

Maurice Roseau

Vibrations in Mechanical Systems

Analytical Methods and Applications



Springer-Verlag

Maurice Roseau

Vibrations in Mechanical Systems

Analytical Methods and Applications

With 112 Figures

Springer-Verlag
Berlin Heidelberg New York
London Paris Tokyo

Maurice Roseau
Université Pierre et Marie Curie (Paris VI)
Mécanique Théorique, Tour 66
4, place Jussieu
F-75230 Paris Cedex 05
France

Translator:

H. L. S. Orde
Bressenden, Biddenden
Ashford, Kent TN27 8DU
England

Title of the French original edition:
Vibrations des systèmes mécaniques. Méthodes analytiques et applications.
© Masson, Editeur, Paris, 1984

Mathematics Subject Classification (1980): 70

ISBN 3-540-17950-X Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-17950-X Springer-Verlag New York Berlin Heidelberg

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, recitation, broadcasting, reproduction on microfilms or in other ways, and storage in data banks. Duplication of this publication or parts thereof is only permitted under the provisions of the German Copyright Law of September 9, 1965, in its version of June 24, 1985, and a copyright fee must always be paid. Violations fall under the prosecution act of the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1987
Printed in Germany

Typesetting: Thomson Press (India) Ltd., New Delhi
Printing: Druckhaus Beltz, Hemsbach/Bergstr.
Bookbinding: J. Schäffer GmbH & Co. KG, Grünstadt
2141/3140-543210

Preface

The familiar concept described by the word “vibrations” suggests the rapid alternating motion of a system about and in the neighbourhood of its equilibrium position, under the action of random or deliberate disturbing forces. It falls within the province of mechanics, the science which deals with the laws of equilibrium, and of motion, and their applications to the theory of machines, to calculate these vibrations and predict their effects.

While it is certainly true that the physical systems which can be the seat of vibrations are many and varied, it appears that they can be studied by methods which are largely indifferent to the nature of the underlying phenomena. It is to the development of such methods that we devote this book which deals with free or induced vibrations in discrete or continuous mechanical structures. The mathematical analysis of ordinary or partial differential equations describing the way in which the values of mechanical variables change over the course of time allows us to develop various theories, linearised or non-linearised, and very often of an asymptotic nature, which take account of conditions governing the stability of the motion, the effects of resonance, and the mechanism of wave interactions or vibratory modes in non-linear systems.

Illustrated by numerous examples chosen for their intrinsic interest, and graduated in its presentation of parts involving difficult or delicate considerations, this work, containing several chapters which have been taught to graduate students at the Pierre and Marie Curie University in Paris, includes unpublished results and throws a new light on several theories.

A glance at the table of contents will convince the reader of the variety of subjects covered. They were selected primarily with an eye to forming a coherent whole, but no doubt the choice also reflects some personal preferences which would be hard to justify, but which we hope may give some grounds for believing that the reader will derive as much pleasure from reading the book as its author had in writing it.

Paris, October 1983

Maurice Roseau

Contents

Chapter I. Forced Vibrations in Systems Having One Degree or Two Degrees of Freedom

Elastic Suspension with a Single Degree of Freedom	1
Torsional Oscillations	2
Natural Oscillations	3
Forced Vibrations	3
Vibration Transmission Factor	5
Elastic Suspension with Two Degrees of Freedom. Vibration Absorber	6
Response Curve of an Elastic System with Two Degrees of Freedom .	7
Vehicle Suspension	11
Whirling Motion of a Rotor-Stator System with Clearance Bearings .	16
Effect of Friction on the Whirling Motion of a Shaft in Rotation;	
Synchronous Precession, Self-sustained Precession	20
Synchronous Motion	24
Self-maintained Precession	24

Chapter II. Vibrations in Lattices

A Simple Mechanical Model	26
The Alternating Lattice Model	28
Vibrations in a One-Dimensional Lattice with Interactive Forces Derived from a Potential	30
Vibrations in a System of Coupled Pendulums	34
Vibrations in Three-Dimensional Lattices	35
Non-Linear Problems	36

Chapter III. Gyroscopic Coupling and Its Applications

1. The Gyroscopic Pendulum	42
Discussion of the Linearised System	45
Appraisal of the Linearisation Process in the Case of Strong Coupling	46
Gyroscopic Stabilisation	46
2. Lagrange's Equations and Their Application to Gyroscopic Systems	49
Example: The Gyroscopic Pendulum	53
3. Applications	53

The Gyrocompass	53
Influence of Relative Motion on the Behaviour of the Gyrocompass	55
Gyroscopic Stabilisation of the Monorail Car	57
4. Routh's Stability Criterion	60
5. The Tuned Gyroscope as Part of an Inertial System for Measuring the Rate of Turn	64
Kinematics of the Multigimbal Suspension	66
a) Orientation of the Rotor	66
b) Co-ordinates of an Intermediate Gimbal	66
c) Relations Between the Parameters θ and ψ	67
The Equations of Motion	68
Inclusion of Damping Terms in the Equations of Motion	71
Dynamic Stability. Undamped System	72
Frequencies of Vibrations of the Free Rotor	73
Motion of the Free Rotor	73
Case of a Multigimbal System Without Damping.	
The Tune Condition	74
Examination of the Two-Gimbal System	75
Chapter IV. Stability of Systems Governed by the Linear Approximation	
Discussion of the Equation $Aq'' + \xi \Gamma q' = 0$	78
Discussion of the Equation $Aq'' + \xi \Gamma q' + Kq = 0$	78
Systems Comprising Both Gyroscopic Forces and Dissipative Forces	81
1. Case $E = 0$	82
A Modified Approach in the Case of Instability	83
2. Case $E \neq 0$	85
Eigenmodes	88
Rayleigh's Method	89
Effect on the Eigenvalues of Changes in Structure	92
An Example	94
Chapter V. The Stability of Operation of Non-Conservative Mechanical Systems	
1. Rolling Motion and Drift Effect	96
2. Yawing of Road Trailers	102
3. Lifting by Air-Cushion	105
The Stationary Regime	106
Case of an Isentropic Expansion	107
Dynamic Stability	108
Chapter VI. Vibrations of Elastic Solids	
I. Flexible Vibrations of Beams	111
1. Equations of Beam Theory	111
2. A Simple Example	114

3. The Energy Equation	116
4. The Modified Equations of Beam Theory; Timoshenko's Model	118
5. Timoshenko's Discretised Model of the Beam	120
6. Rayleigh's Method	122
6.1. Some Elementary Properties of the Spaces $H^1(0, l)$, $H^2(0, l)$	122
6.2. Existence of the Lowest Eigenfrequency	126
6.3. Case of a Beam Supporting Additional Concentrated Loads	130
6.4. Intermediate Conditions Imposed on the Beam	130
6.5. Investigation of Higher Frequencies	133
7. Examples of Applications	134
7.1. Beam Fixed at $x = 0$, Free at $x = l$	134
7.2. Beam Fixed at Both Ends	134
7.3. Beam Free at Both Ends	135
7.4. Beam Hinged at $x = 0$, Free at $x = l$	136
7.5. Beam Fixed at $x = 0$ and Bearing a Point Load at the Other End	136
7.6. Beam Supported at Three Points	137
7.7. Vibration of a Wedge Clamped at $x = 0$. Ritz's Method	137
7.8. Vibrations of a Supported Pipeline	139
7.9. Effect of Longitudinal Stress on the Flexural Vibrations of a Beam and Application to Blade Vibrations in Turbomachinery	141
7.10. Vibrations of Interactive Systems	143
8. Forced Vibrations of Beams Under Flexure	145
9. The Comparison Method	147
9.1. The Functional Operator Associated with the Model of a Beam Under Flexure	147
9.2. The Min-Max Principle	150
9.3. Application to Comparison Theorems	151
10. Forced Excitation of a Beam	154
10.1. Fourier's Method	154
10.2. Boundary Conditions with Elasticity Terms	157
10.3. Forced Vibrations of a Beam Clamped at One End, Bearing a Point Load at the Other End, and Excited at the Clamped End by an Imposed Transverse Motion of Frequency ω	158
II. Longitudinal Vibrations of Bars. Torsional Vibrations	162
1. Equations of the Problem and the Calculation of Eigenvalues.	162
2. The Associated Functional Operator	164
3. The Method of Moments	165
3.1. Introduction	165
3.2. Lanczos's Orthogonalisation Method	166
3.3. Eigenvalues of A_n	167
3.4. Padé's Method	169
3.5. Approximation of the A Operator	170
III. Vibrations of Elastic Solids	174
1. Statement of Problem and General Assumptions	174

2. The Energy Theorem	176
3. Free Vibrations of Elastic Solids	177
3.1. Existence of the Lowest Eigenfrequency	177
3.2. Higher Eigenfrequencies	181
3.3. Case Where There Are No Kinematic Conditions	182
3.4. Properties of Eigenmodes and Eigenfrequencies	182
4. Forced Vibrations of Elastic Solids	186
4.1. Excitation by Periodic Forces Acting on Part of the Boundary	186
4.2. Excitation by Periodic Displacements Imposed on Some Part of the Boundary	191
4.3. Excitation by Periodic Volume Forces	193
5. Vibrations of Non-Linear Elastic Media	196
IV. Vibrations of Plane Elastic Plates	197
1. Description of Stresses; Equations of Motion.	197
2. Potential Energy of a Plate	200
3. Determination of the Law of Behaviour	201
4. Eigenfrequencies and Eigenmodes	203
5. Forced Vibrations	209
6. Eigenfrequencies and Eigenmodes of Vibration of Complex Systems	211
6.1. Free Vibrations of a Plate Supported Elastically over a Part U of Its Area, U Open and $\bar{U} \subset \Omega$	211
6.2. Eigenfrequencies and Eigenmodes of a Rectangular Plate Reinforced by Regularly Spaced Stiffeners	211
V. Vibrations in Periodic Media	212
1. Formulation of the Problem and Some Consequences of Korn's Inequality	212
2. Bloch Waves	214

Chapter VII. Modal Analysis and Vibrations of Structures

I. Vibrations of Structures	217
Free Vibrations	217
Forced Vibrations	218
Random Excitation of Structures	220
II. Vibrations in Suspension Bridges	224
The Equilibrium Configuration	224
The Flexure Equation Assuming Small Disturbances	225
Free Flexural Vibrations in the Absence of Stiffness	227
a) Symmetric Modes: $\eta(x) = \eta(-x)$	228
b) Skew-Symmetric Modes: $\eta(x) = -\eta(-x)$	228
Torsional Vibrations of a Suspension Bridge	230
Symmetric Modes	232
a) Flexure	232
b) Torsion	233
Vibrations Induced by Wind	234
Aerodynamic Forces Exerted on the Deck of the Bridge	236

Discussion Based on a Simplified Model	239
A More Realistic Approach	241

Chapter VIII. Synchronisation Theory

1. Non-Linear Interactions in Vibrating Systems	245
2. Non-Linear Oscillations of a System with One Degree of Freedom	250
2.1. Reduction to Standard Form	250
2.2. The Associated Functions	251
2.3. Choice of the Numbers m and N	252
2.4. Case of an Autonomous System	252
3. Synchronisation of a Non-Linear Oscillator Sustained by a Periodic Couple. Response Curve. Stability	253
4. Oscillations Sustained by Friction	256
5. Parametric Excitation of a Non-Linear System	258
6. Subharmonic Synchronisation	261
7. Non-Linear Excitation of Vibrating Systems. Some Model Equations	265
8. On a Class of Strongly Non-Linear Systems	266
8.1. Periodic Regimes and Stability	266
8.2. Van der Pol's Equation with Amplitude Delay Effect	269
9. Non-Linear Coupling Between the Excitation Forces and the Elastic Reactions of the Structure on Which They Are Exerted	272
Application to Bouasse and Sarda's Regulator	276
10. Stability of Rotation of a Machine Mounted on an Elastic Base and Driven by a Motor with a Steep Characteristic Curve	278
11. Periodic Differential Equations with Singular Perturbation	281
11.1. Study of a Linear System with Singular Perturbation $\mu(dx/dt) = A(t)x + h(t)$	281
11.2. The Non-Linear System	283
11.3. Stability of the Periodic Solution	285
12. Application to the Study of the Stability of a Rotating Machine Mounted on an Elastic Suspension and Driven by a Motor with a Steep Characteristic Curve	287
13. Analysis of Stability	290
14. Rotation of an Unbalanced Shaft Sustained by Alternating Vertical Displacements	297
15. Stability of Rotation of the Shaft	301
16. Synchronisation of the Rotation of an Unbalanced Shaft Sustained by Alternating Vertical Forces	304
16.1. The Non-Resonant Case	304
16.2. Analysis of Stability	307
17. Synchronisation of the Rotation of an Unbalanced Shaft Sustained by Alternating Forces in the Case of Resonance	311
17.1. The Modified Standard System	312
17.2. Synchronisation of Non-Linear System	314

17.3. Stability Criterion for Periodic Solution	318
17.4. Application	323

Chapter IX. Stability of a Column Under Compression – Mathieu's Equation

Buckling of a Column	325
Analysis of Stability	327
A Discretised Model of the Loaded Column	329
The Discretised Model with Slave Load	331
Description of the Asymptotic Nature of the Zones of Instability for the Mathieu Equation	333
Normal Form of Infinite Determinant. Analysis of Convergence	337
Hill's Equation	340

Chapter X. The Method of Amplitude Variation and Its Application to Coupled Oscillators

Posing the Problem	345
Cases Where Certain Oscillations Have the Same Frequency	353
Coupled Oscillators; Non-Autonomous System and Resonance. A Modified Approach	354
Case of Resonance	358
Case Where Certain Eigenmodes Decay (Degeneracy)	358
Case of Oscillators Coupled Through Linear Terms	360
Non-Autonomous Non-Linear System in the General Case; Examination of the Case When Certain Eigenmodes Are Evanescent	362
Gyroscopic Stabiliser with Non-Linear Servomechanism	368

Chapter XI. Rotating Machinery

I. The Simplified Model with Frictionless Bearings	373
Preliminary Study of the Static Bending of a Shaft with Circular Cross- Section	373
Steady Motion of a Disc Rotating on a Flexible Shaft	375
Flexural Vibrations When Shaft Is in Rotation	377
Forced Vibrations	379
II. Effects of Flexibility of the Bearings	380
Hydrodynamics of Thin Films and Reynold's Equation	380
Application to Circular Bearings	382
Unsteady Regime	386
Gas Lubricated Bearings	387
Effects of Bearing Flexibility on the Stability of Rotation of a Disc . .	388
1. Case of an Isotropic Shaft: $b_2 = \tilde{b}_2$, $c_2 = \tilde{c}_2$	390
2. Case Where Shaft and Bearings Are Both Anisotropic	393

Periodic Linear Differential Equation with Reciprocity Property . .	394
Stability of Rotation of Disc Where the System Has Anisotropic Flexibilities	397
An Alternative Approach to the Stability Problem	403
Application to the Problem of the Stability of a Rotating Shaft . .	405
III. Stability of Motion of a Rigid Rotor on Flexible Bearings. Gyroscopic Effects and Stability	411
Notation and Equations of Motion	411
Analysis of Stability in the Isotropic Case	414
Calculating the Critical Speeds of the Rotor	414
Resonant Instability Near $\omega = (\omega_1 + \omega_2)/2$	417
Instability Near the Resonance $\omega = \omega_1$	423
Ground Resonance of the Helicopter Blade Rotor System	425
IV. Whirling Motion of a Shaft in Rotation with Non-Linear Law of Physical Behaviour	428
Calculation of T_y, T_z	431
The Equations of Motion	432
Effect of Hysteresis on Whirling	434
Stability of the Regime $\omega < \omega_0$	434
Analysis of the Rotatory Regime When $\omega > \omega_0$	436
V. Suspension of Rotating Machinery in Magnetic Bearings	439
Principle of Magnetic Suspension	439
Quadratic Functionals and Optimal Control	442
Application to the Model with One Degree of Freedom	445
Characteristics and Applications of Magnetic Bearings	446

Chapter XII. Non-Linear Waves and Solitons

1. Waves in Dispersive or Dissipative Media	449
The Non-Linear Perturbation Equations	451
An Example: Gravity Waves in Shallow Water	453
2. The Inverse Scattering Method	454
The Method of Solution	456
3. The Direct Problem	456
3.1. The Eigenvalue Problem	459
On Some Estimates	460
The Finiteness of the Set of Eigenvalues	463
3.2. Transmission and Reflection Coefficients	465
Eigenvalues (Continued)	467
4. The Inverse Problem	469
The Kernel $K(x, y)$ (Continued)	473
The Gelfand-Levitan Integral Equation	474
An Alternative Definition of the Kernel $K(x, y)$	476
Solving Gelfand-Levitan's Equation	478
5. The Inverse Scattering Method	480
The Evolution Equation	486

Integral Invariants	489
Another Approach to the Evolution Equation	493
6. Solution of the Inverse Problem in the Case Where the Reflection Coefficient is Zero	498
7. The Korteweg-de Vries Equation. Interaction of Solitary Waves . .	503
Investigation of Asymptotic Behaviour for $t \rightarrow +\infty$	505
Asymptotic Behaviour for $t \rightarrow -\infty$	506
 References	 508
Subject Index	511

Chapter I. Forced Vibrations in Systems Having One Degree or Two Degrees of Freedom

The study of linear vibrations of systems with one degree or two degrees of freedom allows certain essential ideas, and in particular the concept of a response curve, to be introduced by means of a few simple calculations. Similarly it allows us to appreciate the influence of damping on the system under conditions in the neighbourhood of resonance. This is of relevance to many widely-used mechanisms, such as shock-absorbers, two-stage suspensions for vehicles or machinery where the characteristics for optimum performance can be determined by using an analytic approach.

Elastic Suspension with a Single Degree of Freedom

A mass m is in contact with a horizontal floor, $z = 0$, through a spring of natural length l_0 , of stiffness k . The length of the spring, in its equilibrium position, under compression is l_1 and

$$(1.1) \quad -mg - k(l_1 - l_0) = 0.$$

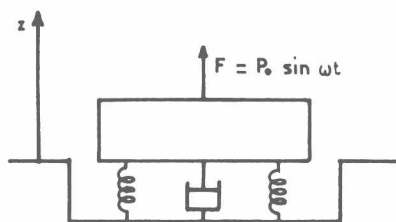


Figure 1.1

We now suppose the mass m to be acted upon by a vertical alternating force of magnitude $P_0 \sin \omega t$; denoting by z the height above ground (the length of the spring) the equation of motion is:

$$(1.2) \quad mz'' + cz' = -mg - k(z - l_0) + P_0 \sin \omega t,$$

where cz' is the viscous damping term. Writing $x = z - l_1$, we obtain, taking (1.1) into account:

$$(1.3) \quad mx'' + cx' + kx = P_0 \sin \omega t.$$

Applying the laws of mechanics to the system comprising the mass, spring and damping device, assuming the masses of the two latter to be negligible, we can write:

$$(1.4) \quad mx'' = -R + P_0 \sin \omega t,$$

where R measures the vertical force exerted by the system on the foundation, or by (1.3):

$$(1.5) \quad R = cx' + kx.$$

Torsional Oscillations

If torsional stresses, whose torque about the axis is $T_0 \sin \omega t$, are exerted on a disc, then in the case where the shaft is embedded, we have

$$I\varphi'' + c\varphi' + k\varphi = T_0 \sin \omega t,$$

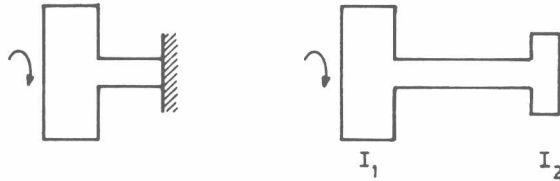


Figure 1.2

where I is the moment of inertia of the disc, k, c are rigidity and damping constants of the shaft, and φ is the angle of rotation, while in the case of a free system with two discs we have:

$$(1.6) \quad \begin{aligned} I_1\varphi_1'' + c(\varphi_1' - \varphi_2') + k(\varphi_1 - \varphi_2) &= T_0 \sin \omega t \\ I_2\varphi_2'' + c(\varphi_2' - \varphi_1') + k(\varphi_2 - \varphi_1) &= 0, \end{aligned}$$

where I_1, I_2 are the moments of inertia of the discs with respect to their common axis, and φ_1, φ_2 their angle of rotation.

The relative motion is described by the co-ordinate $\psi = \varphi_1 - \varphi_2$, which, by (1.6) is the solution of:

$$\frac{I_1 I_2}{I_1 + I_2} \psi'' + c\psi' + k\psi = \frac{I_2 T_0}{I_1 + I_2} \sin \omega t.$$

In the absence of damping ($c = 0$) the natural frequency of the oscillations ($T_0 = 0$) is

$$\omega = \sqrt{k \cdot \frac{I_1 + I_2}{I_1 I_2}}.$$

Natural Oscillations

We consider the model described by (1.3), on the supposition that $P_0 = 0$. In the absence of damping ($c = 0$), the frequency of the natural oscillations is

$$(1.7) \quad \omega_n = \sqrt{\frac{k}{m}}.$$

If $c \neq 0$, the solutions of (1.3) are given by:

$$x = ae^{s_1 t} + be^{s_2 t}, \quad s_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}, \quad c > 0.$$

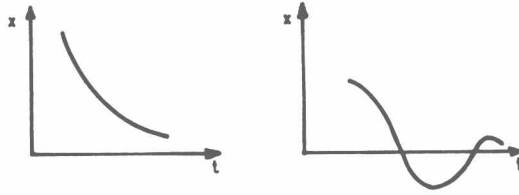


Figure 1.3

In the strongly-damped case where $\left(\frac{c}{2m}\right)^2 > \frac{k}{m}$, there is no oscillation; if the damping is weak and $\left(\frac{c}{2m}\right)^2 < \frac{k}{m}$, we write $s = -\frac{c}{2m} \pm iq$,

$$(1.8) \quad q = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2} \quad \text{and} \quad x = \exp\left(-\frac{ct}{2m}\right) \cdot (a_1 \cos qt + a_2 \sin qt).$$

It will be seen that the amplitudes of the oscillations have relative maxima at intervals of $T = \frac{\pi}{q}$, which decrease like the terms of a geometric progression of ratio $\exp\left(-\frac{\pi c}{2qm}\right)$. The frequency of vibration

$$(1.9) \quad \omega = \omega_n \sqrt{1 - \left(\frac{c}{c_*}\right)^2}, \quad c_* = 2\sqrt{mk}$$

diminishes when the damping factor increases.

Forced Vibrations

We consider once more the model described by (1.3), with a forced excitation due to the load $P_0 \sin \omega t$; ignoring the transient case just discussed, we can write the

periodic solution of (1.3) in the form:

$$(1.10) \quad x = A \sin \omega t + B \cos \omega t = x_0 \sin(\omega t - \varphi)$$

and calculate A, B or x_0, φ from

$$\begin{aligned} (k - m\omega^2)A - c\omega B &= P_0 \\ c\omega A + (k - m\omega^2)B &= 0, \end{aligned}$$

whence

$$A = \frac{P_0(k - m\omega^2)}{(k - m\omega^2)^2 + c^2\omega^2}, \quad B = -\frac{P_0c\omega}{(k - m\omega^2)^2 + c^2\omega^2}$$

and

$$(1.11) \quad \frac{x_0}{x_{st}} = \left(\left(1 - \frac{\omega^2}{\omega_n^2} \right)^2 + \left(\frac{2c}{c_*} \frac{\omega}{\omega_n} \right)^2 \right)^{-1/2}$$

$$(1.12) \quad \operatorname{tg} \varphi = \frac{2c}{c_*} \cdot \frac{\omega}{\omega_n} \cdot \left(1 - \frac{\omega^2}{\omega_n^2} \right)^{-1},$$

where $x_{st} = \frac{P_0}{k}$ is the static deformation which the system would undergo under the effect of a stationary load P_0 .

We deduce from (1.11) that at resonance $\omega = \omega_n$, we have $\frac{x_0}{x_{st}} = \frac{c_*}{2c} = \frac{1}{2\varepsilon}$, with $\varepsilon = \frac{c}{c_*}$. We shall say that $\frac{1}{2\varepsilon}$ is the 'overtension' of the system, which thus appears as the ratio, at resonance, of the amplitude of the forced vibration to the static deformation, when the corresponding ratio of the excitative forces is equal to 1.

The formula (1.11) which expresses for a given c the amplitude $\frac{x_0}{x_{st}}$ of the forced vibration as a function of $\frac{\omega}{\omega_n}$ defines the response curve; the maximum amplitude is obtained when $\frac{\omega}{\omega_n} = \eta$ is such that $(1 - \eta^2)^2 + \left(\frac{2c}{c_*} \eta \right)^2$ is a minimum, i.e. when $\eta^2 = 1 - 2\left(\frac{c}{c_*} \right)^2$, if $\frac{c}{c_*} < 2^{-1/2}$ and when $\eta = 0$ otherwise. The preceding discussion has thus led us to consider in turn:

1. the frequency of the natural or free oscillations:

$$\omega_n = \sqrt{\frac{k}{m}}$$

2. that of the damped oscillations

$$q = \omega_n \sqrt{1 - \left(\frac{c}{c_*} \right)^2}$$

3. that for which the amplitude of the forced vibration is a maximum:

$$\tilde{\omega} = \omega_n \sqrt{1 - 2\left(\frac{c}{c_*}\right)^2}$$

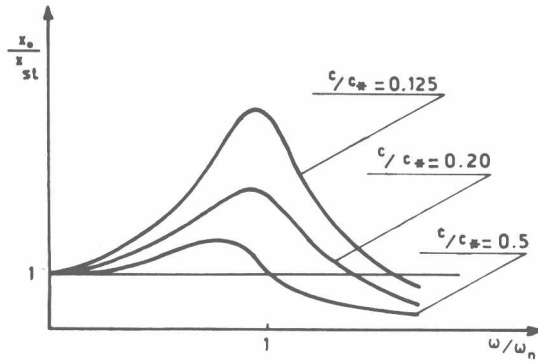


Figure 1.4

Vibration Transmission Factor

We deduce from (1.5) and (1.10) that the amplitude of the periodic force R exerted on the foundation is

$$R_0 = x_0 \sqrt{k^2 + (c\omega)^2},$$

so that by (1.11) the transmittivity coefficient is:

$$(1.13) \quad \frac{R_0}{P_0} = \left(\frac{1 + \left(\frac{2c\omega}{c_*\omega_n} \right)^2}{\left(1 - \frac{\omega^2}{\omega_n^2} \right)^2 + \left(\frac{2c}{c_*} \frac{\omega}{\omega_n} \right)^2} \right)^{1/2}$$

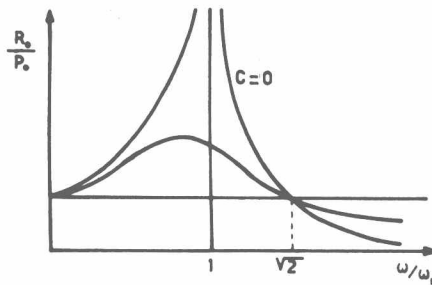


Figure 1.5