Optimum Structural Design

Theory and Applications

Edited by R. H. Gallagher and O. C. Zienkiewicz

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Preface

In the last two decades outstanding progress has been achieved in the field of structural analysis. With the aid of the computer nearly all structural problems can be solved within the limits of our knowledge of the materials. While these achievements are, *per se*, of the greatest importance in allowing the behaviour of a particular design to be assessed, their full benefits for our society will not be materialized until they are reflected in the improved design of structures.

The aim of devising better design solutions which, while satisfying safety and performance 'constraints', do it at least cost, is clearly not a new one. From time immemorial a self-respecting engineer has investigated several alternatives and chosen the 'best' one of these. Unfortunately, cost and time usually limit very severely the number of alternatives that can be investigated. With the 'computerization' of the analysis process, it is natural that a development of more effective and rapid techniques for the search of the 'absolute best', or 'optimum' solution is required. Many possibilities obviously exist, with two clearly defined extremes. One extreme is to utilize the computer capability to the fullest and *automate* this search; the other extreme is to utilize human intuition in an *interactive manner* to guide the computer in its calculations.

Much work has been done in recent years on both approaches by various researchers, and a stage has now been reached at which an appraisal of the developments and of their practical possibilities should be made and presented to the engineering profession. This appraisal is the subject of this book. To achieve a representative picture of the 'state of the art', the editors invited contributions from some leading exponents of both the theoretical and practical aspects of structural optimization, assigning to each a certain coverage of the subject and specifying in some detail the objectives and scope of presentation so that a coherent volume could be obtained.

The major part of this volume, Chapters 1–15, is concerned with approaches in the first of the above classes—'automated' approaches. Chapter 1 outlines the background of automated optimum structural design, broadly classifies the relevant mathematical procedures, and refers to the principal sources of already published information. Chapter 2 sets the stage for most of the theoretical work that follows by defining terminology and presenting certain basic definitions and theorems.

Chapter 3 explores what has been, historically, the most appealing approach to design analysis, the *fully stressed design* philosophy and procedure. These ideas are expanded in Chapter 4 to form an approach, related also to so-called *optimality criteria*, that does not suffer the limitations of fully stressed design, but nevertheless retains its computational economies *vis-à-vis* the more sophisticated procedures.

The largest share of modern activity in optimum structural design revolves about the utilization of mathematical-programming procedures. Chapter 5 outlines these procedures, irrespective of their role in the structural context, and gives a 'road map' for the pursuit of appropriate alternative techniques for the sundry mathematical classification of problems. In the process, this chapter presents a résumé of many mathematical papers and books and gives the structural engineer a comprehensive insight into available techniques.

Chapters 6–13 follow up this review by presenting, from the viewpoint of researchers in structural engineering, the background, fundamental theory and considerations, and some applications experience in various major alternative techniques in mathematical programming. Linear programming (Chapter 6), iterative linear programming (Chapter 7), feasible-direction methods (Chapter 8), penalty-function procedures (Chapter 9), dynamic programming (Chapter 10) and discrete-variable methods (Chapters 11 and 12) are dealt with. Each of these is based on or infers a deterministic design philosophy. There is a considerable trend towards probability-based design philosophy, however, and the implications of this for optimum structural design are treated in Chapter 13. Chapters 14–16 will be of direct interest to the practising civil engineer. Here, present-day application and results for structural steel and concrete show how much has already been achieved in practical engineering and that real benefits are already being achieved. Other practical structural-engineering applications and similar achievements in such fields as aerospace, mechanical engineering and naval architecture are discussed in the prior chapters.

As noted earlier, another approach to optimization is via the *interactive* mode. This approach is specifically dealt with in Chapters 16 and 17, and also enters to some extent in other contributions.

Our thanks go out to the respective authors for entering into this venture with an enthusiasm and a spirit of collaboration which often necessitated considerable patience and rewriting. The editors have endeavoured with their blue pencil to keep the notation consistent, if not uniform, and to avoid excessive repetition. A unified notation has been sought in those chapters that deal almost exclusively with mathematical programming in structural optimization; this was not possible, however, for those chapters with heavy emphasis on both analysis and design technologies, i.e. Chapters 6, 12 and 13. To what extent the objectives of coherence and logical sequence are fully achieved will be for the reader to judge. It is seldom possible for perfection to be approached with a multiauthored text.

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Contents

Chapter 1

Introduction

Richard H. Gallagher

In contrast to analysis technology, optimum-structural-design technology has not yet enjoyed intensive study, and computational aspects of practical design today largely depend on iterative analysis. Modern developments in optimum structural design, represented by attempts to introduce the (then) novel accomplishments of mathematical programming into structure technology, first appeared over ten years ago. Shortly thereafter, the earliest systems for interactive, or computer-aided, design, achieved operational status. Neither of these aspects of the total design technology has enjoyed the utilization predicted of it, although certain features have proved successful.

It is difficult to ascertain the full range of considerations responsible for the slow rate of acceptance of the available design technology. These include such factors as unfamiliarity of the practitioner with mathematical-programming concepts, the bewildering array of alternative paths in mathematical programming and the costs of optimum-design analysis beyond the costs of simple analysis. Nevertheless, it appears that these problems are now being overcome. An extremely large backlog of optimum-structural-design literature has accumulated meanwhile, and, if the practitioner is to make use of it, some evaluation of the alternatives must first be made, followed by detailed study of the applicable procedures. This chapter helps to evaluate the various approaches; subsequent chapters examine techniques in detail.

Four previously independent major areas (and, to some extent, chronological phases) can be identified in the development of optimum-structural-design technology. We shall term these the (a) theory of layout, (b) simultaneous mode of failure, (c) optimality-criteria-based and (d) mathematical-programming formulations.

The first of these was the *theory of layout*, which seeks the arrangement of uniaxial structural members that produces a minimum-volume structure for specified loads and materials. The basic theorems of this approach were established by Maxwell as early as 1854[1], but these ideas were amplified and given their first significant application by Michell[2] in 1904. Such theorems, since they are applied without meaningful constraints on the geometric form of the structure, yield impractical solutions. The theory of layout has been reconsidered in the work of Cox[3] and Hemp[4], however, and many researchers are now developing further the related concepts.

The *simultaneous mode of failure approach* presumes that optimality is achieved when each component element of the complete structure is at its limit of strength as failure of the complete structure impends. The term 'simultaneous' implies a single load condition, and this restriction governs nearly all of the work which flourished during the 1940s and 1950s and is recorded in the books by Shanley[5], Gerard[6] and Cox[3]. These efforts, predating the electronic digital computer, deal with simple structural forms and depend on classical ideas of function minimization. The fact that there are only a small number of simple situations, together with their limited applicability to practical design, has resulted in very little new work in this area during the past decade.

If the concept of the simultaneous mode of failure approach is broadened to admit more than one load condition, and at the same time restricted to strength limitations applying only to stress, the *fully stressed design approach* is produced. This approach generally consists of the iterative application of analysis, leading to a design in which each member is subjected to its limiting stress under at least one of the specified load conditions. Although the result is the designer's traditional view of an optimal structure, the concept has not been subjected to a broad, rational study. Chapter 3 of this book correlates the relevant published information.

The concept of a *criterion of optimality* as the basis of selection of a minimum-volume structure emerged in the early 1960s. This approach derives from the extremum principles of structural mechanics, and for the most part has been limited to simple structural forms and loading conditions. Prager[7] and Taylor[8] have been instrumental in the development of much of this work, and the procedures of Venkayya[9] and Gellatly and Berke[10] are described in this book. Chapter 4, by Gellatly and Berke, elaborates on their procedures.

Finally, we come to procedures characterized as *mathematical-programming formulations*. The basic ideas of mathematical programming are outlined in Chapter 2. To define these concisely, it may be said that they seek the minimum or maximum of a function of many variables subject to limitations (*constraints*) that are expressed as equalities or inequalities. The representation of inequality constraints is of critical importance, since this permits the design to be identified as one in which not all members are subject to limiting conditions under specified loads, avoiding a restriction inherent in certain of the aforementioned approaches. Also, the orientation of mathematical-programming formulations towards many-variable problems fits quite well with the trend in analysis towards finite-element representations which require large-order systems. It should be emphasized, however, that procedures described in subsequent chapters are not, in general, tied to a particular method of analysis.

Mathematical programming was first applied to structural optimization in the late 1950s. Early contributions included Livesley's[11] and Pearson's[12] treatments of limit design as a linear-programming problem, and Schmit's casting[13] of elastic design as the more general non-linear-programming problem. We will not attempt to delineate here the detailed historical development of this avenue of activity in structural optimization, because recent surveys have been published and are cited below for reference, and also because many of the later chapters summarize detailed facets of the total approach.

There are other modern approaches to structural optimization which do not fit in the classifications that have already been mentioned. Certain of these fall in the category of control theory[14], while others are of a special character and have defied classification in a particular mathematical discipline (e.g. the work of Melosh and Luik[15]). These approaches may indeed be highly promising, but they have not yet received widespread attention.

To conclude this chapter, the following comments are presented to direct the reader to literature which amplifies the historical categorization of optimum structural design, fills in additional details of procedures described in subsequent chapters and enables the investigation of approaches not covered by this book.

The first comprehensive survey of related literature to appear in a widely circulated journal was written by Wasiutynski and Brandt[16] in 1963. Subsequently, in the same journal, Prager and Sheu[17] reviewed developments to 1968. More restricted surveys, with differing vantage points, have been prepared by Barnett[18], McNunn and Jorgenson[19] and Gerard[20]. The most complete and authoritative surveys of optimum structural design in the context of mathematical-programming procedures have been written by Schmit[21–23]. These are especially valuable because of their development, in elementary terms and via simple examples, of the basic concepts of these procedures.

Literature which develops the concepts and procedures of mathematical programming from first principles has not taken account, to any significant extent, of the structural design problem. A notable exception is the book by Fox[24], whose interests in applications have been principally associated with structural optimization. Also, the book by Whittle[25] describes the application of linear programming to the theory of layout.

Developments from the opposite direction, in which structural theorists and designers have compiled studies of structural optimization, have appeared. Pope and Schmit[26] edited one such document, published under the aegis of the Advisory Group for Aeronautical Research and Development (AGARD), NATO, and another, edited by Moe and Gisvold[27] emerged from a short course held at the University of Trondheim, Norway. In the same vein, the proceedings of a 1969 AGARD symposium on structural optimization have been published[28], as have a series of seminar lectures at the University of Waterloo, Canada[29]. The book by Spunt[30] covers some ideas relating to mathematical programming, but is principally devoted to the more classical schemes cited earlier.

I believe that the existing literature of structural optimization, although growing rapidly, may be regarded as 'wieldy' and capable of assimilation by the interested individual. In certain respects, the expansion of published literature assists one in gaining acquaintance with the subject, since a large part of new work is expository, rather than investigative, in nature. The references associated with the respective chapters of this book, supplemented by the contents of previously cited surveys, comprise an almost complete bibliography of the topic to date.

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