Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

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P. Eymard J.-P. Pier (Eds.)

Harmonic Analysis

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Harmonic Analysis

Proceedings of the International Symposium held at the Centre Universitaire de Luxembourg
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The International Symposium, held at Centre universitaire of Luxembourg, September 7-11, 1987, was intended to focus on various aspects of abstract harmonic analysis. General surveys aimed at shedding some lights on present trends of the theory, put in the frame of its recent historical evolution. Specialized conferences presented new results.

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SOME VIEWS ON THE EVOLUTION OF HARMONIC ANALYSIS

Jean - Paul PIER
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Harmonic analysis, classical or not, appears in the most diverse fields of mathematics. Is not it then somehow frustrating to observe that the origin of the term "harmonic analysis" remains unclear?

Victor Hugo called Palestrina the "father of harmony". Similarly one could claim Fourier to be the "father of harmonic analysis" or rather the "grandfather of harmonic analysis". But obviously the term was not used by Fourier; it was popularized much later by Wiener introducing the "generalized harmonic analyis". Yet we should quote the following lines due to Fourier:

> "Les questions de la chaleur offrent autant d'exemples de ces dispositions simples et constantes qui naissent des lois générales de la nature; et si l'ordre qui s'établit dans ces phénomènes pouvait être saisi par nos sens, ils nous causeraient une impression comparable à celle des résonances harmoniques." ([18] p. xxii-xxiii).

Poincaré was probably one of the first mathematicians to fully recognize the relevance of Fourier's results for the progress of analysis:

> "La série de Fourier est un instrument précieux dont l'analyse fait un usage continuel, c'est par ce moyen qu'elle a pu représenter des fonctions discontinues; si Fourier l'a inventée, c'est pour résoudre un problème de physique relatif à la propagation de la chaleur. Si ce problème ne s'était posé naturellement, on n'aurait jamais osé rendre au discontinu ses droits; on aurait longtemps encore regardé les fonctions continues comme les seules fonctions véritables ... Derrière la série de Fourier d'autres séries analogues sont entrées dans le domaine de l'analyse; elles y sont entrées par la même porte; elles ont été imaginées en vue des applications"([59] p. 150-151).

In the same vein, Dirichlet [13] had already spoken of the "new carreer" opened up to analysis.

About a later crucial moment in the history of harmonic analysis let us listen to Wiener:

> "For several years the chief demand on me at M.I.T. by the electrical-engineering department was to put the Heaviside calculus on a proper logical foundation. Other people were doing the same thing at the same time in other countries, although I do not think that any of these treatments were more satisfactory than the one I ultimately gave. In performing this task, I had to study harmonic analysis on an extremely general basis, and I found out that Heaviside's work could be translated word for word

into the language of this generalized harmonic analysis" ([83] p. 78).

Concerning his precise motivation, Wiener added:

"In all this there was an interplay between what I was doing on the Heaviside theory and what I had done on the Brownian motion. Previous to my wok there had been no thoroughly satisfactory example given of the sort of motion that would correspond to sound or light with a continuous spectrum ... the harmonic analysis which had already been given corresponded more closely to what one sees when one examines sunlight" (loc. cit.).

To what extend may Wiener have been driven by a more global motivation when attacking that problem? He declared:

"Our main obligation is to establish arbitrary enclaves of order and system" ([83] p. 324).

We feel free to let these lines be followed by Hilbert's statement:

"Auf diesem stets sich wiederholenden Spiel zwischen Denken und Erfahrung beruhen, wie mir scheint, die zahlreichen und überraschenden Analogien und jene scheinbar praestabilirte Harmonie welche der Mathematiker so oft in den Fragestellungen, Methoden und Begriffen verschiedener Wissensgebiete wahrnimmt" ([28] p. 257)

*

We will now try to give a bird's eye view on some highlights in the evolution of harmonic analysis.

Following Galois' fundamental work and the publication of Jordan's treatise, Cayley gave the first definition of an abstract group not necessarily being a transformation group. But Cayley merely considered finite groups; although symmetric elements then exist trivially, he did not mention them explicitly. Even Weber's definition given later lacked the extraordinary simplicity of the present formulation of the group concept. In his Erlanger Programm, Klein acknowledged that in his own definition of transformation group he had not required the existence of symmetric elements; he credited Lie to have been the first to realize that hypothesis to be indispensable.

The notion of a topological group is obviously rooted in that of a Lie group, the latter possessing also a differential manifold structure. After Lie's studies, interest focused on more general topological groups. Hilbert wondered whether via the introduction of new variables or new parameters one could replace Lie's transformation group by a group of functions that are necessarily differentiable. His Fifth Problem provided higher status to the concept of topological group. He wrote:

"Was zunächst die Stetigkeit betrifft, so wird man gewiss an dieser Forderung zunächst festhalten - schon im Hinblick auf die geometrischen und arithmetischen Anwendungen, bei denen die Stetigkeit der in Frage kommenden Functionen als eine Folge des Stetigkeitsaxioms erscheint. Dagegen enthält die Differenzirbarkeit der die Gruppe definirenden Functionen eine Forderung, die sich in den geometrischen Axiomen nur auf recht gezwungene und complicirte Weise zum Ausdruck bringen lässt" ([28] p. 270).

In 1925 Schreier [69] introduced "abstract continuous groups" for which he could neglect considerations on the specific nature of the elements; he just added convergence conditions to the group axioms. Leja [37] was the first mathematician to use the term "topological group" for an object which, as a matter of fact, is a topological semigroup. In 1927 Leja [38] gave the precise definition of a topological group expressing continuity of both the multiplication and the inversion in the language of neighborhoods. Later van Dantzig, von Neumann, van Kampen produced similar formulations [57].

Whereas the introduction of topological groups was motivated by the intention to formalize completely the topological character of the group operations, the distinction between continuous, open, homeomorphic mappings took some time to become rigorous; these problems came up mainly in order to establish the topological versions of van der Waerden's isomorphism theorems. After 1932, van Dantzig [10] started the first systematic study of stability properties in the class of topological groups which he assumed to be metrizable. Freudenthal [20] defined open homomorphisms. All standard facts were stated explicitly in the early monographs, due to Pontrjagin [63] and Weil [78], the first one being devoted mainly to topological abelian groups.

A decisive step in the evolution of harmonic analysis was performed by the introduction of invariant measures on locally compact groups [56]. Poincaré, E. Cartan had studied integral invariants. On groups, Hurwitz [29] produced the first additive set function that is invariant by left translations. Schur and Weyl constructed invariant measures for specific compact Lie groups. Finally, Weyl and Peter [54] defined such measures on arbitrary compact Lie groups. Weyl commented later:

"For myself I can say that the wish to understand what really is the mathematical substance behind the formal apparatus of relativity theory led me to the study of representations and invariants of groups" ([81] p. 400).

In 1933 Haar [26] constructed a left invariant (Haar) measure for any separable, metrizable, locally compact group. Weyl's appreciation reads:

"By an ingenious device A. Haar succeeded in defining a 'good' volume measure on every locally Euclidian compact group. In other words, he got rid of the akward assumptions of differentiability involved in the notion of a Lie group" ([80] p. 193).

Von Neumann [52] established existence and uniqueness of such a measure for every separable compact group; Weil [78] solved the problem

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for arbitrary locally compact groups. Invariant measures were determined precisely for specific locally compact groups.

Invariant measure problems were linked originally to the study of the space of almost periodic functions, i. e., functions for which the orbits of translated functions are relatively compact. These functions had been considered first on ℝ by Bohr [4]. Von Neumann [53] obtained an invariant mean on the space, extending his construction of a biinvariant measure for compact groups. Van Kampen [34] reduced the theory of almost periodic functions to a theory of continuous functions on a compact group. Weil [78] observed precisely that the invariant mean on almost periodic functions corresponds to the normalized invariant measure on the compactification. In his monography Maak expressed some regrets:

> "Durch eine interessante, allerdings naheliegende Bemerkung haben A. Weil und v. Kampen den fastperiodischen Funktionen etwas von der Berechtigung, eine selbstständige Existenz zu führen, wieder geraubt" ([40] p. 225--26).

After the appearance of a left invariant measure on a locally compact group, the transposition of the convolution product to this general situation became possible. That multiplication had been studied more and more systematically by Weierstrass, Volterra, Cayley. On almost periodic functions Weyl and Peter [54] defined the product via the Haar measure. In 1936 Weil [77] came close to the general concept. If G is a locally compact group, f, g $\in L^1(G)$, let $f * g(x) = \int f(y)\, g(y^{-1}x)$, $x \in G$; then $\|f * g\|_1 \leqslant \|f\|_1 \|g\|_1$. Hence $L^1(G)$ becomes a "group algebra" which was examined by Segal [71].

The study of group algebras largely benefitted from the introduction of the important objects constituted by Banach algebras. This concept had been considered in particular situations by Nagumo, Yosida, Kakutani. F. Riesz [68] sytematically investigated the normed algebra of continuous endomorphisms of the space formed by the real sequences $a = (a_n)$ such that $\sum_{n=1}^{\infty} |a_n|^2 < \infty$ with $\|a\|_2 = (\sum_{n=1}^{\infty} |a_n|^2)^{\frac{1}{2}}$. Von Neumann [50] was the first mathematician to consider an abstract Hilbert space H; the algebra of continuous endomorphisms of H forms a "von Neumann algebra", a concept which was in fact introduced earlier than the more general and more elementary notion of a normed algebra.

The theory of Banach algebras is a creation of Gelfand [21] under the designation "normed ring". In the preface of his monograph Naimark wrote:

> "It was the abstract point of view which turned out to be the most fruitful; from this viewpoint, the nature of the elements of the ring does not play any role, so that a normed ring is simply an arbitrary set of elements which forms a ring in the

algebraic sense and which is, furthermore, provided with a norm which satisfies these requirements" ([49] p. viii).

Ambrose [1] introduced the term "Banach algebra" and studied these objects in full generality, without the hypotheses of commutativity and existence of a unit element. Rickart's later monograph [65] , laying down all standard facts, fixed the terminology.

Bourbaki's historical notes state:

"La mesure de Haar et la convolution sont rapidement devenues des outils essentiels dans la tendance à l'algébrisation qui marque si profondément l'Analyse moderne ... Il restait à faire la synthèse de tous ces points de vue, qui s'accomplit dans le livre de A. Weil, préludant aux généralisations ultérieures que constitueront, d'une part les algèbres normées de I. Gelfand, et de l'autre la convolution des distributions" ([5] p. 215).

In 1947 the fundamental properties of the group algebra $L^1(G)$ of a locally compact group G with respect to a Haar measure were given by Segal [72], independently from the work of Gelfand and Raikov. The algebra is commutative if and only if the group is abelian. Let Δ be the modular function: $\int f(xa)\ \Delta(xa)\ dx = \int f(x)\ dx$ for $f \in L^1(G)$, $x, a \in G$; one defines $f^*(x) = \overline{f(x^{-1})}\ \Delta(x^{-1})$ making $L^1(G)$ an involutive Banach algebra. Godement [24] reexposed the convolution product theory insisting especially on the role of the modular function.

Preliminary forms of the convolution product for measures were used on $\mathbb{R}$ by Tchebycheff, Daniell, Wiener and Pitt. Such a multiplication was considered on an arbitrary locally compact group by H. Cartan and Weil; Godement [24] formalized the subject.

The structure of locally compact abelian groups could be discribed via decompositions into direct sums of more elementary groups. Pontrjagin [61] showed an abelian, locally compact, connected, separable group to be the direct sum of a compact group and a group $\mathbb{R}^m$ ($m \in \mathbb{N}$). Weil [78] proved more generally that a locally compact abelian group, which is generated by a compact neighborhood of the identity element, constitutes a projective limit of groups of the type $\mathbb{R}^m \times T^n \times Z^p \times F$, F being a finite group. He attributed the result to van Kampen.

Hilbert's famous Fifth Problem, formulated in 1900 [28] , suggested the approximation of locally compact groups by Lie groups. Von Neumann [51] showed every compact, connected, locally euclidian group to be a Lie group; his result was improved by Pontrjagin [61]. In [8] Chevalley conjectured every topological, separable, locally compact, locally connected group, which contains arbitrary small subgroups, to be a Lie group. Iwasawa [30] studied this class thoroughly; he showed the existence of a maximal solvable connected subgroup, called radical, in every locally compact group. He conjectured every locally compact, connected group to admit a family (H_i) of closed invariant subgroups such that $\bigcap_i H_i = \{e\}$ and any G/H_i is a Lie group. Such groups were going to play an essential role in the investigations performed by Montgomery and Zippin [47] who established the final result: Let G be a locally compact group the quotient of which by the connected component of e is compact (The group is "almost connected"). Then every neighborhood of e contains a compact invariant subgroup H such that G/H is a Lie group admitting no arbitrary small subgroups.

Frobenius and Burnside had inaugurated the theory of representations for finite groups, i. e., the determination of homomorphisms into groups of linear transformations. These properties and H. Cartan's representation theory for semisimple Lie algebras led Weyl to the study of representations of general Lie groups, which meant an algebraic problem to him.

Weyl and Peter [54] established the theory for compact Lie groups. As one of the main ingredients is finite Haar measure, the procedure could be carried over to general compact groups quite rapidly. The prodigious development of the theory is due to its adaptability in physics, more precisely in quantum mechanics.

Concerning possible generalizations to the noncompact case, Mackey comments:

> "The extension to locally compact groups that are not compact did not come at once. The difficulties are great enough so one had to forget Frobenius for awhile and deal first with the commutative case where all irreducible unitary representations are one dimensional" ([45]) p. 83).

To the locally compact abelian group G, Pontrjagin associated all characters, i. e., all continuous homomorphisms into the torus. The set of all characters may also be equipped with a locally compact abelian group structure; it constitutes the dual group $\hat{G}$ [60][62].

The classical Fourier series theory, and more specifically the determination of an orthonormal basis in the Hilbert space $L^2_{\mathbb{R}}([a,b])$ obtained by Fischer [17], F. Riesz [66][67], and by Plancherel [58], could be interpreted in the framework of a general abelian locally compact group G. Weil's monograph [78] showed that these groups form a natural domain for harmonic analysis which had "developed around the Fourier integral". The Fourier transform $\hat{f}$ of $f \in L^1(G)$ is a continuous function on $\hat{G}$ defined by

$$\hat{f}(\chi) = \int_G f(x)\, \overline{\chi(x)}\, dx,$$

$\chi \in \hat{G}$. More specifically, Gelfand's theory [21] enabled the consideration of the "Fourier-Gelfand transform" on the commutative Banach algebra A. Any ideal I, with respect to which the corresponding quotient admits a unit, is called modular; let $\mathcal{M}$ be the set of maximal modular ideals in A. To $a \in A$, one associates

$$\hat{a}(M) = \theta_M(a),$$

$M \in \mathcal{M}$, θ_M being the homomorphism of A into $\mathbb{C}$ admitting M as its kernel. The set $\mathcal{M}$ is equipped with the weakest topology under which all functions $\hat{a}$ are continuous. In case $A = L^1(G)$, there exists a bijection between $\mathcal{M}$ and $\hat{G}$, as was observed by Segal [72]. For the commutative Banach algebra $M^1(G)$ of bounded measures on G, the Fourier-Stieltjes transform $\hat{\mu}$ of $\mu \in M^1(G)$ is defined by

$$\hat{\mu}(\chi) = \int_G \overline{\chi(x)}\, d\mu(x),$$

$\chi \in \hat{G}$; H. Cartan and Godement [7] pointed out that this transformation constitutes a bijection between $M^1(G)$ and the space of bounded uniformly continuous functions defined on $\hat{G}$.

In the theory of locally compact abelian groups a duality is available. Pontrjagin [63] showed that a locally compact abelian group G is isomorphic to its bidual $\hat{\hat{G}}$. Plancherel's formula admits a general version: The Fourier transformation is an isometric isomorphism of $L^2(G)$ onto $L^2(\hat{G})$. Later less simple duality theorems were obtained on other locally compact groups, for instance, on compact groups by Tannaka [74] and Krein [36].

Starting from Frobenius' study of finite groups, the theory of representations of locally compact groups has undergone a long history. The fundamental work is due to Gelfand and Raikov [22]; it was published in 1943. The locally compact group G may be described via the consideration of unitary representations, i. e., homomorphisms into the group of unitary operators over a Hilbert space. These representations are one-dimensional in the abelian case. As had been apparent in the work of Weyl and Peter [54], the representations are finite-dimensional for compact groups, a fact established in its full generality by Nachbin [48]. The decomposition of a unitary representation into a direct sum of irreducible ones constitutes a central problem. The classification of the representations suggests the investigation of their restrictions to subgroups. Conversely, the construction of unitary representations for a group from unitary representations of subgroups is realized by the theory of induced representations due to Mackey [41][42][43].

Positive-definite type properties had been considered by Caratheodory, Toeplitz, Herglotz in particular situations. Bochner generalized the notion on $\mathbb{R}$ [2] and $\mathbb{R}^n$ [3]. The continuous bounded function $f\colon \mathbb{R}^n \to \mathbb{C}$ is said to be positive-definite if $\sum_{j=1}^{m} \sum_{k=1}^{m} \alpha_j \alpha_k \ f(x_k - x_k) \geq 0$ whenever $\alpha_1, \ldots, \alpha_m \in \mathbb{C}$, $x_1, \ldots, x_m \in \mathbb{R}^n$, $m \in \mathbb{N}^*$. Bochner showed these functions to be precisely the Fourier-Stieltjes transforms of bounded measures.Weil [78] carried the situation over to all locally compact abelian groups; he studied positive-definite functions on general locally compact groups. The importance of the subject lies in the following fact established by Gelfand and Raikov [22]: On a locally compact group G, the positive-definite functions are the coefficient functions $\underset{G \to \mathbb{C}}{x \mapsto (U_x \xi | \xi)}$ corresponding to continuous unitary representations U of G over Hilbert spaces H , with $\xi \in H$.

The study of harmonic analysis in the classical case of (periodic) functions defined on the compact group $G = T = \mathbb{R}/2\pi Z$ may be illustrated by the determination of the Fourier series corresponding to the Fourier transforms on the dual group $\hat{G} = Z$. It leads to the problem of harmonic synthesis, i. e., the "reconstruction" of $f \in L^1(T)$ as the limit of the series $x \mapsto \sum_{n \in Z} \hat{f}(n) e^{inx}$.

Wiener's celebrated tauberian theorem [82] established the following statement: Let f be a bounded mesurable function on $\mathbb{R}$. Let $g \in L^1(\mathbb{R})$ such that $\int_{-\infty}^{\infty} g(x)\, e^{ixy}\, dx \neq 0$ whenever $y \in \mathbb{R}$. If

$$\lim_{x \to \infty} \int_{-\infty}^{\infty} f(y)\, g(y-x)\, dy = A \int_{-\infty}^{\infty} g(y)\, dy,$$

then, for all $h \in L^1(\mathbb{R})$,

$$\lim_{x \to \infty} \int_{-\infty}^{\infty} f(y)\, h(y-x)\, dy = A \int_{-\infty}^{\infty} h(y)\, dy.$$

The theory of maximal modular ideals developed by Silov [73] on a commutative Banach algebra, by Segal [72] and Godement [23] on the group algebra of a locally compact abelian group G,gave rise to a very general formalized version of Wiener's result: Let A be a commutative Banach algebra and let M be the set of its maximal modular ideals. Assume A to be semisimple, i. e., $\bigcap_{M \in \mathcal{M}} M = \{0\}$, and regular, i. e., if C is a closed subset in $\mathcal{M}$ and $M_0 \in \mathcal{M} \setminus C$, there exists $x \in A$ such that $\hat{x}(C) = \{0\}$ and $\hat{x}(M_0) \neq 0$. Suppose the set of all $x \in A$, for which $\hat{x}$ has compact support, to be dense in A.Then every proper closed ideal of A is included in a maximal modular ideal. The theorem applies to $L^1(G)$ for a locally compact abelian group G. With Wiener's hypothesis, the ideal generated by the functions g satisfying the given identity cannot be included in a maximal modular ideal; it coincides with $L^1(\mathbb{R})$.

Silov's general study [73] was carried out by Mackey [44] on commutative Banach algebras A not necessarily admitting units. Let $\mathcal{M}$ again be the set of maximal modular ideals in A. If I is an ideal in A, the hull h(I) of I is the set of all elements in $\mathcal{M}$ containing I. In case the algebra is regular, any closed subset E of $\mathcal{M}$ is h(I) for one closed ideal I at least. If E = h(I) for a single closed ideal, it is said to be of spectral synthesis. The Wiener tauberian theorem asserts that ϕ is a spectral synthesis set in $L^1(G)$ for a locally compact abelian group G. The strongest necessary and sufficient condition assuring spectral synthesis is a generalization of a property first considerd by Ditkin [14].

In 1948 Schwartz showed that the unit sphere of $\mathbb{R}^3$ is not of spectral synthesis [70]. In 1959 Malliavin established the failure of spectral synthesis in any locally compact noncompact abelian group [46]. The result was improved by several mathematicians, namely Kahane who produced a complete account of the situation in 1962 [33]. Varopoulos [75] demonstrated Malliavin's theorem via Banach tensor products.

For the locally compact abelian group G, due to the availability of the Fourier transform, the duality, and the Plancherel formula, much information is known on $A(G) = \{\hat{f}: f \in L^1(\hat{G})\}$. The object generalizing this set to an arbitrary locally compact group G was introduced and described by Eymard [15]: The Fourier algebra A(G) is the Banach algebra constituted by the functions $u = f * \check{g}$, where $f, g \in L^2(G)$ with $\check{g}(x) = g(x^{-1})$, $x \in G$. Herz [27] defined, for $1 < p < \infty$, the Banach algebra $A_p(G)$ spanned by the functions $u = \sum_{n=1}^{\infty} f_n * \check{g}_n$, where f_n, g_n are continuous functions with compact supports on **G**; $\|u\|$ is the greatest lower bound of the numbers $\sum_{n=1}^{\infty} \|f_n\|_p \|g_n\|_{p'}$ $(1/p + 1/p' = 1)$ for all possible expressions of u. The algebra $A_2(G)$ coincides with A(G).

For any Banach space X of complex-valued functions one considers the multiplier functions f such that $fX \subset X$ and $g \mapsto fg$ is a continuous endomorphism of X. Multipliers were studied by Wendel [79], Johnson [31]. Convolutors, i. e., multipliers on $A_p(G)$, provide information on the group itself; they were examined, in particular, by Derighetti [11][12]. Of special interest is the Fourier-Stieltjes algebra introduced by Eymard [15]; in the case of a locally compact abelian group G, it consists of the Fourier-Stieltjes transforms of bounded measures on $\hat{G}$. Eymard [16] and Granirer [25] investigated a large variety of Banach function algebras attached to locally compact groups.

Many studies on harmonic analysis aim at getting particular locally compact groups under control. A popular class is constituted

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