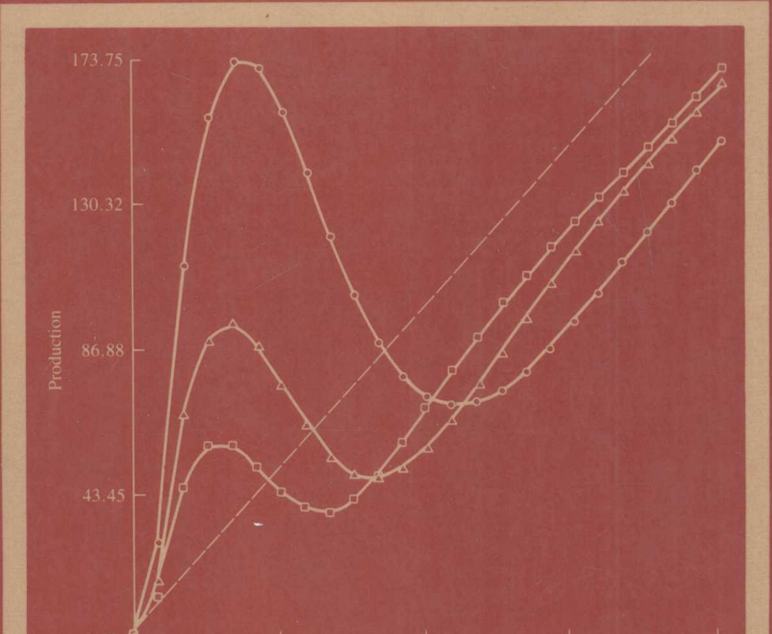
CONTROL SYSTEMS MODELING AND ANALYSIS

Gerard Voland



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This modest work is dedicated to my sister and nephews

Norma Bell
Mark Bell
Steven Bell

to my aunt and uncle

Thelma Burke
John M. Burke

and to the Lord who has given me the opportunity to complete it.

And if any man think that he knoweth any thing,
he knoweth nothing yet as he ought to know.

1 Corinthians 8:2

PREFACE

This text is designed to provide insight and an enhanced appreciation of control systems modeling and analysis. The reader is assumed to be familiar with calculus, physics, and a computer programming language (FORTRAN is used in this text). *The text is structured for use by those with limited familiarity with the mathematical tools and techniques used in control systems analysis and, indeed, in engineering.*

The perspective of the text is that of "classical" control theory, in which behavior in the time domain and the use of transfer functions are emphasized. It is a focused presentation which develops the reader's ability to interpret the physical significance of mathematical results in systems analysis. A novice in control theory can easily be overwhelmed by the abstract nature of the analytical techniques which are used in advanced work; this text carefully *prepares* the reader for more advanced treatments if he/she chooses to delve further into the subject. The last chapter presents the reader with a perspective of such advanced topics as sampled-data systems, state-space modeling, and modern control theory, which will serve as a smooth transition to texts devoted to these topics.

The pedagogical approach of this text distinguishes it from other systems textbooks. Most treatments of systems theory are so comprehensive that the fundamental concepts are lost among the sheer mass of material. The current work focuses on these fundamental concepts; *learning objectives* are given at the beginning of each chapter in order to guide the reader through the material. These objectives are "performance-oriented"; that is, the reader is expected to be able to *demonstrate* his/her achievement of each objective. There is a *review section* at the end of each chapter so that the reader may obtain a perspective of the topics treated in a given chapter. In addition, there are more than 240 *exercises* in the text with which the reader may sharpen his/her skills. Throughout the text, I have included numerous *solved examples* through which the textual material is related to actual contemporary engineering systems.

Chapter 1 presents a brief historical perspective of control theory and its applications in civilization, together with a review of the objectives and guidelines which should form a vital part of any control engineering effort. The next three chapters present the mathematical foundations of systems analysis: differential equations, Laplace transformations, and numerical approximation techniques. Differential equations form the mathematical models of physical systems, whereas Laplace transformations provide the basis of the transfer function approach in classical control theory and are used to simplify the mathematical difficulties in systems analysis. Chapter 4 reviews several numerical approximation techniques which are valuable in systems analysis (particularly if one uses higher-order mathematical models for systems). Such numerical techniques provide the basis for computer simulation software in systems engineering. This chapter on numerical techniques is an unusual inclusion in a systems textbook; most texts present the (very valuable) root-locus technique for graphically determining the roots of the characteristic equation for a system in the complex (Laplace) s-plane, but neglect to consider any other techniques for determining these roots. After studying differential equations and Laplace transformations, students often wonder how one goes about determining the roots of a higher-order equation; however, one must usually delay presentation of the root-locus method until some general systems have been analyzed in terms of stability and accuracy (the root-locus method is presented in Chapter 8 of this text). As a result, there is a period during which the student may question the utility of the differential equation and Laplace transformation models of a physical system. A brief presentation of numerical methods, such as that given in Chapter 4, allows one to emphasize the value of the root-locus method relative to the numerical techniques. Furthermore, such an inclusion introduces the reader to the use of the computer in systems analysis. In addition, many students do not complete a course in numerical techniques or, in fact, are never exposed to numerical techniques during their undergraduate studies. The inclusion of these techniques in a systems course is, therefore, one way in which to introduce such methods to the student in a very appropriate applications environment (systems engineering).

Chapter 5 presents transfer functions and the block diagram representation with which one may specify both system organization and functional relationships between system components. Chapter 6 then applies the mathematical tools developed earlier to analyze various types of systems. Chapters

7 and 8 deal with the two major goals of a control systems effort: accuracy (Chapter 7) and stability (Chapter 8). Control actions, steady-state error, the Routh–Hurwitz stability criterion, and the root-locus method are presented in these two chapters. Finally, Chapter 9 reviews several alternative modeling and analysis schemes, including frequency response, sampled-data systems, the z-transform, system compensation, nonlinearities, modern control theory, and state-state modeling. The appendices review such topics as complex variables, integration by parts, the method of partial fractions, and bond graphing.

This book has been developed through the use of prepublication versions of the textual material in undergraduate and graduate systems courses at Northeastern University. As a result, I firmly believe that the published version is a learning tool of significant value for engineering students. I am grateful to the many students who have provided feedback in my classes during the creation of this text.

I would like to thank several people who have aided immeasurably in the development of this text. First, Dean Harold Lurie of the College of Engineering at Northeastern University and Professor David Freeman, formerly Chairperson of the Industrial Engineering and Information Systems Department at Northeastern University, provided the initial support of this project, for which I am grateful. I also thank Professors Ronald Mourant (current Chairperson of the Department of Industrial Engineering and Information Systems at Northeastern University) and Wilfred Rule for their continued encouragement. In addition, I am grateful to Professors Richard Carter and Stewart Hoover for providing me with the opportunity to teach the undergraduate and graduate courses in control theory which are offered by our department. Mrs. Eleanor Lubin, Director of the Northeastern University Custom Book Program, provided the mechanism through which an earlier test version of this text was used in courses at Northeastern University; I thank her for her support. The efforts of Michael Gunderloy, a student in one of my systems classes who volunteered to proofread the manuscript, are also very much appreciated.

Any textbook author is very dependent on the support he/she receives from the publisher. I am very grateful for the guidance and enthusiastic sponsorship which I have received from Prentice-Hall. A totally professional staff has ensured that the production of this text meets the highest standards. In particular, I thank Mr. Matthew I. Fox, Editor-in-Chief and Assistant Vice President of Prentice-Hall's College Book Division, for his unwavering support.

Dr. David M. Pepper of Hughes Research Laboratories in Malibu, California, suggested specific examples of feedback control applied to research efforts; I am grateful for his interest and suggestions.

I have adopted a stylistic aspect of the excellent book on advanced mathematical methods by Bender and Orszag (1978): the inclusion of quotes taken from the work of Sir Arthur Conan Doyle to illuminate textual material. I, too, am a fan of the Sherlock Holmes stories. I thank Messieurs Bender, Orszag, and (of course) Doyle.

Professor William Crochetiere, Chairperson of the Department of Engineering Design at Tufts University, continues to guide me through the world of systems modeling, analysis, and design with his broad expertise in both theory and applications. I happily take this opportunity to thank him for his guidance and patience.

An author of a textbook—and the book itself—are particularly dependent upon the professional ability and personal enthusiasm of the production editor. I have been most fortunate to work with an editor who has been consistently supportive and totally professional: Mr. David Ershun of Total Concept Associates in Brattleboro, Vermont.

Finally, I thank my wife and colleague, Margaret, for her love, understanding, and assistance in this work (particularly in the development of the computer programs), and my mother, Eleanor, for a lifetime of encouragement and support and for proofreading the galleys for this book.

I would like to conclude with a special request to the reader: Please write to me concerning any errors which may appear in the text. In addition, I will be most grateful for any suggestions which will ensure that the next edition of this text is an even more effective learning tool. All contributions which are adopted will be acknowledged in future editions.

Systems modeling and analysis must be mastered by professional engineers if they are to be truly effective within their chosen discipline. I trust that this text will aid students in the development of their modeling and analytical abilities. In addition, I trust that this text will convince the reader that systems modeling and analysis is indeed fun!

Gerard Voland
Boston, Massachusetts

CONTENTS

1

INTRODUCTION TO SYSTEMS ANALYSIS AND DESIGN

The world is full of obvious things which nobody by any chance ever observes.

Sherlock Holmes, The Hound of the Baskervilles
by Sir Arthur Conan Doyle

1.1 OBJECTIVES

Upon completion of this chapter, the reader should be able to:

- Identify the characteristics of control systems which are desirable.
- Explain the reasons for introducing control in systems designs.
- Distinguish between open-loop and closed-loop systems.
- Explain the use of feedback in control systems.
- Identify examples of systems in which feedback is used in control efforts.
- Define the two major goals of control applications.
- Explain the significance and the effect of large time lags and large sensitivity within a closed-loop feedback control system.
- State the factors that must be considered by the systems analyst/designer as he/she develops a systems model.

1.2 MODELING, ANALYSIS, DESIGN, AND CONTROL

Systems modeling, analysis, and design efforts seek to produce specific responses from the operation of the device, process, or other portion of the universe that forms the "system" under consideration. Control systems can be *manipulated* to produce the desired response or output. *Automatic* control systems do not require human or manual actions for proper control.

Why do we seek automatic control in systems design? The answers are numerous and varied: automatic control systems allow us to produce results (e.g., products, behavior) which are *more consistent* than would be obtained from manually controlled systems. Furthermore, automatic control allows us to free human beings from tasks that are monotonous and dangerous; that is, automatic control systems can increase the *safety* of plant operations and provide the opportunity for people to perform more interesting tasks. The *quality* of the system output is also enhanced through the introduction of automatic control. The operation of automatically controlled systems is generally *faster* and more *accurate* than that of manually controlled systems. We could generate several other reasons to justify the introduction of automatic control in the design of a system.

Why do we seek control in systems design? Consider the system shown in block diagram notation in Figure 1.1. A system or process has been designed, together with a control mechanism or "controller," to produce a desired response. The blocks represent components within the entire design between which are transmitted signals (indicated by lines) in the direction indicated by arrowheads. (More correctly, blocks represent the functional relationships that exist between the signals.) The design shown in Figure 1.1 is known as an *open-loop system*, in which there is no monitoring of the *actual* system response.

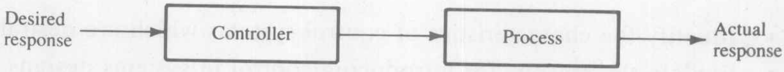

Figure 1.1 Open-loop system.

Figure 1.2 presents the general structure of a *closed-loop system*, in which the actual system response $C(t)$ is measured and then compared to the *desired* output. (Open-loop systems actually have no "loop.") The advantage of such an introduction of "feedback"—in which one monitors the performance of a system and uses the measured performance as information for the proper control or manipulation of the system—is that it allows the system to respond to any disturbances that act upon it from the environment. (The environment

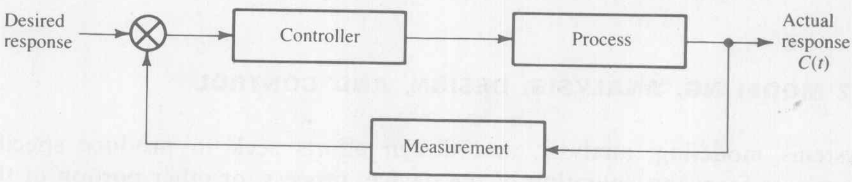

Figure 1.2 Closed-loop system.

is that portion of the universe which is not included in the definition of the system.) An open-loop system behaves properly only if all design specifications are maintained; any disturbance can produce incorrect results. A closed-loop system constantly monitors the output of the process under control; any disturbance that affects the system response will result in an appropriate modification of the system's operation so that the desired output should be obtained.

As an example of a (manually controlled) closed-loop system, consider the case of an automobile being steered by a driver. The driver acts as the monitoring device which determines if the automobile is traveling in the correct direction (e.g., on the highway); he/she also acts as the comparison device which determines if the actual response (direction of travel) is identical to the desired response (desired direction of travel). The driver, together with the steering mechanism of the automobile, also serves as the controller of the system, thereby introducing any changes in the direction of travel so that the desired response is achieved. If the road curves in a certain direction, this information is relayed to the driver through his/her vision and the needed modification in the direction of travel is introduced.

An open-loop system of this type could be achieved if the driver now closes his/her eyes; any disturbances in the operation of the system (e.g., a curve in the road) may now lead to disaster because of the lack of feedback within the system.

Feedback has been extremely important throughout the development of technology (Mayr, 1970). James Watt's centrifugal governor for steam engines (1788) maintained the speed of rotation of the engine by monitoring the actual speed and adjusting the steam inlet valve as needed; any disturbances (e.g., changes in the load or the steam pressure) did not produce incorrect behavior except during short intervals during which the needed adjustment was made. The success of Watt's design provided momentum to the use of feedback in systems design; however, feedback had been used by Ktesibios (in approximately 250 B.C.) in Alexandria in the design of water clocks (in which a float is used to measure the liquid level and thereby control the time-measuring mechanisms). Other float-valve regulators were designed in succeeding generations. A temperature regulator was designed by Cornelis Drebbel (1572–1633) of Alkmaar, Holland, for use in "thermostatic furnaces"; Drebbel's device is recognized as the first feedback system invented in modern Europe that was independent of ancient designs (see Mayr, 1970). Feedback control was also popularly applied to mill designs during the eighteenth and nineteenth centuries.

The interested reader is referred to Mayr's excellent treatment of the early history of feedback control for detailed information about these devices and others designed by creative early engineers (Mayr, 1970).

Bennett (1979) continues to trace the development of control engineering from 1800 through 1930 in his book. *Servomechanisms*, in which the input and output signals are mechanical positions, were developed. Navigational systems, electronic devices, hydraulic components, and so on, were designed to include some form of feedback control. In 1947, Norbert Wiener called the growing field of control and communications theory *cybernetics*, after the Greek word for "steersman" (Wiener, 1948).

Today, we find feedback control in innumerable devices and situations, including the use of robotics in manufacturing processes, thermostats in heating systems, speed control mechanisms in automobiles, automatic pilot devices in airplanes, the vapor pressure control used in gasoline gauges/nozzles, and myoelectrically controlled prosthetic devices. Closed-loop designs are also significant tools in research efforts [for examples of such applications of feedback, see Pepper (1982), Hori et al. (1983), O'Meara (1977), Swigert and Forward (1981), and Forward (1981)].

1.3 STABILITY AND ACCURACY

The two *major* goals of control applications are to produce system behavior which is *stable*, together with a system response which is *accurate* (i.e., identical to the desired system response). These two goals of stability and accuracy are often in conflict, forcing the systems designer to accept a compromise between a response that is accurate and system behavior that is stable.

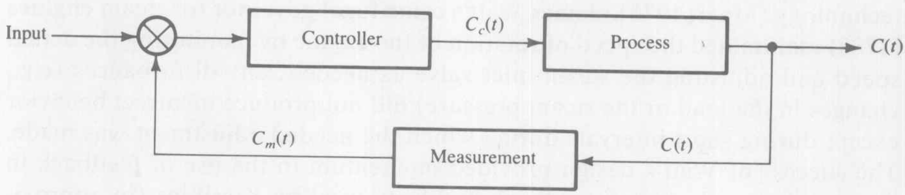

Figure 1.3 Example of a control system.

Instability is the result of *time lags* within the closed loop of the system coupled with relatively large *sensitivity.* (Sensitivity is the amount of *corrective effort per unit error* that is produced within a system in response to a deviation of the actual system output from the desired output.) Figure 1.3 presents a simple closed-loop control system in block diagram form; there is an output signal from each of the components within the loop. Each component requires a finite time interval to generate a response (output signal) to its input signal from the preceding component in the loop. As a result, each component (controller, process, measurement device) produces an *incorrect* output signal—relative to the behavior of the system—during such a time interval. Figure 1.4 illustrates the effect of these time lags or delays on the control of the system output $C(t)$. If the sensitivity of the controller (e.g., the magnitude of its

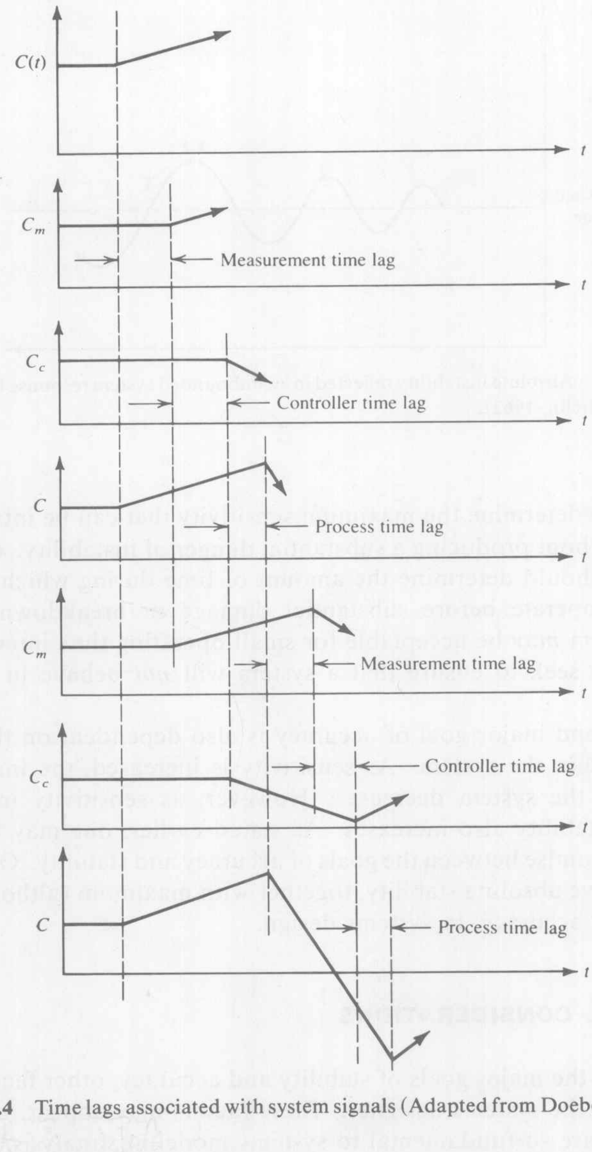

Figure 1.4 Time lags associated with system signals (Adapted from Doebelin, 1962).

output signal) is large *relative* to the time lags within the system, ***absolute instability*** may occur (Figure 1.5) in which the system response $C(t)$ increases without bound until (for real systems) the system shatters or components are worn beyond use (see Doebelin, 1962).

One attempts to minimize the time lags contained within a system; however, finite delays are inevitable within real systems in which energy (in the form of signals) is transmitted through the loop. As a result, the systems

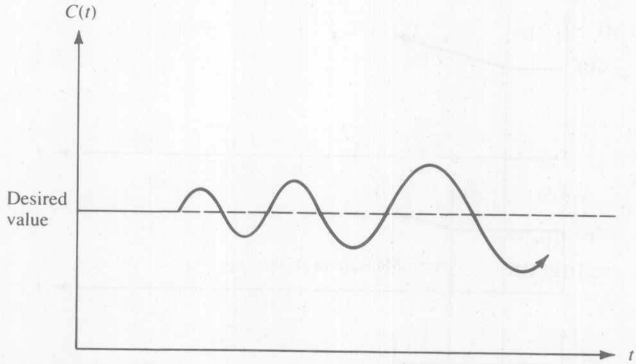

Figure 1.5 Absolute instability reflected in an unbounded system response (Adapted from Doebelin, 1962).

designer must determine the maximum sensitivity that can be introduced into the system without producing a substantial danger of instability. (In addition, the designer should determine the amount of time during which an unstable system may operate before substantial damage or breakdown occurs; an unstable system *may* be acceptable for small operating time intervals. In our treatment, we seek to ensure that a system will *not* behave in an unstable manner.)

The second major goal of accuracy is also dependent on the degree of sensitivity within the system. As sensitivity is increased, the inaccuracy (or error) within the system decreases. However, as sensitivity increases, the danger of instability also increases. As stated earlier, one may be forced to select a compromise between the goals of accuracy and stability. Our treatment seeks to achieve absolute stability, together with maximum (although perhaps not complete) accuracy, in systems design.

1.4 GENERAL CONSIDERATIONS

In addition to the major goals of stability and accuracy, other factors must be considered by the systems designer. These factors may appear to be obvious because they are so fundamental to systems modeling, analysis, design, and simulation; however, one can lose his/her perspective of the problem, the extent of analysis required, and the analytical approach that should be used if he/she fails to consider these additional factors.

Systems modeling (and computer simulation) allows one to experiment with different system designs. The general goal of a modeling effort is to optimize system performance for a given set of realistic operating parameters or for a range of such parameters. The objectives associated with a systems