Sanctioned for Use in the Royal Navy

NAUTICAL TABLES

DESIGNED FOR

THE USE OF BRITISH SEAMEN

BY THE

REV JAMES INMAN, D.D.

RE-EDITED AND ADAPTED TO MODERN NEEDS BY

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CHAPLAIN AND NAVAL INSTRUCTOR, R.N.

A SECOND EDITION OF THE ABOVE

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BY THE LORDS COMMISSIONERS OF THE ADMIRALTY
FOR USE IN THE ROYAL NAVY
FORMERLY EXAMINER IN NAVIGATION AND NAUTICAL ASTRONOMY
AT THE ROYAL NAVAL COLLEGE, GREENWICH, AND TO
H.M. BOARD OF EDUCATION

1948

REVISER'S PREFACE TO SECOND EDITION

INMAN'S NAUTICAL TABLES, first published in 1821, were in the year 1910 thoroughly revised and brought up to modern requirements by the Rev. W. Hall, Chaplain and Naval Instructor, R.N., in whose premature death upon active service a short time since the Naval Service has suffered a severe loss. So completely was this revision carried out that no material changes seem now to be either necessary or desirable, and the principal alteration in the present edition consists in a slight re-grouping of the tables, so that tables similar to one another in character may be as far as possible placed in the same part of the book.

The additions to the tables consist chiefly of two, both introduced in accordance with suggestions in notes left by Mr Hall:—

(*a*) A table giving the principal corrections of the altitude of the Moon, similar to the tables given in the 1910 edition for ***Sun and Star***. The importance of a lunar altitude taken in the daytime, which, when combined with a Sun altitude, affords an opportunity for a complete "fix" by simultaneous observations, is now very generally recognised, and it is proper, therefore, that the correction of the Moon's altitude, as well as that of the Sun, should be effected in the most short and simple manner which circumstances permit.

(*b*) Navigating officers are continually called upon to construct small sections of a Mercator Chart, involving the calculation of the actual length in inches of each degree of latitude involved.

The table giving *the actual length of the several degrees* is intended to save unnecessary labour of computation.

The tables of approximate values of elements of heavenly bodies at given epochs, which require revision from time to time, have been carefully corrected for the latest dates available.

In the "Explanation of the Tables" particular attention has been directed to principles of construction upon which the tables are based. This is especially the case with reference to tables which, like those relating to the Ex-Meridian observation, are peculiar to the Inman collection. The process employed in the tables mentioned is an original one, deduced by the late Mr Hall from the standard formula employed in the Royal Navy, and is believed to offer one of the simplest and most accurate methods of dealing with this important problem in existence. In accordance with suggestions from highly competent authorities as to the practical utility of these tables, their limits have in the present edition been largely extended.

The "Note on Lunars." Within the last few years a simple but excellent method of clearing the distance has been introduced into the *Abridged Nautical Almanac*. In the Note upon the subject included in the present volume advantage is taken of the fact that the clearing of the lunar distance, and the reduction of the altitude of the pole star to latitude of place, are in substance examples of one and the same problem. While, therefore, the general features of the process in the Note are similar to those of the *Almanac* method, the adaptation of Table II., given in the *Almanac* for the pole star, to the requirements of the allied problem of clearing the lunar distance, materially shortens and simplifies the necessary calculations.

Before bringing these remarks to a close, the Reviser would like to express his sense of obligation to Chief Naval Instructor S. F. Card, of the Royal Naval College, Greenwich, for valuable suggestions in connection with the extension and arrangement of certain of the Tables.

H. B. GOODWIN.

1918.

EXPLANATION OF THE TABLES

Page 1. **Divisions of Compass Card.**—The circumference of the compass card is divided into 360°, beginning from north, the angle increasing clockwise. The card is also divided into thirty-two points, each containing $11\frac{1}{4}°$. The names by which these points, and their subdivisions, are known are shown in the table, together with the equivalent values in degrees, correct to $\frac{1}{4}'$.

Pages 2 *and* 3. **Time Courses, &c.**—These supply at sight the distance run in any time at a given speed, or, conversely, the time in which a given distance will be covered at a particular speed.

Pages 4 *and* 5. **Position by Two Bearings of an Object and the Run between Observations.**—The table furnishes the results of the solution of a number of plane triangles, the object being to find the distance of a point from the ship by means of a pair of observations of the "Angle on the Bow." The following example will serve to illustrate the use of the table:—

Ex.—A light was sighted 29° on the port bow, and after the ship had run for 12 minutes at 12 knots it bore 62° on the port bow.

The angle from bow at first bearing (top of page) is 29°.
The change of bearing (left-hand index column) is 33°.
The factor from table (p. 4) = ·89.
The run is $\frac{1}{5}(12) = 2{\cdot}4$ miles.
Therefore the distance at second bearing is (2·4) (·89) = 2·1 miles.

Note.—This table is correct only if there is no cross-current or leeway.

Pages 6–11. The corrections in altitude for the various heavenly bodies are here given in a single term. In the case of the Moon (pages 10, 11) the table has been specially computed for the present edition.

Ex.—June 7th. Obs. Alt. Sun's L.L. 33° 50′. Height of eye 36 feet.

The correction falls in the middle of a block $\left|\begin{array}{c|c} 8{\cdot}8 & 8{\cdot}6 \\ 9{\cdot}0 & 8{\cdot}8 \end{array}\right|$, and may be taken as 8·8′. For June 7th the small table at foot gives the correction −0′·2, so that 8′·6 has to be added to the altitude observed.

Page 14. **Dip and Distance of the Sea Horizon.**—The *theoretical* Angle of Dip, in minutes, for a height of h feet is

$$\text{Dip}\ (') = 1{\cdot}063\ \sqrt{h},$$

and, in theory, the distance of the sea-horizon, in nautical miles, is the same.

The effect of atmospheric refraction is to reduce the dip, and to increase the distance, by about 8 per cent. of the theoretical value.

The formulæ employed in these tables are:

$$\text{Dip} = {\cdot}98\ \sqrt{h}. \qquad \text{Distance} = 1{\cdot}15\ \sqrt{h}.$$

The distance of the sea-horizon is the distance at which an object at sea-level first becomes visible, and this table helps to find approximately the distance at which a light is sighted.

Thus, a light 120 feet above sea-level could just be seen at a distance of 12·6 miles by an eye on the sea surface. But at a height of 50 feet from water-line the distance of visibility would be increased by the observer's range of vision, namely, 8·1 miles, so that the light would be 20·7 miles distant.

Mean Refraction.—The effect of refraction is to increase the altitude of a heavenly body, and the table gives the amount of arc to be subtracted from the apparent altitude in order to allow for this effect. The table has been calculated for average atmospheric conditions of pressure and temperature, as represented by 1015 millibars, =29·94 inches, for height of barometer, and by 50° Fahrenheit for height of thermometer.

It may be well to draw attention here to the fact that at low altitudes the amount of refraction is always uncertain.

The question of Abnormal Refraction has been the subject of Admiralty investigation and in the result the table hitherto appearing on this page is deleted. This investigation has shown that the allowance made for refraction in the standard dip tables is the correct average value, that variations on either side are equally likely, and that the chance that the error exceeds 2′ is considered to be about one in two hundred.

Page 17. **Pole Star Azimuths.**—By the use of the True Azimuth here tabulated for the Argument "Sidereal Time" the pole star becomes available for determination of compass error whenever visible at a suitable altitude—that is, whenever the observer's latitude is not too great.

Page 18. **Correction of Moon's Meridian Passage.**—The Moon crosses the meridian about 48 minutes later on an average day by day. The actual amount of retardation for a particular day is found by inspection of the Times of Transit in the *Almanac*. The difference of Transit on successive days is equivalent to a difference of 360° of longitude. If the retardation for the day is 60 minutes, and the longitude 90° W., the Moon will transit the meridian of the place later than at Greenwich by $\frac{90}{360} \times 60$ $=15$ minutes. Had the longitude been 90° E., the passage would have been 15 minutes earlier in local time than at Greenwich. The table gives the quantity to be added to the *Almanac* time for West, but to be subtracted for East longitude. The Ship Time of transit being thus found, the Greenwich Date is found by applying the longitude in time.

Ex.—Find the Greenwich Date of the Moon's Meridian Passage on 18th April 1918 in longitude 84° W., having given from the *Almanac*—

		h.	m.
Meridian Passage	18th April	6	41
,, ,,	19th ,,	7	26

The difference in time of transit for 24 hours is 45^m. The correction from table for change in time of passage 45^m., longitude 84° W., is $+10^m \cdot 5$. Then—

	h.	m.
Transit at Greenwich	6	41
Correction		10·5 +
Longitude in Time	5	36 +
Greenwich Date	12	27·5

Page 19. **Augmentation of the Moon's Horizontal Semidiameter.**—The Moon being distant from the earth about sixty earth-radii, is nearer when in the zenith than in the horizon by the distance of the earth's radius. It follows that the semidiameter in the zenith subtends a greater angle than when in the horizon, and for any given altitude has a value between these extreme limits. The quantity tabulated in the *Almanac* is the horizontal value, and the correction necessary is taken from the table.

Page 19. **Reduction of Semidiameter on account of Refraction.**—This small correction results from the fact that the upper and lower limbs are unequally affected by refraction. The correction is only appreciable at low altitudes, and in practice need not be taken into account in ordinary observations at sea.

Page 19. **Reduction of Horizontal Parallax, and Latitude of Place.**—Parallax of the Moon is defined as the angle subtended at the Moon by the observer's earth-radius. The figure of the earth is not exactly spherical, and the parallax given in the *Almanac* is calculated for the earth's equatorial radius. The correction in the table reduces the value to that proper for the latitude of the observer.

Page 19. **Reduction of Latitude.**—On account of the spheroidal figure of the earth, the observer's earth-radius is not in general strictly perpendicular to his horizon. Latitude calculated from observations taken from the " geographical " horizon must therefore be reduced by the amount taken from the table to give the " geocentric " latitude. See also the table, p. 489.

Page 20. **Moon's Parallax-in-Altitude.**—If from the Moon the angle subtended by the earth's semi-diameter could be measured, it would be the Moon's Parallax, previously defined as the angle subtended at the Moon's centre by the observer's earth-radius. When the Moon is on the horizon the earth-radius is at right angles to the line of sight, and the angle is a maximum (called Horizontal Parallax). When the Moon is in the zenith, the earth-radius lies along the line of sight, and the angle vanishes. At intermediate altitudes the angle (called Parallax-in-Altitude) is calculated from the formula

$$\text{Par.-in-Alt.} = (\text{Hor. Par.}) \times (\cos \text{Alt.}),$$

and is always additive to apparent altitude.

This table is printed on the same principle as the " Total Correction in Altitude " of Sun and star. The numbers found in the table differ by ·2′ throughout, and the value is estimated correct to ·1′ by interpolation at sight for the altitude and the whole minute of parallax. Then the odd seconds of parallax are allowed for by the aid of the small table on the right.

In practice the correction for the Moon's parallax is treated in a single correction, in combination with those for semidiameter and refraction by means of the special table introduced at page 10, as shown in the example worked in an earlier part of the " Explanation."

Page 25. **Planet's Parallax-in-Altitude.**—This is similar in character to the table for the Moon. In practice, however, it is hardly ever of sufficient importance to be brought into account, and the correction of a planet's altitude may be effected by means of the table for star's altitudes to be found on page 8.

Page 26. **Traverse Table.**—The Traverse Table furnishes the results of the solution of a large number of plane right-angled triangles, its primary object being for use in connection with two important formulæ constantly employed in navigation, namely,

$$\text{True Diff. Lat.} = \text{Distance} \times \text{Cos Course.}$$
$$\text{Departure} = \text{Distance} \times \text{Sin Course.}$$

With Distance from one mile to 600 miles as one Argument, and Course for each whole degree from 1° to 89° as a second Argument, the table supplies at sight the values for the corresponding Departure and Difference of Latitude.

Thus for a Distance of 363 miles sailed on a Course N. 37° W. we have

$$\text{Diff. Lat.} = 363' \cos 37° = 289'{\cdot}9.$$
$$\text{Departure} = 363' \sin 37° = 218'{\cdot}5.$$

The use of this table is not restricted to navigation, since, as is obvious from the principle of its construction, it is obvious that it may be employed for right-angled triangles in general whenever an approximate solution only is required.

Page 104, &c. **Conversion of Departure into d. Longitude.**—After the use of the Traverse Table to resolve the run into d. LAT. and DEP., it is necessary to convert the DEP. into d. LONG. For this purpose the table is entered at the top with the Middle Latitude, that is, with the half-sum of the LAT. "*from*," and the LAT. "*in*." Then the d. LONG. for any number of miles of DEP. from 1 to 100 is taken out.

When crossing the Equator, or when very near to the Equator, DEP. = d. LONG.

Ex.—From LAT. 34° 10′ S. to LAT. 30° 50′ S., departure was 172′.

Mid-LAT. = 32½° and for	100	we have	118·5	d. Long.
,,	72	,,	85·4	,,
so for	172	,,	204	,,

Since the formula is—

$$\text{d. Long.} = \text{Dep.} \times \text{Sec (Mid-Lat.)},$$

it follows that the table, when read from the top, is a multiplication table of numbers by Secants. Thus-

$$69 \sec 38° = 87{\cdot}6.$$

And reading from the bottom it is a multiplication table of Cosecants. Thus—

$$29 \operatorname{cosec} 54° = 35{\cdot}8.$$

Page 114. **Correction of Middle Latitude.**—The preceding table, " Dep. into d. Long.," is absolutely accurate only when the departure in question is taken wholly on one parallel of latitude. For all practical purposes no sensible error can arise from assuming it to be true for a day's run of a fast ship. The correction here given renders it mathematically accurate.

Page 115. **The length in inches of the several degrees on a Mercator Chart.**—In this table is supplied in a convenient form, the length of each degree of a Mercator Chart, to 70° of latitude, upon a scale of 1 inch to the degree of longitude.

The length in question is based upon the proportion

$$x = \frac{\text{Difference of Meridional Parts}}{60} \text{ 1 inch.}$$

Ex.—Required the length of the degree from Lat. 36° to Lat. 37°. Mer. Parts (table, p. 116).

Lat. 37°	2392·63	$x = \frac{74{\cdot}64}{60} = 1{\cdot}244$ inches.
„ 36°	2317·99	
	74·64	

The length for any other scale of longitude can easily be derived from the table by ordinary proportion.

Page 116, &c. **Meridional (or Mercatorial) Parts.**—A Mercator's Chart is not, strictly speaking, a projection, but a map on which the ship's track, cutting each meridian in succession at a constant angle, is a straight line. To secure this advantage the meridians are drawn as parallel straight lines, and the chart-length of 1′ of longitude is constant all over the chart. It follows that the chart-length of 1′ of latitude must vary with the latitude, so as to satisfy the fundamental relation—

$$\text{Chart-length of } 1' \text{ Lat.} = (\text{chart-length of } 1' \text{ Long.}) \times (\sec \text{Lat.}).$$

Using the notation of the calculus, let dl be an element of latitude in latitude l, and let the chart-length of 1′ of longitude be the unit, then—

$$\text{chart-length of } dl = dl \,.\, \sec l,$$

and the distance from the equator to the parallel of latitude l will be—

$$\text{Mer. parts for Lat. } l = \int \sec l \,.\, dl = \text{gd}^{-1} l = \log_e \cot (45° - \tfrac{1}{2}l).$$

Using ordinary logarithms, and considering the earth as a true sphere, this gives as the formula for computation—

$$\text{Mer. parts for } l = 2{\cdot}302585 \times 3437{\cdot}747 \times \log. \cot \tfrac{1}{2} \text{ co-lat.}$$

But if it be desired to take account of the spheroidal shape of the earth, then the table must be entered not with the ordinary " geographical " latitude, but with that latitude reduced to " geocentric " latitude by means of the table on p. 19, or, more accurately, by p. 489.

Ex.—Calculate the Meridional parts for latitude 60°.

Log. cot ½ co-lat 60° = Log. cot 15°	·571948
Log. ·571948	$\bar{1}$·757356
Log. 2·3026	·362218
Log. 3437·75	3·536273
Log.	3·655847
Nat. No.	4527·4

Page 125. **Acceleration.**—Owing to the earth's yearly revolution round the Sun, and the consequent apparent orbit described by the Sun in the heavens, the right ascension of the Mean Sun increases uniformly at the rate of 3m. 56·55s. per mean solar day. At Mean Noon on each day the hour angle of the First Point of Aries, which is known as the Sidereal Time, corresponds with the right ascension of the Mean Sun. At this instant of Mean Noon, therefore, we may obtain the right ascension of Mean Sun from the column of the *Almanac* headed "Sidereal Time." The right ascension for any other epoch may be deduced by adding quantities taken from this table. The Argument Mean Time proceeds by intervals of ten minutes, the adjustment for odd minutes and seconds being effected by means of the table to the extreme right of the page.

Ex.—Required R.A. Mean Sun for Greenwich Date 11h. 46m. 50s., 16th April 1918.

	h.	m.	s.
Sidereal Time at Mean Noon, 16th April	1	35	16·7
Correction for 11h. 40m. 0s.		1	55·1
,, ,, 6m. 50s.			1·1
R.A. Mean Sun	1	37	12·9

A second use of the table is to convert an interval of Time, expressed in Mean Time, into Sidereal.

Retardation.—This table may be regarded as the converse of the preceding one, being intended for the conversion of a portion of Sidereal Time into Mean Time, the quantities taken out being subtractive instead of additive.

Pages 127, &c. **Longitude Correction.**—This table is an enlargement of a very useful table printed in former editions by permission of its author, the late Mr. A. C. Johnson, R.N. It is now arranged so that the quantities given differ uniformly by about 4 per cent. Mental interpolation will give a result correct to 1 per cent., which is as close as is required. Thus for Latitude 49° and Azimuth 39° we can read 1·9 as the factor *F*.

It is well recognised that the results of all "Sights" taken at sea must be interpreted by position lines. The calculations of a "Chronometer Sight" or "Ex-Meridian Sight" give as their final result nothing more than the information that the ship *may* be at a certain point called a position point, and *must* be on a line drawn through the position point at right angles to the bearing of the body observed.

If a chronometer sight be worked with a dead reckoning latitude, the position point has that latitude and the longitude given by the calculations, and the position line can be drawn on the chart with the certainty that the ship is *somewhere* on it if the sight be correct. But it is not known exactly where on the position line this is, unless the latitude be exactly known.

This table gives a correction to the longitude for any error in the latitude used in the calculations.

Ex.—Suppose that ship's position, as determined by Chronometer **Sights taken when the Sun's Azimuth** was S. 39° E., was 49° 6′ N., 25° 11′ W. when run up to noon. Suppose that at noon the latitude was found to be 49° 10′ N.

The Chronometer Sights must have been worked with a latitude 4′ too much to the southward. In other words, the correction to D. R. latitude is 4′ N.

The table tells us that under these conditions there is a correction of 1′·9 of longitude due to each mile of error in latitude. The longitude is therefore wrong by 4×1′·9=7′·6.

To name this correction. The position line (perpendicular to the bearing S. 39° E.) runs N. 51° E. or S. 51° W., whichever we choose to call it. But it is known that the latitude correction is N., so the longitude correction must be E. Mr. Johnson suggested the easy diagrammatic method of naming the correction shown here. Write the bearing (S.E.) with its opposite (N.W.) underneath it. The position line is named by one of the diagonals of the figure. From what we know of the latitude correction, it must be the N.E. diagonal.

S	E
N	W

A convenient form of work, is as follows :—

Dead Reckoning from Chron. Sights	49° 6′ N.,	25° 11′ W.	Azimuth.
Correct latitude by Noon Sights	49° 10′ N.		S 39° E
Correction to latitude	4′ N.		N W
longitude	=1·9×4	7′·6 E.	Factor *F* 1·9
Noon Position, corrected	49° 10′ N.,	25° 3′·4 W.	

No apology is needed for treating this question at length, for on its proper understanding depends the whole theory and practice of navigation. For the further use of the Factor *F* in combining two chronometer sights, reference must be made to textbooks on navigation.

Page 130. **Latitude Correction.**—This original table has been computed as a complement to Mr. Johnson's. If an Ex-Meridian Sight be worked with a longitude afterwards found to be in error, then the table shows what is the correction to latitude due to each minute of error in longitude.

Suppose that an Ex-Meridian Sight, taken on a bearing S. 14° E., gave a latitude of 35° N. when worked with a longitude of 31° W. Suppose, further, that it was afterwards found that the longitude should have been 31° 10′ W.

		Azimuth.
Correction to longitude	10′ W.	S 14° E
,, latitude	=·20′×10	
	=2′ S.	
Thus correct latitude at Sights was	34° 58′ N.	N W
		Factor *f* ·20

Page 133. **Hour Angle and Altitude on the Prime Vertical.**—From what has been said above, it is clear that if a body is observed upon the Prime Vertical—that is, due E. or due W.—the position-line runs due N. and S., or, in other words, the longitude found is correct. When possible, therefore, the altitude should be observed when the body is near this favourable position. The table supplies the hour angle and altitude which enable the observer to secure this advantageous condition.

Suppose the latitude to be 40° N. and the Sun's declination 12° N. Then the table gives H.A.=5h. and Alt.=19°, about. Thus sights for longitude taken at 7 A.M. or 5 P.M. will be independent of error in latitude, and if an altitude of about 19° be set on the sextant, that will ensure the proper moment being caught.

Of course if the declination be of contrary name to latitude the body never crosses the prime vertical. If declination be of the same name but greater than latitude, the table gives the moment of *maximum* Azimuth which is then the best available. Thus in latitude 11° N. for an observation of Castor (32° N.) for longitude, the altitude to be set on the sextant is about 21°.

Page 138. **Amplitudes.**—At rising and setting the difference between 6h. and the hour angle is called Time-Amplitude ; and that between 90° and the Azimuth is called Bearing-Amplitude. It is evident that a body of N. declination must rise N. of E., and set N. of W. ; and also that when declination is of the same name as latitude the hour angles of rising and setting are both greater than 6h. There can never, then, be any doubt about which way to name the Amplitudes.

Ex.—In LAT. 50° N., when Sun's DEC. is 19° N.

Time of Sunrise	6h. − 1h. 37m., = 4h. 23m.	of Sunset	6h. + 1h. 37m. = 7h. 37m.
Bearing at Sunrise	= 30°·4 N. of E., = N. 59°·6 E.	at Sunset	= 30°·4 N. of W. = N. 59°·6 W.

Page 143. **Correction to Observed Amplitude.**—The preceding table is calculated for the moment when the sun's or body's *true* altitude = 0°. The *apparent* altitude of sun or star is then about 34′, owing to the effect of parallax and refraction. The *apparent* altitude of the moon is about 20′ *below* the horizon.

If, then, the sun or star be observed when its centre *appears* to be on the horizon, an error is introduced which can be corrected by this table. The correction is (for sun or star) always additive to the observed Azimuth reckoned from the elevated pole. For the moon, $\frac{2}{3}$ of the correction is to be applied in the opposite way.

Page 144. **Ex Meridian Tables.**—These tables offer a short and handy method of obtaining the Reduction to the Meridian.

If in the ordinary "triangle of position" P X Z, p, c, z denote the polar distance, co-latitude and zenith distance which form the three sides, and h the hour angle, we have by a fundamental formula

$$\text{vers } z = \text{vers } (p - c) + \sin p \sin c \text{ vers } h.$$

Let x denote the First, or principal, Correction, which subtracted from z, the observed zenith distance, gives $(p - c)$ the meridian zenith distance, and the expression becomes

$$\text{vers } z = \text{vers } (z - x) + \sin p \sin c \text{ vers } h.$$

Whence may be deduced the relation

$$\sin \frac{x}{2} = \frac{\sin p \sin c}{\sin z} \text{ hav } h.$$

Table No. 1 gives the logarithm of $\frac{\sin p \sin c}{\sin z}$ for each of three cases that may occur, viz. (1) Latitude and Declination of Contrary Name, (2) Latitude and Declination of Same Name, and (3) Observations below Pole.

Ex.—LAT. and DEC. (Smaller). Contrary Name. LAT. 50° N., DEC. 8°. Here $p = 98°$, $c = 40°$, $(p - c) = z = 58°$.

L. Sin	98°	9·99575
L. Sin	40°	9·80807
L. Cosec	58°	·07158
Tab. Log.		9·87540

Table No. 2 supplies the Tabular Logarithm of the Haversine of the Hour Angle. Thus if H h = 25m. 42s., L Hav 25m. 42s., which is 7·49699, is given to three places of decimals as 7·497.

Table No. 3 gives the values corresponding to Sin $\frac{x}{2}$ in minutes and tenths of a minute. To avoid multiplication by two, the Tabular Logarithm for a given angle is that of the Sine of one half the angle.

Thus for 0° 44′ as Argument we have the Tabular Logarithm 7·806, which is L Sin 0° 22′.

The following example illustrates the method of working:—

Ex.—LAT. 47° 20′ N., by Dead Reckoning. Sun's DEC., 14° 11′ N. Ship A.T. 30m. 18s.

Table No. 1 gives	Tab. Log. C	·080
„ „ 2 „	Tab. Log. H	7·640
(Sum)		7·720

From Table No. 3 we have the First Correction 36′·1, additive to the altitude.

Construction of Table No. 4, the "Second Correction" Table.—If x denote the value of the First Correction, obtained as above by means of Tables (1), (2), (3), and y the more exact value of the Reduction when a Second Correction has been applied, it may be shown that

$$\text{Second Correction} = y - x = -\frac{x^2}{2} \tan \text{alt.} \sin 1'.$$

To obtain the value of a term, therefore, we require as Arguments the First Correction x and the altitude.

The Rule will be—

Add the logarithm of one half the square of the First Correction to that of the tangent of the altitude and of Sin 1′. The sum of the three is the logarithm of the Second Correction in minutes.

Ex.—Altitude = 50°, First Correction = 80′.

Log. 3200′ (½ 6400)	3·50515
Log. Tan 50°	·07619
L. Sin 1′	6·46373
(Sum)	10·04507
Second Correction	1′·1

In the example worked above we have the First Correction 36′·1, altitude 57°, we have −·3′ as the Second Correction. The net value of the Reduction, therefore, is 36′·1 − ·3′ = 35′·8.

Pages 158, &c. **Astronomical Tables.**—These tables have been carefully corrected and brought up to date. Although only under exceptional circumstances to be used for finding positions, they are convenient for occasional use in such problems as finding Azimuth, Time of Sunrise, &c.

The list of stars arranged according to right ascension indicates the order in which the several bodies arrive at the meridian, while that which follows the values of declination shows what stars will be above the horizon in any latitude. The latter table, viz. the list of principal stars arranged in order of declination, should be particularly useful in discovering the identity of an unknown star from the approximate values of its altitude and azimuth by means of Alt-Azimuth Tables.

Page 168, &c. **Mast Head Angles.**—The title indicates only one of the uses of the table. The angles given are those subtended by a base line (here called height) at various distances, the base line being perpendicular to the line joining its middle point to the point of sight. Two sets of tables are provided, one ranging to 2000 yards, and the other to 8000 yards. Finally, on page 171 will be found a table whereby a base line may be measured with a theodolite and a 10-foot pole.

Page 176. **Table of Positions.**—With reference to this table it should be observed that the navigator should, when possible, rely upon his charts and sailing directions rather than on these positions.

Page 200. **Logarithms of Numbers.**—The user of a book such as the present is supposed to understand the nature of a logarithm and the rules for the use of logarithms. The following remarks are intended simply to explain the special arrangement here adopted.

The numbers are in black-faced type, the logarithms are ordinary type. Numbers proceed from 0 up to 10800. For 5-figure work interpolation is necessary from 0 to 9999. Consecutive logarithms are differenced in the column "*D*," and proportional parts are given for each difference. But for 5-figure numbers lying between 10000 and 10800 the logs. are given in full, so as to save interpolation for large differences. Reference should be made to the *Note*, p. xxi.

Experienced computers are much divided with regard to some points in connection with the use of logarithms. The Reviser records here his personal opinion.

1. Use of Co-Logarithms.

$$\log. \frac{1}{39\cdot25} = \log. 1 - \log. 39\cdot25 = 0\cdot00000 - 1\cdot59384$$
$$= \bar{2}\cdot40616 = \text{co-log. } 39\cdot25.$$

Here the arithmetical complement of ·59384, namely ·40616, is the co-log. of 39·25, that is, the log. of $\frac{1}{39\cdot25}$. It can be read from the tables just as easily as the log. itself, proceeding *from left to right* and subtracting each digit from 9 until the last, which is taken from 10. As regards the characteristic preceding the decimal point, it is seen to be MINUS *the number of digits before the point* in the number.

$$\log. \frac{1}{\cdot003925} = 0 - (\bar{3}\cdot59384) = 2\cdot40616 = \text{co-log. } \cdot003925,$$

and it is seen that here the characteristic is PLUS *the number of noughts after the point* in the decimal.

It is worth while to use co-logs. so long as no interpolation is required.

Ex.—$N = \dfrac{57\cdot92 \times \cdot0173}{\cdot9491 \times 1\cdot0695}$	log. 57·92	1·76283
	log. ·0173	$\bar{2}$·23805
	co-log. ·9491	0·02269
	co-log. 1·0695	$\bar{1}$·97082
= ·98716	log. *N*	= $\bar{1}$·99439

2. Use of the Negative Characteristic ($\bar{2}$, $\bar{3}$, &c.).—It seems to be accepted as the best practice to write 9 for $\bar{1}$, 8 for $\bar{2}$, &c., whenever log. sines, cosines, &c., are involved in combination with numbers. In systematic calculation of results which are known beforehand to have negative characteristics, the positive numbers 9, 8, &c., should certainly be used.

Page 236. **Log. Sines of Small Angles.**—For very small angles the differences of log. sines and log. tangents are both large and variable, so that interpolation is inconvenient or impossible, and this table provides, to each 10", the functions needed.

In some astronomical formulæ, such as those, for instance, which determine the correction for refraction, the logarithm of Sine 1" is sometimes required. It should be noted that the sines of small angles practically varying with the angle, L Sin $1''$ $=$ L Sin $10''$ $-$ Log. 10 $= 5{\cdot}68557 - 1 = 4{\cdot}68557$.

Page 240. **Logarithmic Trigonometric Functions.**—In the arrangement of these tables the usual convention has been observed of increasing the characteristics of such functions as are less than unity by 10. By this means the use of negative characteristics is avoided.

A new departure has been made in separating the functions from one another. A considerable saving of time will accrue from the fact that only 18 pages are thus devoted to any one function, and as 10 degrees are visible at each opening, it is very easy to place the left-index finger on the desired part of the page without any search.

The direct functions, *sine, tangent, secant,* all read downwards ; the complementary functions, *cosine, cotangent, cosecant,* read upwards, and their proportional parts are subtractive.

The small tables of proportional parts (P.P.) read to tenths of $1'$ or $6''$ of arc, and give *at sight* all the accuracy desirable in ordinary calculations.

The *Note* on Interpolation (p. xxi) should be consulted.

Pages 294–394. **Logarithmic and Natural Haversines.**—The Haversine (half-versine) is a function whose utility has been too much ignored by writers on trigonometry. It simplifies the solution of triangles, both plane and spherical, in a remarkable manner, and eliminates all ambiguity from the results. It is defined by

$$\text{hav } A = \tfrac{1}{2} \text{ vers } A = \tfrac{1}{2}(1 - \cos A) = \sin^2 \tfrac{1}{2} A.$$

The saving of time and labour effected by the use of versines or haversines will easily be appreciated on reference to any of the text-books on Trigonometry or Nautical Astronomy used in the Royal Navy.

Attention may be here drawn to an improvement recently introduced in the arrangement of the Haversine Table, whereby the L Haversine and the Natural Haversine are shown in adjacent columns, the Natural Haversine being printed in heavier type. By this means the process of finding the third side in the solution of a spherical triangle, when two sides and the included angle are given, is greatly facilitated. As this particular case constantly occurs in determining position lines on the Marcq-Saint-Hilaire principle, now a standard method followed in our own and in foreign Navies, this arrangement of the Haversine Tables is of considerable importance. The simplification thus effected may be illustrated by the following practical example :—

Ex.—Solve the spherical triangle ABC, where $A = 102^\circ\ 20'$, $b = 50^\circ$, $c = 40^\circ\ 15'$.

A	$102^\circ\ 20'$	L. hav	9·78305		
b	$50^\circ\ 0'$	L. sin	9·88425		
c	$40^\circ\ 15'$	L. sin	9·81032		
		L. hav	9·47762	N. hav	·30035
$b \sim c$	$9^\circ\ 45'$			N. hav	·00722
a	$67^\circ\ 21'{\cdot}9$			N. hav	·30757

It will be noticed that when the value for L. Hav θ is once obtained, viz. 9·47762, that of N. Hav θ, ·30035, is found in the table by the side of it, the actual value of the auxiliary angle θ being immaterial. When the tables of logarithmic haversines were given separately it was necessary, having taken the value of θ, in this case

66° 28′, from the logarithmic table, to enter the table of natural values with this angle as argument, and thus arrive at the required natural function. A wholly unnecessary operation is thus avoided by combining the two functions, logarithmic and natural, in a single table.*

Page 400. **Half Log Haversines.**—Some apology may seem to be necessary for assigning so many pages of tabular matter to a table which is practically obtained by dividing the values given in the Log. Haversine Table by two. To justify the inclusion of this table, however, it is only necessary to cite an important problem of navigation, that of finding Hour Angle from the data, Latitude, Declination, and Zenith Distance.

This is a special case of the general problem of finding an angle of a spherical triangle, in which we have given three sides, and the typical formula is

$$\text{Hav } A = \sqrt{\text{Hav }(a+b-c)}\ \sqrt{\text{Hav }(a-b+c)}\ \text{Cosec } b \text{ Cosec } c.$$

If we were limited to the ordinary table of Logarithmic Haversines, division of the logarithm would be necessary in two cases. The table of half Haversines not only saves this unnecessary labour, but secures also greater accuracy in computation, for it was a matter of common experience that before the introduction of this table more errors were made in this process of division than in all the other parts of the operation together.

The example which follows, from Goodwin's *Plane and Spherical Trigonometry*,† may serve to illustrate the use of the table.‡

Ex.—In the spherical triangle *ABC*, $a=124°\ 10'$, $b=89°\ 0'\ 15''$, $c=108°\ 40'$ Find the angle *A*.

c	108° 40′ 0″	Log cosec c	·02347
b	89° 0′ 15″	Log cosec b	·00007
		½ L. hav $(a+\overline{c-b})$	4·97800
$c-b$	19° 39′ 45″	½ L. hav $(a-\overline{c-b})$	4·89801
a	124° 10′ 0″		
		(Sum) L. hav A	9·89955
$a+\overline{c-b}$	143° 49′ 45″		
$a-\overline{c-b}$	104° 30′ 15″	$A=125°\ 56'{\cdot}7.$	

Pages 430–483. **Natural Trigonometrical Functions.**—These are in every respect similar to the Logarithmic Functions, and are read in the same way.

Page 484. **Circular (or Radian) Measure.**—The mathematical Unit of Angle is that angle whose arc is equal to the radius of the circle on which the arc is measured. The table gives lengths of arc for radius unity, for whole degrees, minutes, and seconds to five places, with a supplementary sixth figure to strengthen the result.

Ex.—Find the Radian Measure of 83° 27′ 33″.

For	83°	1·44862,3
	27′	·00785,4
	33″	·00016,0
Radian Measure of 83° 27′ 33″		1·45664

Page 485, &c. Contains a variety of miscellaneous information.

* The combined form of table (the copyright of which is strictly reserved) first appeared in the *Requisite Tables*, by Mr Percy L. H. Davis, published by J. D. Potter in 1905, and is included in this volume by permission.

† *Plane and Spherical Trigonometry.* By H. B. Goodwin, M.A., Naval Instructor, Royal Navy. Longmans, Green & Co.

‡ In the *Admiralty Manual of Navigation* the use of the Haversine table is recommended for this problem, and the solution is as follows :—

	° ′ ″				
a	124 10 0	N. hav a	·78080		
b	89 0 15			Log cosec b	·00007
c	108 40 0			Log cosec c	·02347
$c-b$	19 39 45	N. hav $(c-b)$	·02915		
		Difference	·75165	Log difference	9·87602
	$A=125°\ 56'{\cdot}8.$			Sum (L. hav A)	9·89956

NOTE ON MATHEMATICAL TABLES AND INTERPOLATION.

QUANTITIES such as those tabulated in this book, namely, logarithms, trigonometrical ratios, etc., are, as a rule, incommensurable—that is, they cannot be exactly expressed by any finite number of figures. The final figure as printed is the nearer to the truth, and may be in excess or defect by half-a-unit. This is shown by the examples below.

Number.	Log. to 7 fig.	Log. to 5 fig.	Error in 5th fig.
2	·30103,00	·30103	·00
2·25	·35218,25	·35218	·25 *defect.*
2·369	·37456,51	·37457	·49 *excess.*
2·431	·38578,50	·38578	·50 *defect.*

Technically, we say that log. 2·369 is "forced over," but log. 2·431 is "forced under."

It is necessary to have correct ideas about the error which forced figures may introduce into results obtained by using 5-figure tables.

In *taking out a logarithm* or natural sine, etc., there *may* be an error of $\frac{1}{2}$ in the last place. For convenience this is written $\pm$, 5. If, then, two or more logs. are combined by addition or subtraction, the combined error may be anything between 0 and $\pm$1. Using the logs. given above:—

$2·431 \times 2·369$	·76035,01	·76035	·01 *defect.*
$2·431 \div 2·369$	·01121,99	·01121	·99 *defect.*

And if any number of logs. (say 4) be combined by sums or differences, there *may* be an error in the last place of *half* that number, *i.e.*, an error of 2 for a block of four logs. But the error may just as likely be 0, and its probable value for a set of four logs. is about 1.

In *taking out the number* corresponding to a given log., there are two sources of error, namely the $\pm$, 5 in the tabulated log. and the $\pm$, 5 in the calculated log. At the worst, there is 1 in the 5th place of logarithms to cause error in the result.

This corresponds accurately to an error of 1 in 43,430 or 23 in 1,000,000. Hence the percentage error inherent in 5-figure work is about ·0023 per cent. as a maximum.

The above remarks have been written to forestall any objection to the Reviser's carefully considered decision to reduce the logarithms in Inman's Tables to 5-figure quantities. It is a mathematical fact that 5-figure calculation is amply sufficient for the needs of the seaman in particular, and in general for all computers except those concerned with the most refined results of the astronomer or surveyor.

A few concrete examples will strengthen this statement. Take the case of determining Hour-Angle from a given Log. Haversine, say 9·69955 ±1, the ±1 allowing for errors:—

	h	m	s	
For 9·69954 we have	6	0	18	} or possible error ± ·3 sec.
9·69955 ,,	6	0	18·3	
9·69956 ,,	6	0	18·7	
At 4h. the possible error is reduced to				± ·2 sec.
,, 3h. ,, ,, ,,				± ·1 sec.

It is necessary to point out that there are certain cases where no number of figures can be expected to give satisfactory results. Take the case of finding an angle from its Log. Sine 9·99989,00.

> Using 5 figures, the angle may be anything between 88° 41′ and 88° 44′;
> Using 7 figures, the limits are 88° 42′ 38″ ± 1″.

This last result means that in order to find the angle within 1″ we must know its log. sine correct to 1 in 10 million.

Cases like this are called "ill-conditioned," because the smallest error in the data is magnified many times in the result. The computer must lay to heart the general rule that, where any choice is possible, he should choose "well-conditioned" cases; but if an "ill-conditioned" problem be forced upon him, he should distrust his results.

INTERPOLATION.

26°	L. Tan.	D.
0′	9·68818	
		+32
1	9·68850	
		32
2	9·68882	
		32
3	9·68914	

In technical language the index quantities in a Table such as the specimen are called "*Arguments*," and the tabulated quantities are "*Resultants*." The column headed "D" is called the "*Difference Column*," and contains the differences between successive Resultants, marked + when the Resultants increase numerically downwards.

In the specimen, then, we "enter" with an "Argument" 26° 1′, and "take out" a "Resultant" L. tan. 9·68850, noting that the "difference" is +32 for 1′. It happens that here the Resultants proceed by a uniform difference of +32.

Suppose that L. tan. 26° 2′·4 be required, it is got by "*Interpolation*" between the Resultants for 26° 2′ and 26° 3′, and the matter is one of simple proportion. Thus:—

> L. tan 26° 2′·0 = 9·68882 diff. + 32 for 1′.
> Add (·4) (32) = 12,8
>
> L. tan 26° 2′·4 = 9·68894,8, or 9·68895 to 5 figures.

Now, as explained above, L. tan 26° 2′ is really 9·68882 ±,5; so if we reduce to five figures by calling 12,8 = 13, we involve an error of anything from −,7 to +,3 as chance may determine.

It is a standing rule, therefore, that to secure the utmost accuracy when combining logs. the interpolation must be carried out to an extra place of decimals so that there shall remain no error except that due to forced figures in the tabulated logs.

A practical case is that of finding Hour-Angle for rating chronometers. Here the navigator should extend to a 6th place, separated from his block of 5 figures by a comma.