ELEMENTS OF STATISTICAL METHOD

Albert E. Waugh

Professor of Economics
University of Connecticut

New York Toronto London

McGRAW-HILL BOOK COMPANY, INC.

1952

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Library of Congress Catalog Card Number: 51-13600

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This book is affectionately
dedicated to
ALEXANDER E. CANCE
and
IRVING G. DAVIS
inspired teachers, tireless seekers after
new things, gentlemen

PREFACE

It is now fourteen years since the first edition of this book appeared. I was gratified at the time by the reception accorded it by the reviewers, and I have been pleased ever since to find a continuing demand, both for the original and for the second edition, which indicates that my colleagues have found the book useful. I am deeply indebted to those who have offered suggestions or criticisms. Particularly am I indebted to Professor Roy W. Jastram of Stanford University and to Professor Frederick E. Croxton of Columbia University who read the entire manuscript of earlier editions and gave invaluable suggestions for improvement.

Those familiar with the book in earlier editions will discover that the major revisions are in the last half of the book. A chapter has been added introducing the student to the ideas of analysis of variance. Chapters on curve fitting have been entirely reworked and made introductory to the chapters on correlation. The elementary work on graphics and the collection of data which appeared in earlier editions have been omitted.

In the main, however, the purpose and method of the book remain unchanged. As the first edition announced, "This book is planned for the beginner in the field who has yet to learn 'what it is all about.' No attempt has been made to treat any aspect of the field exhaustively, and advanced students will find it necessary to consult other books and, particularly, to acquaint themselves with articles in the current technical statistical journals. The aim of this book is to introduce the student to statistical concepts and statistical nomenclature and to get him to think in statistical terms."

The great difficulty in writing any textbook is to know what to omit—to keep in mind about how large a body of material a beginning student can be reasonably expected to assimilate in a semester. In many cases where the limitations of space and the student's time make it impossible to cover a subject in detail (in advanced correlation analysis, for example) a special effort has been made to cover the basic ideas in such a way that the thoughtful student will find it easy to continue under his own power. Every effort has been made to keep the book on the beginner's level. It has been written primarily to help the beginner, and not to impress the mature statistician.

ALBERT E. WAUGH

STORRS, CONN.
February, 1952

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CHAPTER I

THE NATURE OF STATISTICS

1.1. Scientific Method.—Men have discovered facts in many ways. Some things have been learned by chance. Wisdom, so it is said, has been imparted to men in dreams and through miraculous revelation. There are those who assert that they are gifted with clairvoyance, by means of which they are able to discern things hidden from the ordinary mortal. And, of course, a large portion of all human knowledge has come down to us from unknown sources. The method by which it was originally discovered is not known.

The tremendous advances in human knowledge that have characterized the past century and a half have not come, in the main, from any of the sources above mentioned. Nowadays a great preponderance of the additions to our information are the result of plan and not the product of chance. We learn new things because we have gone about the learning process methodically, and not haphazardly. The methods which are used in acquiring knowledge are called *scientific methods*, and it should be remembered that they do involve the following of a definite plan.

1.2. Experimental Method.—The best known of the scientific methods, and the one which has been most fruitful, is called the *experimental method*. Galileo, we are told, was attracted by the swinging of lamps in the cathedral. He noted that the period of the swing varied, and he wished to determine what factors influenced the length of the period. He did not rely on dreams, nor did he, so far as we know, patronize the local fortuneteller. He began to experiment. He went at the problem methodically, in an attempt to determine what forces were at work in the pendulum.

Now Galileo might have taken the first half-dozen pendulums that he encountered and studied them. If he had done so he would have found that the pendulums differed in many respects. In some the bob would be heavier than in others. In some the length would be greater than in others. In some cases there might be air currents which were not present in others. At the points where the pendulums were located there might be differences in air pressure, relative humidity, the attraction of gravity, etc. And under such circumstances when Galileo found that pendulum 1 oscillated more rapidly than pendulum 2 he would be unable to tell whether the difference was due to differences in length, weight, humidity, air pressure, or some combination of these forces. Hence Galileo was very careful to construct pendulums that differed in but one respect; that is, he would make several pendulums of the same length; he would protect them all from currents of air that might affect the rate of swinging; he would operate them all at the same time and place so that there would be no differences in barometric pressure and the like. In fact, these pendulums would be exactly the same and would be operated under identical conditions, except, let us say, that there would be variations in the weight of the bob. Under these circumstances if Galileo found that there were regular variations in the period of oscillation which corresponded to differences in the weight of the bob, he would know that the cause must be bob weight, since no other factors differed. If, on the other hand, these pendulums did not differ in period, he would know that bob weight did not affect the rate of swinging.

Having discovered the effect, if any, which was associated with the weight of the bob, Galileo would now construct pendulums which differed in nothing except length, and would ascertain the relationship of length to period. He would then, in turn, discover the effect of gravitational attraction, barometric pressure, etc. The important thing to note is that the method consists in keeping all forces save one constant, and in varying that force in order that the scientist may discover its effect, if any. This method of investigation is called the experimental method, and where it is applicable scientists prefer it to all other methods.

1.3. Statistical Method.—But often men wish to discover facts in fields where the experimental method cannot be applied. Let us suppose, for example, that you wish to discover the forces that determine the price of milk in New York City. You would like to apply the experimental method. This means that you would have to try one thing at a time, keeping all other things constant, and note the effect of the changes. First you would make changes in the quantity of milk offered on the market, to determine whether or not the price varied as you varied the amount offered. But it would be necessary for you to see that there were no changes in the other factors. You would have to establish entire uniformity in people's wages, since presumably the amount that they will pay for milk depends partly on the amount of their incomes; you would have to make sure that the tastes of consumers remained constant, since changes in their desires would perhaps change the amount that they would pay; you would find that you were forced to fix the general price level so that there would be no changes from variations in the purchasing power of money; etc. But this is manifestly out of the question. Here the experimental method cannot be applied, because the many factors cannot be held constant as the scientist varies the one force in which he is interested for the moment. Thus in the social and biological sciences it is often impossible to make use of the experimental method.

We should be foolish, however, to neglect entirely those fields in which experimental method is out of the question. To be sure, it is very difficult to discover facts in such fields, but it is none the less desirable that facts be discovered, and the scientist must do what he can in the face of difficulties. He could, of course, fall back on chance or revelation. In such a case he would have left the field of science entirely, since he would be following no plan. As a matter of fact he is likely to fall back on another method, as a poor second choice. This other method (or body of methods) we call *statistical method.* When we apply statistical method to a problem we go at the problem systematically, as in the experimental method; but the system used is not the same. Being unable to hold forces constant, we perforce let them vary. But now we record the variations in all the forces operating and attempt to determine the separate part which each plays in influencing the result. Under ordinary circumstances this method is much more difficult than the experimental method, and the results obtained are usually less accurate and less satisfying. But they are much better than no results at all.

The classification of scientific methods into those which are experimental and those which are statistical, like most classifications, is formal and arbitrary and not entirely realistic. When the scientist comes to work on a problem in practice, he almost always combines elements of both the statistical and the experimental approach. Many of the most important of the statistical methods were originated in the fields of physics and astronomy—fields that we usually think of as "exact" sciences. Even in these fields the scientist has to contend with errors of observation, and in addition he usually finds it impossible to record the values of all the variables which are involved.[1] Under such circumstances the "exact" scientist is forced to combine statistical methods with his experimental procedures. On the other hand, even the social scientist can and does use a certain amount of control in his investigations.

1.4. Statistics.—The word "statistics" is used in two senses which differ materially. It is sometimes used in the singular and sometimes in the plural. When used in the plural, it refers to numerical data. Thus if we say that there *are* statistics in the "World Almanac" we mean that there are numerical data there. When we use the word in the singular, we refer to a body of methods which are used in summarizing such numerical data. Statistics *is* a body of methods which are used when we wish to study masses of numerical data and to extract from them a few simple facts.

Originally statistics were gathered for public purposes. In fact, the word "statistics" and the word "state" come from the same root. Statistics were gathered for purposes of taxation and for military purposes. But nowadays there is almost no field in which statistics are not useful. Every science depends to some extent on the gathering of data and on the application to them of statistical methods. In some fields, as has been pointed out above, the statistical methods are almost the only methods that can be used, while in other fields they are a minor supplement to other scientific methods.

1.5. Preliminary Admonitions.—It is important to remember that the purpose of statistical method is to simplify great bodies of numerical data. If you were shown a table containing 1000 figures, each figure representing the weight of a newborn baby, you would be confused by the very mass of material itself.

[1] See Chap. II for a discussion of these problems.