Andreas Brandstädt
Dieter Kratsch
Haiko Müller (Eds.)

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Graph-Theoretic Concepts in Computer Science

33rd International Workshop, WG 2007
Dornburg, Germany, June 2007
Revised Papers

Springer

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Preface

The 33rd International Conference "Workshop on Graph-Theoretic Concepts in Computer Science" (WG 2007) took place in the Conference Center in old castle in Dornburg near Jena, Germany, June 21–23, 2007. The approximately 80 participants came from various countries all over the world, among them Brazil, Canada, the Czech Republic, France, UK, Greece, Hungary, Italy, Japan, The Netherlands, Norway, Sweden, Taiwan, and the USA.

WG 2007 continued the series of 32 previous WG conferences. Since 1975, the WG conference has taken place 20 times in Germany, four times in The Netherlands, twice in Austria as well as once in Italy, Slovakia, Switzerland, the Czech Republic, France and in Norway.

The WG conference traditionally aims at uniting theory and practice by demonstrating how graph-theoretic concepts can be applied to various areas in computer science, or by extracting new problems from applications. The goal is to present recent research results and to identify and explore directions of future research.

The continuing interest in the WG conferences was reflected in the high number of submissions; 99 papers were submitted and in an evaluation process with four reports per submission, 30 papers were accepted by the Program Committee for the conference. Due to the high number of submissions and the limited schedule of 3 days, various good papers could not be accepted.

There were invited talks by Ming-Yang Kao (Evanston, Illinois) on algorithmic DNA assembly, and by Klaus Jansen (Kiel, Germany) on approximation algorithms for geometric intersection graphs.

We are grateful to all those who contributed to WG 2007: First of all, the authors submitting so many good papers, the numerous referees, the speakers, the Program Committee, the Organizing Committee (special thanks go to Katrin Erdmann and Roswitha Fengler, Rostock, to Mathieu Liedloff, Metz, as well as to Nadja Betzler, Falk Hüffner and Marita Venth, Jena, and the whole group of Rolf Niedermeier for hosting WG 2007 in the wonderful Conference Center in the old castle in Dornburg) and last but not least, to our sponsors the German Research Council (DFG), Land Thüringen, Universität Jena, and Stiftung für Innovation, Technologie und Forschung Thüringen (STIFT) as well as to the Universities of Leeds, Metz and Rostock.

October 2007

Andreas Brandstädt
Dieter Kratsch
Haiko Müller

Organization

The Tradition of WG

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1979	U. Pape – Berlin, Germany
1980	H. Noltemeier – Bad Honnef, Germany
1981	J. Mühlbacher – Linz, Austria
1982	H.J. Schneider, H. Göttler – Neuenkirchen, Germany
1983	M. Nagl, J. Perl – Haus Ohrbeck near Osnabrück, Germany
1984	U. Pape – Berlin, Germany
1985	H. Noltemeier – Castle Schwanenberg near Würzburg, Germany
1986	G. Tinhofer, G. Schmidt – Bernried near Munich, Germany
1987	H. Göttler, H.J. Schneider – Kloster Banz near Bamberg, Germany
1988	J. van Leeuwen – Amsterdam, The Netherlands
1989	M. Nagl – Castle Rolduc, The Netherlands
1990	R.H. Möhring – Berlin, Germany
1991	G. Schmidt, R. Berghammer – Fischbachau near Munich, Germany
1992	E.W. Mayr – Wiesbaden-Naurod, Germany
1993	J. van Leeuwen – Utrecht, The Netherlands
1994	G. Tinhofer, E.W. Mayr, G. Schmidt – Herrsching near Munich, Germany
1995	M. Nagl – Aachen, Germany
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1997	R.H. Möhring – Berlin, Germany
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1999	P. Widmayer – Ascona, Switzerland
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2001	A. Brandstädt – Boltenhagen near Rostock, Germany
2002	L. Kučera – Cesky Krumlov, Czech Republic
2003	H.L. Bodlaender – Elspeet, The Netherlands
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Table of Contents

Computational Complexity of Generalized Domination: A Complete Dichotomy for Chordal Graphs

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Abstract. The so called (σ, ρ)-domination, introduced by J.A. Telle, is a concept which provides a unifying generalization for many variants of domination in graphs. (A set S of vertices of a graph G is called (σ, ρ)-*dominating* if for every vertex $v \in S$, $|S \cap N(v)| \in \sigma$, and for every $v \notin S$, $|S \cap N(v)| \in \rho$, where σ and ρ are sets of nonnegative integers and $N(v)$ denotes the open neighborhood of the vertex v in G.) It was known that for any two nonempty finite sets σ and ρ (such that $0 \notin \rho$), the decision problem whether an input graph contains a (σ, ρ)-dominating set is NP-complete, but that when restricted to chordal graphs, some polynomial time solvable instances occur. We show that for chordal graphs, the problem performs a complete dichotomy: it is polynomial time solvable if σ, ρ are such that every chordal graph contains at most one (σ, ρ)-dominating set, and NP-complete otherwise. The proof involves certain flavor of existentionality - we are not able to characterize such pairs (σ, ρ) by a structural description, but at least we can provide a recursive algorithm for their recognition. If ρ contains the 0 element, every graph contains a (σ, ρ)-dominating set (the empty one), and so the nontrivial question here is to ask for a maximum such set. We show that MAX-(σ, ρ)-domination problem is NP-complete for chordal graphs whenever ρ contains, besides 0, at least one more integer.

Keywords: Computational complexity, graph algorithms.

1 Introduction and Overview of Results

We consider finite undirected graphs without loops or multiple edges. The vertex set of a graph G is denoted by $V(G)$ and its edge set by $E(G)$. The open neighborhood of a vertex is denoted by $N(u) = \{v\colon uv \in E(G)\}$. A graph is chordal if it does not contain an induced cycle of length greater than three.

⋆ On leave from Department of Applied Mathematics, Syktyvkar State University, Syktyvkar, Russia. Most of the results were obtained during the research stay of the first author at DIMATIA Prague in 2006.

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1.1 (σ, ρ)-Domination

Let σ, ρ be a pair of nonempty sets of nonnegative integers. A set of vertices of G is called *(σ, ρ)-dominating* if for every vertex $v \in S$, $|S \cap N(v)| \in \sigma$, and for every $v \notin S$, $|S \cap N(v)| \in \rho$. The concept of (σ, ρ)-domination was introduced by J.A. Telle [14,15] (and further elaborated on in [12,9]) as a unifying generalization of many previously studied variants of the notion of dominating sets (see [8] for an extensive bibliography on domination in graphs). In particular, ($\mathbb{N}_0$,$\mathbb{N}$)-dominating sets are ordinary dominating sets, ($\{0\}$,$\mathbb{N}_0$)-dominating sets are independent sets, ($\mathbb{N}_0$,$\{1\}$)-dominating sets are efficient dominating sets, ($\{0\}$,$\{1\}$)-dominating sets are 1-perfect codes (or independent efficient dominating sets), ($\{0\}$,$\{0,1\}$)-dominating sets are strong stable sets, ($\{0\}$,$\mathbb{N}$)-dominating sets are independent dominating set, ($\{1\}$,$\{1\}$)-dominating sets are total perfect dominating set, or ($\{r\}$,$\mathbb{N}_0$)-dominating sets are induced r-regular subgraphs ($\mathbb{N}$ and $\mathbb{N}_0$ denote the sets of positive and nonnegative integers, respectively).

We are interested in the complexity of the problem of existence of a (σ, ρ)-dominating set in an input graph, and we denote this problem by $\exists(\sigma, \rho)$-DOMINATION. It can be easily seen that if $0 \in \rho$, then the $\exists(\sigma, \rho)$-DOMINATION problem has a trivial solution $S = \emptyset$. So throughout the main part of the paper (and unless not explicitly stated otherwise) we suppose that $0 \notin \rho$.

1.2 Our Results

In view of the above given examples, it is not surprising that for any nontrivial combination of finite sets σ and ρ (considered as fixed parameters of the problem), $\exists(\sigma, \rho)$-DOMINATION is NP-complete [14]. It is then natural to pay attention to restricted graph classes for inputs of the problem. It was observed in [11] that for any pair of finite sets σ and ρ, the problem is solvable in polynomial time for interval graphs, but that it becomes NP-complete when restricted to chordal graphs (for some parameter sets σ and ρ). In particular, it was shown that for one-element sets $\sigma = \{p\}, \rho = \{q\}$, $\exists(\sigma, \rho)$-DOMINATION is polynomial time solvable if $q > 2p + 1$ and NP-complete if $q \leq p + 1$. We close this gap by showing that all the remaining cases are also polynomial time solvable. Moreover, we extend this polytime/NP-completeness dichotomy to any pair of finite sets σ, ρ by showing the following characterization:

Theorem A. *For finite sets σ, ρ, $\exists(\sigma, \rho)$-DOMINATION is polynomial time solvable for chordal graphs if every chordal graph has at most one (σ, ρ)-dominating set, and it is NP-complete otherwise.*

This theorem provides a full characterization and dichotomy, with both the polynomial time solvable and NP-complete cases including nontrivial and interesting samples (as we show by discussing some examples in Section 4). Dichotomy results are valued and intensively looked for (e.g., the classification of Boolean satisfiability by Schaefer [13], further dichotomy results for larger classes of the Constraint Satisfaction Problem by Bulatov et al. [2] paving the way to the utmost CSP dichotomy conjecture of Feder and Vardi [4], or several results for graph homomorphisms [10,3,6,5].) The characterization is nonconstructive in the sense that we are not able to provide a structural description of ambivalent (or non-ambivalent) pairs σ, ρ (we call a pair σ, ρ *ambivalent* if there exists a chordal graph containing two different (σ, ρ)-dominating sets), and there is indication that such a description will not be simple. Indeed, for any pair of σ and ρ, there are infinitely many chordal graphs to be checked if any of them, by chance, contains two different (σ, ρ)-dominating sets. Perhaps somewhat surprisingly we show that this fact can be overcome at least from the computational point of view:

Theorem B. *It can be decided in finite time (i.e., by a recursive algorithm) whether for a given pair of finite sets σ, ρ, there exists a chordal graph containing two different (σ, ρ)-dominating sets.*

The NP-hardness part of Theorem A is proved in Section 2 by a reduction from a variant of the EXACT COVER problem. Its polynomial part is proved in Section 3 by providing an explicit dynamic programming algorithm. Theorem B is proved by providing an explicit upper bound on the minimum size of an ambivalent graph in Section 4. In Section 5, we discuss the case when $0 \in \rho$. As we have already mentioned, the $\exists(\sigma, \rho)$-DOMINATION problem is then trivial (the empty set is always (σ, ρ)-dominating), and the natural question here is the optimization variant. However, we show this is always a hard problem:

Theorem C. *Given a chordal graph graph G and a number k, it is NP-complete to decide if G contains a (σ, ρ)-dominating set of size at least k, provided σ, ρ are finite sets of nonnegative integers and $\rho \neq \{0\}$.*

Throughout the paper $n = |V(G)|$, $p_{\min} = \min \sigma$, $p_{\max} = \max \sigma$, $q_{\min} = \min \rho$ and $q_{\max} = \max \rho$, where G is the graph and σ, ρ the sets under consideration. In case of single-element sets σ or ρ, we write simply $p = p_{\min} = p_{\max}$ and $q = q_{\min} = q_{\max}$.

2 NP-Complete Cases

This section is devoted to the proof of the following theorem.

Theorem 1. *Let σ, ρ be finite sets of nonnegative integers, $0 \notin \rho$. If there is a chordal graph with at least two different (σ, ρ)-dominating sets, then the $\exists(\sigma, \rho)$-DOMINATION problem is NP-complete for chordal graphs.*

2.1 An Auxiliary Complexity Lemma

We are going to reduce from a special variant of the COVER BY TRIPLES problem (or EXACT COVER)(see [7]).

Let r be a positive integer. An instance of the COVER BY NO MORE THAN r TRIPLES is a pair (X, M), where X is a nonempty finite set and M is a set of triples of elements of X. We ask about the existence of a set $M' \subset M$ such that every element of X belongs to at least one and to at most r triples of M'. Such a set we call a *cover of X by no more than r triples*. For space limitations the proof of the following auxiliary lemma is omitted.

Lemma 1. *For every fixed $r \geq 1$, the* Cover by no more than r triples *problem is NP-complete.*

2.2 The Forcing Gadget

Our next step of the proof is the construction of a gadget which "enforces" on a given vertex the property of "not belonging to any (σ, ρ)-dominating set".

It is known (cf. [11]) that if $q_{\min} \geq 2p_{\max}+2$, then every chordal graph contains at most one (σ, ρ)-dominating set. Hence we assume that $q_{\min} \leq 2p_{\max} + 1$. We construct a rooted graph F as follows.

Suppose first that $q_{\min} \leq p_{\max} + 1$. We start with a complete graph $K_{p_{\max}+1}$ with vertices $u_1, u_2, \ldots, u_{p_{\max}+1}$. Let $\{S_1, S_2, \ldots, S_t\}$ be a set of $q_{\min}$-tuples which covers the set $\{u_1, u_2, \ldots, u_{p_{\max}+1}\}$ (i.e., each u_j belongs to at least one S_i). For every $i = 1, 2, \ldots, t$, we add $q_{\max} + 1$ new vertices $v_1^{(i)}, v_2^{(i)}, \ldots, v_{q_{\max}+1}^{(i)}$ and connect them to all vertices of S_i by edges.

If $q_{\min} > p_{\max} + 1$, the construction is slightly different. We again start with a complete graph $K_{p_{\max}+1}$ with vertices $u_1, u_2, \ldots, u_{p_{\max}+1}$. We add $q_{\max} + 1$ new vertices $v_1, v_2, \ldots, v_{q_{\max}+1}$ and $q_{\max}+1$ copies of $K_{p_{\max}+1}$, say $Q_1, Q_2, \ldots, Q_{q_{\max}+1}$, and connect every v_j by edges to all vertices $u_1, u_2, \ldots, u_{p_{\max}+1}$ and to $q_{\min}-p_{\max}+1$ vertices of the corresponding Q_j.

In both cases the vertex u_1 is the root of F.

Lemma 2. *The graph F has at least one (σ, ρ)-dominating set, and for every (σ, ρ)-dominating set S in F, $u_1, u_2, \ldots, u_{p_{\max}+1} \in S$. Moreover, if F is an induced subgraph of a graph F' such that u_1 is the only vertex of F adjacent to vertices of $F' \setminus F$, then the vertices of $F' \setminus F$ that are adjacent to u_1 do not belong to any (σ, ρ)-dominating set in F'.*

Proof. Suppose that $q_{\min} \leq p_{\max} + 1$. Obviously $\{u_1, u_2, \ldots, u_{p_{\max}+1}\}$ is a (σ, ρ)-dominating set in F. For the second statement, assume that S is a (σ, ρ)-dominating set in F and $u_i \notin S$ for some i. Let S_j be a $q_{\min}$-tuple which contains u_i. It is readily seen that $v_1^{(j)}, v_2^{(j)}, \ldots, v_{q_{\max}+1}^{(j)} \in S$. But then u_i is adjacent to at least $q_{\max} + 1$ vertices of S, a contradiction.

If $q_{\min} > p_{\max} + 1$, the proof of the second statement is similar. For the first part, note that the vertices $u_1, u_2, \ldots, u_{p_{\max}+1}$ and all vertices of the added cliques Q_j form a (σ, ρ)-dominating set.

For the last statement, note that we have proved that in both cases u_1 is in S and has $p_{\max}$ neighbors in S, for any (σ, ρ)-dominating set S in F, but the argument survives for any (σ, ρ)-dominating set in F' as well. □

2.3 The Reduction

Let H be a graph which has at least two different (σ, ρ)-dominating sets $S, \widetilde{S}$. We choose a vertex $u \in S \div \widetilde{S}$, where $\div$ denotes the symmetric difference of sets,