# Theory of Machines and Computations

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# PREFACE

This volume consists of the papers presented at the International Symposium on the Theory of Machines and Computations, held at Technion–Israel Institute of Technology, Haifa, Israel in August 1971. They are representative of a wide variety of subjects in the areas of computability theory, formal languages, automata theory, and switching theory.

The papers were chosen by an international program committee from a large number of submitted contributions. Each author was responsible for the typing and preparation of his manuscript for the photocopying process by which this volume is printed.

The editors would like to thank the committees' members for their help in the organization of the symposium and the screening of the manuscripts.

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# COMPUTABILITY THEORY

# DECIDABLE PROPERTIES OF MONADIC FUNCTIONAL SCHEMAS

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## Abstract

We define a class of (monadic) functional schemas which properly includes 'Ianov' flowchart schemas. We show that the termination, divergence and freedom problems for functional schemas are decidable. Although it is possible to translate a large class of non-free functional schemas into equivalent free functional schemas, we show that this cannot be done in general. We show also that the equivalence problem for free functional schemas is decidable. Most of the results are obtained from well-known results in Formal Languages and Automata Theory.

## I. Monadic Functional Schemas

An _alphabet_ $\Sigma_S$ of a (monadic) functional schema S consists of one individual variable x, a finite set of monadic function variables $\{F_i\}$ (with a designated _initial_ function variable $F_0$), a finite set of monadic function constants $\{f_i\}$, and a finite set of monadic predicate constants $\{p_i\}$. Note that individual constants are not allowed.

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A term over $\Sigma_S$ is any term in the normal sense constructed from the monadic function variables $\{F_i\}$, monadic function constants $\{f_i\}$ and the variable $x$, e.g., $f_1(F_0(f_2(x)))$. A conditional term over $\Sigma_S$ is any finite expression of the form

$$\underline{\text{if}}\ p_i(x)\ \underline{\text{then}}\ \tau_1\ \underline{\text{else}}\ \tau_2\ ,$$

where $p_i$ is any predicate constant of $\Sigma_S$, and $\tau_1$ and $\tau_2$ are any terms or conditional terms over $\Sigma_S$. A definition of $F_i$ over $\Sigma_S$ is of the form

$$F_i(x) \Leftarrow \tau\ ,$$

where $\tau$ is any term or conditional term over $\Sigma_S$. A (monadic) functional schema $S$ (over an alphabet $\Sigma_S$) consists of a finite set of definitions over $\Sigma_S$, one for each function variable $F_i$ in $\Sigma_S$. Whenever the special function variable $F_\infty$ is used, its definition is considered to be $F_\infty(x) \Leftarrow F_\infty(x)$. This definition is usually omitted.

Example 1: Let us consider the functional schema $S_1$:

$$F_0(x) \Leftarrow \underline{\text{if}}\ p_1(x)\ \underline{\text{then}}\ \underline{\text{if}}\ p_2(x)\ \underline{\text{then}}\ F_\infty(x)\ \underline{\text{else}}\ F_1(f_1(x))$$
$$\underline{\text{else}}\ x$$

$$F_1(x) \Leftarrow \underline{\text{if}}\ p_3(x)\ \underline{\text{then}}\ F_0(f_2(x))\ \underline{\text{else}}\ f_1(x)\ .$$

Since we are using a very restricted alphabet, parentheses and the individual variable $x$ may be omitted without causing any confusion. Therefore the functional schema $S_1$ can be rewritten as:

$$F_0 \Leftarrow \underline{\text{if}}\ p_1\ \underline{\text{then}}\ \underline{\text{if}}\ p_2\ \underline{\text{then}}\ F_\infty\ \underline{\text{else}}\ F_1f_1$$
$$\underline{\text{else}}\ I$$

$$F_1 \Leftarrow \underline{\text{if}}\ p\ \ \underline{\text{then}}\ F_0f_2\ \underline{\text{else}}\ f_1\ ,$$

where $I$ stands for the 'identity function'.*/

*/ It is worth noting that most of the results in this paper would be trivial if we did not allow the 'identity function'.

An interpretation $\mathcal{I}$ of a functional schema S consists of:

1. a non-empty set of elements D (called the domain),
2. an element $\xi_o$ of D used as the initial value of x, and
3. assignments to the constants of $\Sigma_S$ :
   (i) a total monadic function (from D into D ) for each function constant $f_i$ , and
   (ii) a total monadic predicate (from D into {T,F}) for each predicate constant $p_i$ .

For a given interpretation $\mathcal{I}$ , the pair $(S,\mathcal{I})$ , called a functional program, can be computed by evaluating $F_o$ with input $\xi_o$ in the usual way (see McCarthy [1963]). The computation either (i) terminates yielding an element of D denoted by val$(S,\mathcal{I})$ , or (ii) diverges (i.e., does not terminate) in which case val$(S,\mathcal{I})$ is said to be undefined. The method of computation is described more fully later for special types of interpretations called 'Herbrand interpretations'.

A functional schema S is said to terminate/diverge if for every interpretation $\mathcal{I}$ , val$(S,\mathcal{I})$ is defined/undefined. Two functional schemas $S_1$ and $S_2$ are said to be equivalent if for every interpretation $\mathcal{I}$ either both val$(S_1,\mathcal{I})$ and val$(S_2,\mathcal{I})$ are undefined or both are defined and val$(S_1,\mathcal{I})$ = val$(S_2,\mathcal{I})$ .

The same class of functional schemas has been discussed by DeBakker and Scott [1969].

It is straightforward to show that every functional schema in which any term contains at most one function variable, occurring on the left-hand side of the term (as in Example 1 above), can be translated to an equivalent 'Ianov' flowchart schema (Ianov [1960], see also Rutledge [1964]). However, such simple functional schemas as

$$F_o \Leftarrow \underline{\text{if}}\ p\ \underline{\text{then}}\ I\ \underline{\text{else}}\ f_1F_of_2$$

have no equivalent Ianov flowchart schema. Hence, the results in this paper generalize known results about Ianov flowchart schemas.