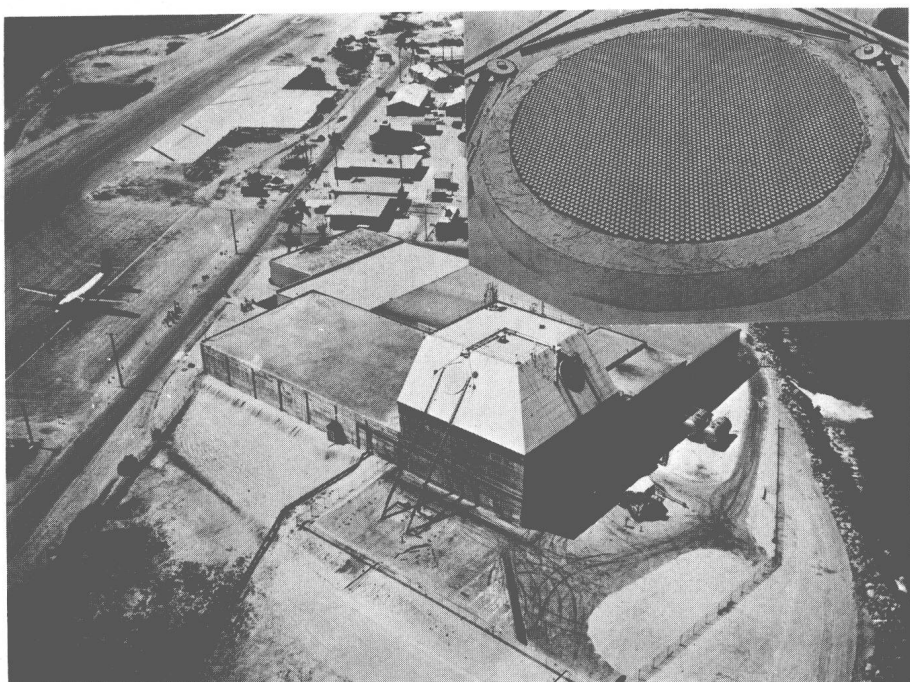


Fig. 1.4. MSR: Missile site radar installed at Meck Island (courtesy Bell Telephone Laboratories).

edge of the array. In the case of “large” planar arrays, however, the bulk of the inner core elements behave almost uniformly, and the important aspects of this behavior may be approximated well by the uniform behavior of elements in an infinite array. Thus the infinite array can serve as a useful model for the analysis of large planar arrays. In this case the array radiation pattern can still be expressed as a product of the array factor and the element radiation pattern, the latter being determined in an infinite array environment. The effects of mutual coupling are taken care of by the element radiation pattern (excited in the array environment with the rest of the elements being properly terminated). Certain general expressions which relate the mutual coupling coefficients, the reflection coefficients, and the radiation power pattern of an element can be derived. In addition the concept of ideally matched arrays is expressed in terms of an ideal element power radiation pattern.

Infinite arrays of narrow slots, dipoles, and current elements have been analyzed in previous years. However, these models, as well as the approximations involved in their analysis, tend to be limited in their applicability in