Maurice Roseau

Vibrations in Mechanical Systems

Analytical Methods and Applications

Springer-Verlag

Maurice Roseau

Vibrations in Mechanical Systems

Analytical Methods and Applications

With 112 Figures

Springer-Verlag
Berlin Heidelberg New York
London Paris Tokyo

Maurice Roseau
Université Pierre et Marie Curie (Paris VI)
Mécanique Théorique, Tour 66
4, place Jussieu
F-75230 Paris Cedex 05
France

Translator:
H. L. S. Orde
Bressenden, Biddenden
Ashford, Kent TN27 8DU
England

Title of the French original edition:
Vibrations des systèmes mécaniques. Méthodes analytiques et applications.
© Masson, Editeur, Paris, 1984

Mathematics Subject Classification (1980): 70

ISBN 3-540-17950-X Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-17950-X Springer-Verlag New York Berlin Heidelberg

Typesetting: Thomson Press (India) Ltd., New Delhi
Printing: Druckhaus Beltz, Hemsbach/Bergstr.
Bookbinding: J. Schäffer GmbH & Co. KG, Grünstadt
2141/3140-543210

Preface

The familiar concept described by the word "vibrations" suggests the rapid alternating motion of a system about and in the neighbourhood of its equilibrium position, under the action of random or deliberate disturbing forces. It falls within the province of mechanics, the science which deals with the laws of equilibrium, and of motion, and their applications to the theory of machines, to calculate these vibrations and predict their effects.

While it is certainly true that the physical systems which can be the seat of vibrations are many and varied, it appears that they can be studied by methods which are largely indifferent to the nature of the underlying phenomena. It is to the development of such methods that we devote this book which deals with free or induced vibrations in discrete or continuous mechanical structures. The mathematical analysis of ordinary or partial differential equations describing the way in which the values of mechanical variables change over the course of time allows us to develop various theories, linearised or non-linearised, and very often of an asymptotic nature, which take account of conditions governing the stability of the motion, the effects of resonance, and the mechanism of wave interactions or vibratory modes in non-linear systems.

Illustrated by numerous examples chosen for their intrinsic interest, and graduated in its presentation of parts involving difficult or delicate considerations, this work, containing several chapters which have been taught to graduate students at the Pierre and Marie Curie University in Paris, includes unpublished results and throws a new light on several theories.

A glance at the table of contents will convince the reader of the variety of subjects covered. They were selected primarily with an eye to forming a coherent whole, but no doubt the choice also reflects some personal preferences which would be hard to justify, but which we hope may give some grounds for believing that the reader will derive as much pleasure from reading the book as its author had in writing it.

Paris, October 1983 *Maurice Roseau*

Contents

Chapter VII. Modal Analysis and Vibrations of Structures

Chapter VIII. Synchronisation Theory

Chapter I. Forced Vibrations in Systems Having One Degree or Two Degrees of Freedom

The study of linear vibrations of systems with one degree or two degrees of freedom allows certain essential ideas, and in particular the concept of a response curve, to be introduced by means of a few simple calculations. Similarly it allows us to appreciate the influence of damping on the system under conditions in the neighbourhood of resonance. This is of relevance to many widely-used mechanisms, such as shock-absorbers, two-stage suspensions for vehicles or machinery where the characteristics for optimum performance can be determined by using an analytic approach.

Elastic Suspension with a Single Degree of Freedom

A mass m is in contact with a horizontal floor, $z = 0$, through a spring of natural length l_0, of stiffness k. The length of the spring, in its equilibrium position, under compression is l_1 and

(1.1) $$-mg - k(l_1 - l_0) = 0.$$

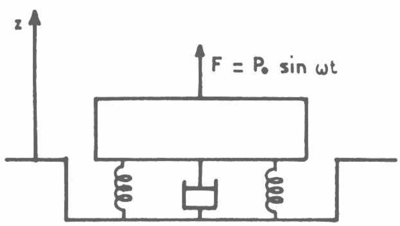

Figure 1.1

We now suppose the mass m to be acted upon by a vertical alternating force of magnitude $P_0 \sin \omega t$; denoting by z the height above ground (the length of the spring) the equation of motion is:

(1.2) $$mz'' + cz' = -mg - k(z - l_0) + P_0 \sin \omega t,$$

where cz' is the viscous damping term. Writing $x = z - l_1$, we obtain, taking (1.1) into account:

(1.3) $$mx'' + cx' + kx = P_0 \sin \omega t.$$

Applying the laws of mechanics to the system comprising the mass, spring and damping device, assuming the masses of the two latter to be negligible, we can write:

$$mx'' = -R + P_0 \sin \omega t, \tag{1.4}$$

where R measures the vertical force exerted by the system on the foundation, or by (1.3):

$$R = cx' + kx. \tag{1.5}$$

Torsional Oscillations

If torsional stresses, whose torque about the axis is $T_0 \sin \omega t$, are exerted on a disc, then in the case where the shaft is embedded, we have

$$I\varphi'' + c\varphi' + k\varphi = T_0 \sin \omega t,$$

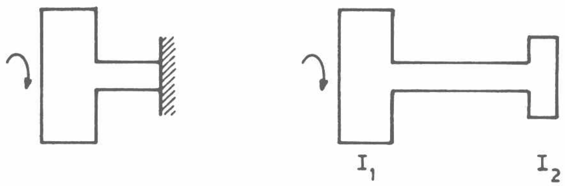

Figure 1.2

where I is the moment of inertia of the disc, k, c are rigidity and damping constants of the shaft, and φ is the angle of rotation, while in the case of a free system with two discs we have:

$$\begin{aligned} I_1\varphi_1'' + c(\varphi_1' - \varphi_2') + k(\varphi_1 - \varphi_2) &= T_0 \sin \omega t \\ I_2\varphi_2'' + c(\varphi_2' - \varphi_1') + k(\varphi_2 - \varphi_1) &= 0, \end{aligned} \tag{1.6}$$

where I_1, I_2 are the moments of inertia of the discs with respect to their common axis, and φ_1, φ_2 their angle of rotation.

The relative motion is described by the co-ordinate $\psi = \varphi_1 - \varphi_2$, which, by (1.6) is the solution of:

$$\frac{I_1 I_2}{I_1 + I_2}\psi'' + c\psi' + k\psi = \frac{I_2 T_0}{I_1 + I_2}\sin \omega t.$$

In the absence of damping ($c = 0$) the natural frequency of the oscillations ($T_0 = 0$) is

$$\omega = \sqrt{k \cdot \frac{I_1 + I_2}{I_1 I_2}}.$$

Natural Oscillations

We consider the model described by (1.3), on the supposition that $P_0 = 0$. In the absence of damping ($c = 0$), the frequency of the natural oscillations is

$$\omega_n = \sqrt{\frac{k}{m}}. \tag{1.7}$$

If $c \neq 0$, the solutions of (1.3) are given by:

$$x = ae^{s_1 t} + be^{s_2 t}, \quad s_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}, \quad c > 0.$$

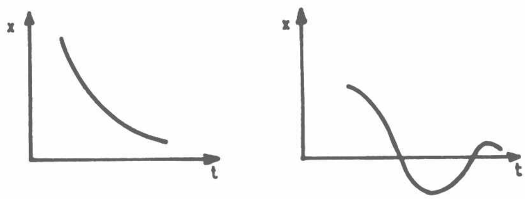

Figure 1.3

In the strongly-damped case where $\left(\frac{c}{2m}\right)^2 > \frac{k}{m}$, there is no oscillation; if the damping is weak and $\left(\frac{c}{2m}\right)^2 < \frac{k}{m}$, we write $s = -\frac{c}{2m} \pm iq$,

$$q = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2} \quad \text{and} \quad x = \exp\left(-\frac{ct}{2m}\right) \cdot (a_1 \cos qt + a_2 \sin qt). \tag{1.8}$$

It will be seen that the amplitudes of the oscillations have relative maxima at intervals of $T = \frac{\pi}{q}$, which decrease like the terms of a geometric progression of ratio $\exp\left(-\frac{\pi c}{2qm}\right)$. The frequency of vibration

$$\omega = \omega_n \sqrt{1 - \left(\frac{c}{c_*}\right)^2}, \quad c_* = 2\sqrt{mk} \tag{1.9}$$

diminishes when the damping factor increases.

Forced Vibrations

We consider once more the model described by (1.3), with a forced excitation due to the load $P_0 \sin \omega t$; ignoring the transient case just discussed, we can write the periodic solution of (1.3) in the form:

(1.10) $$x = A\sin\omega t + B\cos\omega t = x_0\sin(\omega t - \varphi)$$

and calculate A, B or x_0, φ from

$$(k - m\omega^2)A - c\omega B = P_0$$
$$c\omega A + (k - m\omega^2)B = 0,$$

whence

$$A = \frac{P_0(k - m\omega^2)}{(k - m\omega^2)^2 + c^2\omega^2}, \quad B = -\frac{P_0 c\omega}{(k - m\omega^2)^2 + c^2\omega^2}$$

and

(1.11) $$\frac{x_0}{x_{st}} = \left(\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(\frac{2c}{c_*}\frac{\omega}{\omega_n}\right)^2\right)^{-1/2}$$

(1.12) $$\operatorname{tg}\varphi = \frac{2c}{c_*}\cdot\frac{\omega}{\omega_n}\cdot\left(1 - \frac{\omega^2}{\omega_n^2}\right)^{-1},$$

where $x_{st} = \dfrac{P_0}{k}$ is the static deformation which the system would undergo under the effect of a stationary load P_0.

We deduce from (1.11) that at resonance $\omega = \omega_n$, we have $\dfrac{x_0}{x_{st}} = \dfrac{c_*}{2c} = \dfrac{1}{2\varepsilon}$, with $\varepsilon = \dfrac{c}{c_*}$. We shall say that $\dfrac{1}{2\varepsilon}$ is the 'overtension' of the system, which thus appears as the ratio, at resonance, of the amplitude of the forced vibration to the static deformation, when the corresponding ratio of the excitative forces is equal to 1.

The formula (1.11) which expresses for a given c the amplitude $\dfrac{x_0}{x_{st}}$ of the forced vibration as a function of $\dfrac{\omega}{\omega_n}$ defines the response curve; the maximum amplitude is obtained when $\dfrac{\omega}{\omega_n} = \eta$ is such that $(1 - \eta^2)^2 + \left(\dfrac{2c}{c_*}\eta\right)^2$ is a minimum, i.e. when $\eta^2 = 1 - 2\left(\dfrac{c}{c_*}\right)^2$, if $\dfrac{c}{c_*} < 2^{-1/2}$ and when $\eta = 0$ otherwise. The preceding discussion has thus led us to consider in turn:

1. the frequency of the natural or free oscillations:

$$\omega_n = \sqrt{\frac{k}{m}}$$

2. that of the damped oscillations

$$q = \omega_n\sqrt{1 - \left(\frac{c}{c_*}\right)^2}$$

3. that for which the amplitude of the forced vibration is a maximum:

$$\tilde{\omega} = \omega_n \sqrt{1 - 2\left(\frac{c}{c_*}\right)^2}$$

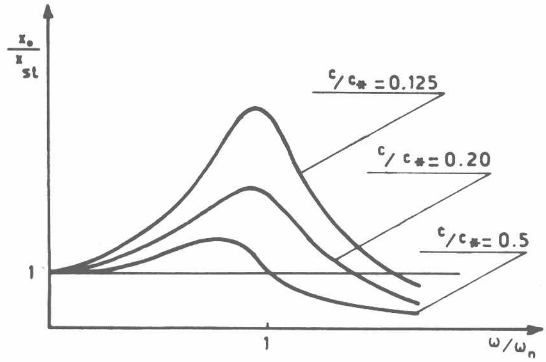

Figure 1.4

Vibration Transmission Factor

We deduce from (1.5) and (1.10) that the amplitude of the periodic force R exerted on the foundation is

$$R_0 = x_0 \sqrt{k^2 + (c\omega)^2},$$

so that by (1.11) the transmittivity coefficient is:

(1.13)
$$\frac{R_0}{P_0} = \left(\frac{1 + \left(\dfrac{2c\omega}{c_* \omega_n}\right)^2}{\left(1 - \dfrac{\omega^2}{\omega_n^2}\right)^2 + \left(\dfrac{2c}{c_*}\dfrac{\omega}{\omega_n}\right)^2} \right)^{1/2}$$

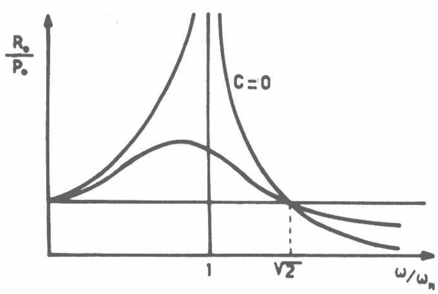

Figure 1.5