CAVITATION AND HYDRAULIC MACHINERY

EDITED BY

F. NUMACHI

PROCEEDINGS OF IAHR-SYMPOSIUM, SENDAI, JAPAN, 1962

LA CAVITATION ET LES MACHINES HYDRAULIQUES

PAR

F. NUMACHI

COMPTES RENDUS DE L'AIRH-SYMPOSIUM, SENDAI, JAPON, 1962
DU 3 AU 5 SEPTEMBRE, 1962

COMITÉ LOCAL D'ORGANISATION
SYMPOSIUM DE L'AIRH SUR LA CAVITATION ET
LES MACHINES HYDRAULIQUES, JAPON, 1962
C/O INSTITUT DE LA MÉCANIQUE DE GRANDE VITESSE
UNIVERSITÉ DE TÔHOKU, SENDAI, JAPON
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Preface

The IAHR-Symposium on Hydraulic Machinery and Cavitation was held in Sendai, Japan, from September 3 to 5, 1962, and the visiting tours to places of technical interest took place in Sendai and Tokyo from September 6 to 8.

Prof. James W. Daily, Chairman of Committee on Hydraulic Machinery, Equipment and Cavitation of IAHR, wrote us in 1959 asking if we had any intention to hold the next Symposium in Sendai and gave us various suggestions in his succeeding letters. The decision to hold the Symposium in Sendai was formally made in the Ninth Convention of IAHR, from September 4 to 7, 1961, in Dubrovnik.

In Japan, preparations were initiated in 1960 and the Local Organizing Committee was organized on July 3, 1961, for studying the plans prepared in advance [refer to § 2 as to the list of members].

The first Managing Council started its functions on August 14, 1961. Its members were composed of 12 persons elected out of the Local Organizing Committee and they deliberated on plans to be discussed afterward by the Local Organizing Committee [refer to § 3 as to the list of members].

Furthermore, the Local Subcommittees were established in the Local Organizing Committee on October 27, 1961 [which see § 4 as to their appellations and the lists of members]. The chiefs' duties of those Subcommittees were entrusted to professors of the Institute of High Speed Mechanics, Tōhoku University, for the convenience of keeping in contact with one another.

In opening this meeting, we received contributions from companies and academic societies in Japan. Chief contributors among them were the member companies of Japan Electric Machine Industry Association, the member companies of Japan Society of Industrial Machinery Manufacturers and those of the Federation of Electric Companies [refer to § 5 as to the list of those Companies and Societies]. We express our heartfelt gratitude to those companies and academic societies.

As to the announcement of the Symposium, the first to third Circulars were sent to the members of IAHR and others. The first Circular was shipped out on October 20, 1961, in which the invitation and stipulations for contribution of papers were made. The acceptance of papers was closed at the end of February, 1962.

Manuscripts of papers received were 45 in number, of which 28 were finally adopted. The reasons for this procedure were, first, the fixed three day period for the Technical Sessions and, secondly, allocation of time for each presentation of papers being determined to 35 minutes (15 minutes for delivery, 20 minutes for discussion) based upon Professor Daily's valuable suggestion. That made the number of presenters definitely 28. There is one thing which we would like to mention here for future benefit is that we received a few papers written in German in spite of our notification in the first Circular that the official languages of IAHR are limited to English or French and, though those papers were up to the standard we felt like adopting, we had to send them back to authors.

Review of papers was conducted not only by the members of Review Committee [§4-1] but also by proper researchers out of the members of Local Organizing Committee, that is, each paper was studied by three reviewers.

The Opening Ceremony and Technical Sessions were held at Denryoku Hall, Sendai, attended by 235 participants from 10 countries of the world [the list of their names and their photographs are shown in §6].

At the Opening Ceremony, the proxy of the President of Japan Science Council, the Honorary President of Local Organizing Committee [§2], the proxy of the Honorary Vice-President of the same Committee [§2], the President of Tōhoku University and the representatives of donating companies and academic societies were present and the opening addresses were delivered by Prof. Emerit. F. Numachi, Chairman of the Local Organizing Committee; Prof. James W. Daily, Chairman of the Committee on Hydraulic Machinery, Equipment and Cavitation, and Prof. Dr. M. Mutō, proxy of the President of Japan Science Council.

Ladies program was also drawn up: ladies were invited to tea ceremony, exhibition and schooling of flower arrangement and a trip to Matsushima, a scenic spot. Also at the intervals of Technical Sessions, a tea party and a banquet were given and Vice-President Prof. Dr. L. Escande and others made speeches.

Excursions were carried out as shown in §8 at the end of this copy. We are thankful to those companies and research institutes that received us on those occasions.

We express our appreciation to the cooperation of many organs and individuals in various fields with the members of Local Organizing Committee in facilitating the Symposium. Especially our thanks are due to Mr. S. Uchigasaki, Ex-President of Tōhoku Electric Company, Mr. T. Hori, one-time President and Mr. Y. Hirai, Ex-Vice-President of the same company for their efforts to materialize the above mentioned donations. It is to be acknowledged, too, that all the staff of the Institute of High Speed Mechanics, from professors, technicians in the attached factories to office workers, made efforts cooperatively to make the Symposium a success.

In conclusion, we believe with delight that our meeting gave us an occasion not only to contribute to advancing our knowledge, but also to cementing valuable friendship among people with common interests and problems.

Sendai, August 1963

F. Numachi

Préface

Le Symposium de l'AIRH sur la Cavitation et les Machines hydrauliques s'est tenu à Sendai (Japon) du 3 au 5 septembre 1962, et du 6 au 8 les participants ont visité des usines et des laboratoires à Sendai et à Tokyo.

En 1959, Monsieur le Professeur James W. Daily, président du Comité pour les Machines hydrauliques, les Equipements et la Cavitation de l'AIRH, nous avons écrit pour nous demander si nous avions l'intention d'organiser un symposium à Sendai. Depuis lors, il a bien voulu nous prodiguer ses conseils précieux. Ce fut à la neuvième assemblée générale de l'AIRH, du 4 au 7 septembre 1961 à Dubrovnik, qu'on a décidé officiellement de tenir à Sendai le symposium.

Les préparatifs avaient commencé au Japon dès 1960 et le Comité local d'organisation [cf. la liste des membres au § 2], créé le 3 juillet 1961, se mit à examiner les projets.

Le 14 août 1961 a été constitué le premier Comité d'exécution dont les 12 membres [cf. § 3] ont été choisis parmis ceux du Comité local d'organisation. Ce Comité a procédé au premier examen des projets qui devaient être mis en discussion au Comité local d'organisation.

Le 27 octobre 1961, on a créé les Sous-Comités [cf. la liste des noms et des membres au § 4] au sein du Comité local d'organisation. Pour faciliter les contacts, on a nommé comme chefs de ces Sous-Comités des professeurs, maîtres de recherches à l'Institut de la Mécanique de Grande Vitesse.

Diverses compagnies privées ainsi que des sociétés savantes ont bien voulu contribuer par des dons importants à résoudre les problèmes pécuniaires du symposium. La plupart de ces compagnies et de ces sociétés sont membres de l'Association de l'Industrie de Machines électriques du Japon, ou de la Société des Fabriquants des Machines industrielles du Japon ou de la Fédération des Compagnies électrique [cf. les noms au § 5]. A toutes ces sociétés nous disons ici notre vive gratitude.

A titre d'information nous avons adressé trois circulaires aux membres de l'AIRH; et dans la première, expédiée le 20 octobre 1961, nous précisions pour ceux qui seraient désireux de prononcer une conférence à l'occasion du symposium les règles concernant tant la rédaction que l'envoi des textes. L'on avait arrêté que seraient seules recevables les demandes qui nous parviendraient au plus tard à la fin du mois de février 1962.

Sur 45 textes écrits qui nous avaient été envoyés nous n'avons pu en retenir que 28 étant donné la courte durée de la session (3 jours); sur la suggestion de Monsieur le Professeur Daily, nous avons accordé à chacun des conférenciers 35 minutes dont 15 pour leur exposé et 20 pour la discussion. Il ne sera peut-être pas sans intérêt pour l'avenir de noter ici que certains textes nous sont parvenus rédigés en allemand, en dépit des instructions de la première circulaire qui fixait pour seules langues officielles du Congrès l'anglais et le français. Nous avons été contraints bien à regret de renvoyer ces textes pourtant dignes d'être retenus.

Le Comité qui a effectué le choix des textes [cf. § 4-1] était assisté par des spécialistes appartenant au Comité local d'organisation. Chaque texte été soumis à l'examen de trois savants.

La cérémonie d'ouverture et les sessions se sont tenues au *Denryoku Hall* à Sendai où étaient réunis les 235 participants venus de 10 pays du monde [la liste et les photos des participants se trouvent au § 6].

A l'ouverture du Congrès des allocutions ont été prononcé par MM. F. Numachi, président du Comité local d'organisation, James W. Daily, président du Comité pour les Machines hydrauliques, les Equipements et la Cavitation de l'AIRH, M. Mutō, qui représentait Monsieur le Président du Conseil des Sciences du Japon en présence de MM. I. Oki, président honoraire du Comité local d'organisation, les Représentants des Messieurs les Vice-présidents honoraires [cf. § 2] et T. Kurokawa, recteur d'Université de Tōhoku. Etaient également présents Messieurs les Représentants des compagnies et des sociétés qui nous ont apporté leur contribution pécuniaire.

On avait organisé un programme spécial pour les dames. Sous la direction de maîtres japonais elles ont pu s'initier à la cérémonie du thé et à la pratique de *l'Ikebana* (art de disposer des fleurs dans un vase). L'on avait aussi organisé une excurtion à Matsushima, site renommé. Un cocktail et un banquet ont réuni les participants et leurs épouses et à cette occasion Monsieur le Professeur L. Escande, vice-président, ainsi que d'autres personnalités ont prononcé des discours.

Des visites ont été faites comme on voit à la fin du volume au § 8. Nous remercions ici de tout coeur les usines et les laboratoires qui ont bien voulu accepter de nous recevoir pour les visites.

Enfin, nous tenons à exprimer notre entière gratitude aux divers établissements, aux membres du Comité local d'organisation et à ceux qui nous ont contribué à l'organisation de notre symposium. Tout particulièrement pour Messieurs S. Uchigasaki et T. Hori, ex-présidents de la Compagnie d'Electricité de Tōhoku, et Monsieur Y. Hirai, ex-vice-président de la même compagnie, qui tous nous ont aidé en facilitant la collecte dont nous avons parlé plus haut. Notre reconnaissance va également au personnel de l'Institut de la Mécanique de Grande Vitesse, professeurs, ouvriers de l'usine et tous les employés qui nous ont prêté leur concours.

Nous aimons à croire que ce Symposium a été pour nous tous une excellente occasion non seulement d'approfondir nos connaissances mais aussi de renforcer l'amitié entre des peuples qui sont liés par des intérêts et des problèmes communs.

Sendai, Août 1963

F. Numachi

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Thermal Effect on the Growth and Collapse of Cavities

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Synopsis

When a vapor cavity collapses, the released latent heat raises the temperature and therefore the pressure at the interface. For a similar reason, the interface temperature and pressure are lowered during the growth of a cavity. The net thermal effect is the reduction of the size of the cavity and the interface speed on closure. To demonstrate this thermal effect, solutions are obtained for the growth and collapse of a one-dimensional cavity. It is found that the heat flow to or from the interface takes place mainly in the liquid in a thickness of the order of $\sqrt{\kappa_1 t}$.

Résumé

Quand une cavité de vapeur s'écrase, la chaleur latente libérée augmente la température et par conséquent la pression à l'interface. Pour la même raison, la température et la pression à l'interface sont réduites pendent la formation de la cavité. L'effet thermique net est la réduction de dimension de la cavité et de vitesse de l'interface à l'écrasement. Pour démontrer cet effet thermique, des solutions représentant la développement et l'écrasement d'une cavité à une dimension sont présentées. On trouve que le débit de chaleur de ou vers l'interface se manifeste principalement dans une couche de liquide d'une épaisseur de l'ordre de $\sqrt{\kappa_1 t}$.

Introduction

When a vapor cavity collapses in a zone of high pressure, latent heat is released at the surface of condensation. The temperature of the interface is thus raised, and, as a result, the pressure there becomes higher. On the other hand, the temperature and the pressure at the interface are lowered during the growth of a cavity in a zone of low pressure. This variation of pressure at the interface will influence the rate of growth and collapse of the cavity.

This thermal problem is the subject matter of this paper. In order to facilitate ease of solution, one-dimensional vapor cavities are considered, instead of spherical ones. Condensation and evaporation are assumed to take place only at the interface. This approach will demonstrate the physical mechanism involved with relative ease. Furthermore, one-dimensional cavities do occur in some hydraulic systems. The solutions presented in this paper will be useful for such cases under suitable conditions.

The following symbols are used in this paper.

$A(t)$	=	acceleration of the origin $= dU/dt$
a	=	deceleration of liquid column
B	=	coefficient in Eq. 20
$b(t)$	=	variable in Eq. 28
c	=	specific heat
$D(t)$	=	displacement of interface
K	=	heat conductivity
$L(T_i)$	=	latent heat per unit mass
M^2	=	$\rho_{20} U^2 / p_0$
m, n	=	constants in Eq. 11
P	=	$c_2 \mu / K_2$
$p(x, t)$	=	absolute pressure of vapor
R	=	$p / (\rho_2 T_2)$
r	=	$BD / (c_1 \rho_1 \sqrt{\kappa_1 t})$
s	=	variable in definition of $\overline{T}$
$T(x, t)$	=	absolute temperature
$\overline{T}$	=	$\int_0^\infty e^{-st} T\, dt$
t	=	time
$U(t)$	=	absolute speed of interface
$u(x, t)$	=	speed of vapor relative to interface, in x-direction
x	=	coordinate with origin at interface
κ	=	$K / (c \rho)$
μ	=	viscosity of vapor
ρ	=	density

Subscripts $_0$, $_1$, $_2$ and $_i$ indicate values at $t = 0$, in the liquid, in the vapor, and at the interface, respectively.

Collapse of a Vapor Column

I. General Equations

The first case to be considered is shown in Fig. 1. A liquid and its vapor is in equilibrium at temperature T_0 at time $t=0$. At $t>0$, the interface moves so that the vapor moves with $u(x,t)$ relative to the interface as shown in Fig. 1 b. Under the condition that $\rho_2 \ll \rho_1$, the velocity of the liquid relative to the interface is negligible. The problem is to find the pressure p_i at the interface so that the mechanical condition in the liquid can be ascertained for a certain motion of the interface, and vice versa. As p_i at condensation is determined by the temperature T_i at the interface, it is necessary to find the temperature distribution in the fluid.

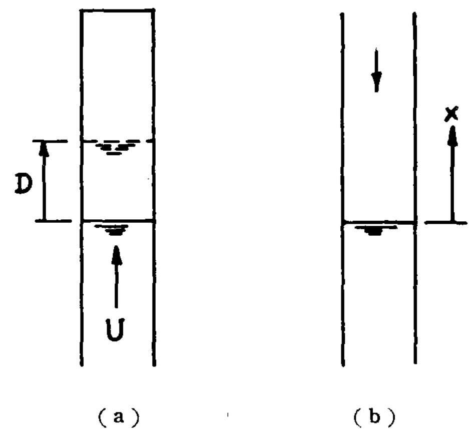

Fig. 1. Relative Motion in the Collapse of a Vapor Column

The equation of heat conduction for the liquid is

$$\frac{\partial T_1}{\partial t} = \kappa_1 \frac{\partial^2 T_1}{\partial x^2}, \qquad (x \leqq 0) \qquad \cdots\cdots (1)$$

For the vapor, we have the energy equation, the equation of continuity, the momentum equation, and the equation of state [1]:

$$\frac{\partial T_2}{\partial t} + u \frac{\partial T_2}{\partial x} = \kappa_2 \frac{\partial^2 T_2}{\partial x^2} - \frac{p}{\rho_2 c_2} \cdot \frac{\partial u}{\partial x} + \frac{4}{3} \cdot \frac{\mu}{c_2 \rho_2} \left(\frac{\partial u}{\partial x} \right)^2, \qquad (x \geqq 0) \quad \cdots (2)$$

$$\frac{\partial \rho_2}{\partial t} + u \frac{\partial \rho_2}{dx} + \rho_2 \frac{\partial u}{\partial x} = 0 \qquad \cdots\cdots (3)$$

$$A + \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{\rho_2} \cdot \frac{\partial p}{\partial x} - \frac{4}{3} \cdot \frac{\mu}{\rho_2} \cdot \frac{\partial^2 u}{\partial x^2} = 0 \qquad \cdots\cdots (4)$$

$$p = R \rho_2 T_2 \qquad \cdots\cdots (5)$$

The thermal boundary conditions are

$$T_1 = T_2 = T_0, \qquad \text{at } t = 0 \qquad \cdots\cdots (6)$$

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[1] Numbers in brackets designate Reference at the end of the paper.

$$T_1 = T_2 = T_i(t), \quad \text{at } x = 0 \qquad \cdots\cdots (7)$$

$$K_1 \frac{\partial T_1}{\partial x} - K_2 \frac{\partial T_2}{\partial x} = -L \rho_{2i} u, \quad \text{at } x = 0 \qquad \cdots\cdots (8)$$

$$T_1 \text{ and } T_2 \text{ remain finite at } x = \pm \infty \qquad \cdots\cdots (9)$$

The mechanical boundary conditions are: $u = -U(t)$ and $\partial u/\partial x = 0$ at $x = \infty$; $p = p_i(t)$ and $\rho_2 = \rho_{2i}(t)$ at $x = 0$. For a given fluid, p_i and ρ_{2i} are known functions of T_i of the interface. Solutions of these equations are to be obtained for cases where the changes of the dependent variables are small so that the equations can be linearized. (No effect of turbulence is being considered. Otherwise, some effective values of K and μ must be used).

II. Temperature of Liquid with $B = 0$ as a Limiting Case

It will be shown in Section IV that $K_2 \cdot \partial T_2 / \partial x$ is negligible compared with $K_1 \cdot \partial T_1 / \partial x$ at $x = 0$. Upon linearizing, Eq. 8 becomes

$$K_1 \frac{\partial T_1}{\partial x} = -L\rho_{2i} u = L_0 \rho_{20} U + \beta (T_i - T_0) U + L_0 \rho_{20} \gamma (T_i - T_0) U$$
$$= L_0 \rho_{20} U + B U (T_i - T_0), \quad \text{at } x = 0 \qquad \cdots\cdots (10)$$

where $\beta = d(L \rho_{2i}) / dT_i$ at T_0, γ will be seen to be approximately $1 / T_0$ (see Eqs. 25 and 29), and $B = \beta + L_0 \rho_{20} \gamma$. The terms with β and γ account for the difference between $L\rho_{2i}$ and $L_0\rho_{20}$, and between $-u$ and U respectively. As a limiting case, first consider $B = 0$. Then

$$K_1 \frac{\partial T_1}{\partial x} = L_0 \rho_{20} U \qquad \cdots\cdots (10a)$$

The temperature $T_1(x, t)$ in the liquid is to be found for the following motion of the interface:

$$U(t) = m t^n \qquad \cdots\cdots (11)$$

where m and n are constants ($n = 0$ for constant speed, and $n = 1$ for constant acceleration). We shall make use of the displacement $D(t)$ of the interface:

$$D(t) = \int_0^t U \, dt = \frac{m}{n+1} t^{n+1} \qquad \cdots\cdots (12)$$

Using the Laplace transform T_1 with the conditions in Eqs. 6 and 9, we obtain from Eq. 1

$$\overline{T}_1 = \frac{T_0}{s} + \alpha \exp\left(\sqrt{\frac{s}{\kappa_1}}\, x\right) \qquad \cdots\cdots(13)$$

where α is to be determined from Eq. 10a. Thus

$$\overline{T}_1 = \frac{T_0}{s} + \frac{L_0 \rho_{20} m \Gamma(n+1)}{c_1 \rho_1 \sqrt{\kappa_1}}\, s^{-\left(n+\frac{3}{2}\right)} \exp\left(\sqrt{\frac{s}{\kappa_1}}\, x\right), \qquad (x \leqq 0)$$

We shall find $T_1(x, t)$ for three different cases with $n = 0$, $1/2$ and 1 respectively [2].

(a) $n = 0$:

$$\overline{T}_1 = \frac{T_0}{s} + \frac{L_0 \rho_{20} m}{c_1 \rho_1 \sqrt{\kappa_1}} s^{-3/2} \exp\left(\frac{x}{\sqrt{\kappa_1}} \sqrt{s}\right), \qquad (x \leqq 0)$$

$$T_1 = T_0 + \frac{L_0 \rho_{20} D}{c_1 \rho_1 \sqrt{\kappa_1 t}} \left[\frac{2}{\sqrt{\pi}} \exp\left(-\frac{x^2}{4\kappa_1 t}\right) + \frac{x}{\sqrt{\kappa_1 t}} \operatorname{erfc}\left(-\frac{x}{2\sqrt{\kappa_1 t}}\right)\right] \cdots(14)$$

where $D = Ut = mt$. At the interface,

$$T_i = T_0 + \frac{2}{\sqrt{\pi}} \cdot \frac{L_0 \rho_{20} D}{c_1 \rho_1 \sqrt{\kappa_1 t}} \qquad \cdots\cdots(15)$$

(b) $n = \frac{1}{2}$:

$$\overline{T}_1 = \frac{T_0}{s} + \frac{L_0 \rho_{20} m}{c_1 \rho_1 \sqrt{\kappa_1}} \frac{\sqrt{\pi}}{2} s^{-2} \exp\left(\frac{x}{\sqrt{\kappa_1}} \sqrt{s}\right), \qquad (x \leqq 0)$$

$$\begin{aligned} T_1 &= T_0 + \frac{\sqrt{\pi}}{2} \frac{L_0 \rho_{20} m}{c_1 \rho_1 \sqrt{\kappa_1}} \int_0^t \operatorname{erfc}\left(-\frac{x}{2\sqrt{\kappa_1 \tau}}\right) d\tau \\ &= T_0 + \frac{L_0 \rho_{20} D}{c_1 \rho_1 \sqrt{\kappa_1 t}} \left[\frac{3\sqrt{\pi}}{4} \left(1 + \frac{x^2}{2\kappa_1 t}\right) \operatorname{erfc}\left(-\frac{x}{2\sqrt{\kappa_1 t}}\right) \right. \\ &\qquad \left. + \frac{3}{4} \cdot \frac{x}{\sqrt{\kappa_1 t}} \exp\left(-\frac{x^2}{4\kappa_1 t}\right)\right] \end{aligned} \qquad \cdots\cdots(16)$$

where $D = \frac{2}{3} m t^{3/2}$. At the interface,

$$T_i = T_0 + \frac{3\sqrt{\pi}}{4} \cdot \frac{L_0 \rho_{20} D}{c_1 \rho_1 \sqrt{\kappa_1 t}} \qquad \cdots\cdots(17)$$

(c) $n = 1$:

$$\bar{T}_1 = \frac{T_0}{s} + \frac{L_0 \rho_{20} m}{c_1 \rho_1 \sqrt{\kappa_1}} s^{-5/2} \exp\left(\frac{x}{\sqrt{\kappa_1}} \sqrt{s} \right), \quad (x \leqq 0)$$

$$T_1 = T_0 + \frac{L_0 \rho_{20} m}{c_1 \rho_1 \sqrt{\kappa_1}} \int_0^t \left[\frac{2}{\sqrt{\pi}} \sqrt{\tau} \exp\left(-\frac{x^2}{4\kappa_1 \tau} \right) + \frac{x}{\sqrt{\kappa_1}} \operatorname{erfc}\left(-\frac{x}{2\sqrt{\kappa_1 \tau}} \right) \right] d\tau$$

$$= T_0 + \frac{L_0 \rho_{20} D}{c_1 \rho_1 \sqrt{\kappa_1 t}} \left[\frac{8}{3\sqrt{\pi}} \left(1 + \frac{x^2}{4\kappa_1 t} \right) \exp\left(-\frac{x^2}{4\kappa_1 t} \right) + \frac{2x}{\sqrt{\kappa_1 t}} \left(1 + \frac{x^2}{6\kappa_1 t} \right) \operatorname{erfc}\left(-\frac{x}{2\sqrt{\kappa_1 t}} \right) \right] \quad \cdots\cdots(18)$$

where $D = \frac{1}{2} m t^2$. At the interface,

$$T_i = T_0 + \frac{8}{3\sqrt{\pi}} \cdot \frac{L_0 \rho_{20} D}{c_1 \rho_1 \sqrt{\kappa_1 t}} \quad \cdots\cdots(19)$$

Eqs. 14, 16 and 18 are plotted as the lowest curves in Figs. 2, 3 and 4. Note that the increase of temperature in the liquid is confined mainly to a thickness of $\sqrt{\kappa_1 t}$. The interface temperature T_i is plotted as the lowest curve in Fig. 5. In all these cases, the value of $c_1 \rho_1 \sqrt{\kappa_1 t} (T_i - T_0)/(L_0 \rho_{20} D)$ is of the order of unity because $L_0 \rho_{20} D$ is the latent heat released (per unit cross-sectional area) which is absorbed by a volume of the order of $\sqrt{\kappa_1 t}$.

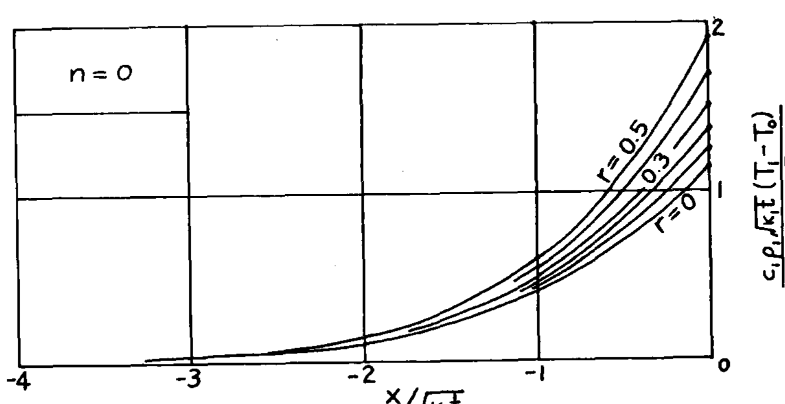

Fig. 2. Temperature in Liquid for Constant Interface Speed

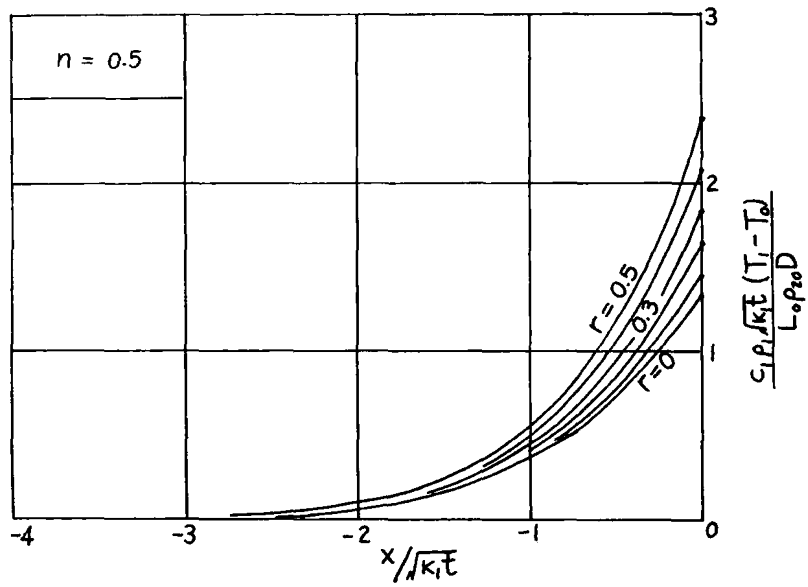

Fig. 3. Temperature in Liquid for $n = 0.5$

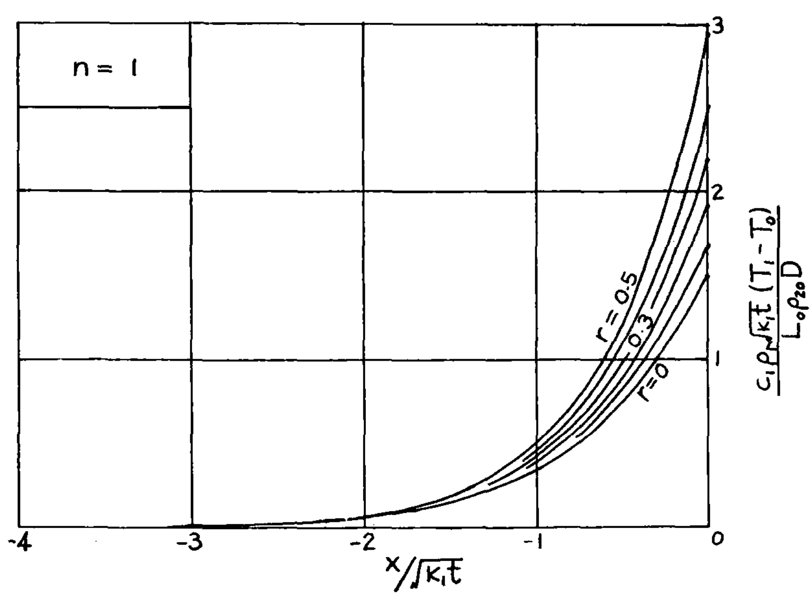

Fig. 4. Temperature in Liquid for Constant Interface Acceleration