ARNOLD D. PICKAR

PREPARING FOR GENERAL PHYSICS

Math Skill Drills and Other Useful Help

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To The Instructor

Although this book is primarily meant to be used for self study, it is an outgrowth of my experience teaching abbreviated courses in remedial and preparatory skills for students about to enroll in general physics classes. In teaching these classes, my greatest frustration arose out of a lack of suitable text materials.

Whereas several books have been published in the past aimed at such students, and some of them have much to commend them, none quite serve all the important objectives of skill preparation as I see them. This is especially critical if the book is to serve students trying to prepare without benefit of a formal class. Some of the books are too verbose, or too inclusive, or too mathematically oriented. Above all, the lack of sufficient and appropriate solved practice problems is a common difficulty. A great deal of the material taught in prerequisite math courses is not needed in beginning physics courses. What *is* needed is guidance and lots of drill in a context which emphasizes the use, rather than the theory, of the mathematics. Just as important, a suitable text, by exercising students in appropriate verbal and pictorial skills, should help develop in them the art of thinking about physics.

The primary audience for this book is meant to be students about to enter or just beginning a college non-calculus level course in general physics, especially those students whose basic mathematical skills are weak or dated . However all of the same skills required by these students are needed, and are often poorly developed, in students attempting calculus level physics; for them, working with this book would certainly not be detrimental.

Arnold D. Pickar
Portland, Oregon
October 1991

To the Student

IS THIS BOOK FOR YOU?

If you are about to begin a college course in introductory physics for science majors, or perhaps have already begun, you may be feeling anxious about what you are getting into. If that is the case, this book may be for you.

For many college students physics appears to be a "different" type of course. Unlike so much of the learning you have had to master over the years — even in science courses — the material in a college-level physics course minimizes the importance of memorizing of facts and standard recipes for solving problems. The emphasis is on understanding concepts and the ability to apply this understanding in solving a wide range of problems.

The obstacles to mastering the art of solving physics problems are many but you can overcome them. Factors which lead to success include the following:

- the competence to use mathematics (usually fairly simple mathematics) with ease and accuracy;

- the ability to visualize the situation a problem describes and the ability to comprehend what is being asked;

- the knowledge required to translate, when necessary, the words of a problem and images they bring to mind into an appropriate mathematical form;

- the sensitivity required to distinguish between those problems which do not require a mathematical solution from those which do;

- the confidence that the use of unfamiliar words will not be an obstacle to learning new ideas.

This is a list of basic skills and habits of thought which hopefully you have developed in the past through contact with other science and mathematics courses. But unless you are completely confident that you have achieved these skills and habits, going through this book is likely to prove worthwhile.

You should recognize that this is *not* a book whose primary purpose is to teach you physics or to introduce you for the first time to the prerequisite skills. Rather, it is a book to help you hone the skills you already have in order to make the study of physics as effective a possible. The experience of physics is both beautiful and challenging; the challenge is most successfully met, and the beauty most enjoyed when the student is not distracted and inhibited by a less than adequate mastery of basic skills.

IF THIS BOOK IS FOR ME, WHEN SHOULD I BEGIN — AND HOW LONG WILL IT TAKE?

The best time to begin preparing for general physics using this book is 4 to 6 weeks before the course begins. However if you should realize only later that you need this preparation — even after the course has begun — you cannot help but profit from working with this book, providing you are willing to devote the time and effort necessary to do most of it before the physics course gets too far along.*

You can probably finish this book with 20 to 30 hours of work. Preferably this should be done in sessions of several hours, spread out evenly over as much as a month or, if necessary, as little as a week. Moreover, the several pretests may indicate you are already strong in some aspects of the work and can go quickly over the related material. On the other hand, if the post-test (at the end of each "Round" of work) indicates a continuing weakness with some skills, you may wish to review some of the material again.

But remember — whatever the amount of time you require to get through this preparation for physics, don't put it off. Physics courses start out in full stride — and every step forward depends to a greater or lesser extent upon the ones that precede it. The object of "Preparing for General Physics" is to assist you in keeping up with the course so that you never get behind.

HOW THIS BOOK IS ORGANIZED AND HOW TO WORK WITH IT

1. Take the pretest.

Each of the five major sections ("Rounds") in this book begins with a timed pretest. Taking this test should give you an understanding of your strengths and weaknesses with respect to the skills emphasized in that round of material. In the reviews and drills that follow you can pay special attention to those areas in which you are weakest. However it would not be unwise to do most of the drills, including ones related to skills you seem to have well in hand. This can only improve your speed and accuracy and help you apply the material in a physics context.

2. Review.

Each round of material is broken into several skill areas for which a brief Review, including fully explained examples, is given. You will discover that many of the examples have a physics-like context. But not to worry — the emphasis is on the basic skills, not on any physics to which the problem may be related. The purpose is to reinforce your ability to think about word problems, while familiarizing you with some of the vocabulary of physics.

(There is also a short essay on physics following each Round which you may find interesting to read. These are not required for skills preparation, but they can help give you some insight into the subject in which you are about to invest so much of your time and effort.)

3. Do the "skill drill".

This is the heart of your preparation for general physics. Space is provided beneath or alongside each problem for your solution. After you have done your best to answer some of the problems on one page, you may compare your own responses with the book's solutions; simply turn over the page and look at the reverse side. Avoid the temptation to look at a solution before you have tried to find your own solution.

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*NOTE: The last part of the book (Round Five) can be delayed, if necessary, since the material it contains is not usually needed during the first two months of a typical physics course. However it is wise to finish this work long before you actually need it.

Examine the suggested solutions carefully. Do you understand the methods used? How do your answers compare with the book's answers? Are you ready to go on to more drill questions, or should you look back at the Review section? If you are satisfied with your understanding of the problems you have just completed, go on to the next questions, trying to build up speed without sacrificing accuracy. Continue until you have finished all the drill problems for that skill area.

4. Take the post-test.

After you have finished the drills for all the skills areas in a particular Round of work, take the post-test. Try to do this in the allotted amount of time; then check the answers at the end of the test.

The results of the post-test should show whether you need to enhance some skill areas. Go back and review the pertinent discussions; try to do the example problems on your own; work certain drill and test problems again. Then, when you are ready, move on to the next round of skill areas until you have completed all five rounds.

TOOLS OF THE TRADE

Before you start working, supply yourself with the following:

1. **A calculator.** Much of the numerical work in the Drills (but not all of it) can be aided using a handheld electronic calculator. An *inexpensive scientific calculator* which can handle numbers in both decimal and exponential form is recommended. Besides the usual arithmetic operations, you will need at least $\sqrt{x}$, **sinx**, **cosx**, **tanx**, $\sin^{-1}x$, $\cos^{-1}x$, and $\tan^{-1}x$ functions. Also **1/x** and x^2 are useful. For the last Round e^x, **logx**, and **lnx** keys are required.

2. **Scratch paper and pencil.** Although space is provided for your work in the drills, it is helpful (as it will be with actual physics assignments) to use scratch paper to help yourself organize your thoughts and make preliminary attempts. (In an actual physics course, the solutions you submit for grading should be neat and organized — this book's solutions are meant to be a reasonable model.) Except perhaps for laboratory work, pencil is the writing instrument of choice. Neat erasures are preferable to ugly scratching-out, so be sure to have a good eraser.

FINAL WORDS BEFORE YOU BEGIN

In doing the work outlined above you should keep in mind what are the goals and what are not the goals of this book:

- This book is not meant to be a physics text. Moreover, although ideas and concepts of physics are referred to in many places, you need not to have studied physics previously.

- The use of this book is not meant to substitute for the mathematics courses which are prerequisites for your physics course. The mathematics reviewed here is much more restricted in scope and is treated with far less rigor. The emphasis is on enhancing your *facility* with mathematics skills frequently used in a beginning physics course.

- Although the organization of this book is largely built around a sequence of mathematical topics, the skills which are useful in doing physics encompass more than pure mathematics. Among other things the reviews and drills in this book should help you deal with word problems, think pictorially, make diagrams, use short cuts, and be discriminating about your methods and answers.

Physics is a fascinating science which is fundamental to all other sciences — and you can master it if you work at it seriously and do not get behind. You are more likely to keep up your progress if you are not distracted and slowed down by difficulties which have little to do with the physics per se.

To help you build up the background which will permit you to concentrate on the physics itself is the purpose of this book.

Now go to the next page and begin!

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Table Of Contents

First Round — Words and Numbers

The first part of this book deals with very basic skills which you will need over and over throughout your study of physics. Some of these things—doing arithmetic, interpreting word statements, using units—are probably very familiar to you, as they are taught from the earliest grades in school. Some related topics may not be as familiar. They may even be new to you. The pretest below will give you an opportunity to see whether you can deal with this sort of material with comfort, speed, and accuracy.

PRETEST - Optimum test time: 18 minutes or less.

Have paper, pencil, and calculator on hand, although these are not required for every question. Note your starting time, then work each problem, putting your answer in the space provided. When you are done, note your ending time before checking your results against the answers given following the test.

STARTING TIME________ ANSWERS

1. Express 15,621 using scientific notation assuming the number is accurate to three significant figures. ____________

2. Express 2.34×10^{-3} in decimal form. ____________

3. Do an "order of magnitude" mental calculation (no paper, pencil, or calculator) to find the range of values in which the following expression lies: ____________

$\frac{(7.15)(8604)}{0.0702}$ lies between

(a) 1,000 and 10,000
(b) 10,000 and 100,000
(c) 100,000 and 1,000,000
(d) 1,000,000 and 10,000,000

4. Without using a calculator quickly find the approximate value (within a factor of two) of the following expression:

$$\frac{(4.52)(76.1)}{(581)(0.413)}$$ ____________

5. Use a calculator to evaluate the following numerical expression. Your answer should be expressed in scientific notation to the appropriate number of significant figures:

$$\frac{(160.)(3.169 \times 10^3)}{72.}$$ ____________

6. Without a calculator determine the value of $\sqrt{90. \times 10^{-5}}$. ____________

7. In a European supermarket you buy 10 "kilos" (kilograms) of sugar. You might guess the weight of the sugar to be close to:

(a) 10 pounds (b) 20 pounds (c) 50 pounds (d) 100 pounds ____________

8. You drive for 2 hours at a steady speed of 55 miles per hour. If you use up 5 gallons of gasoline, what is your fuel mileage (in miles per gallon)?

____

9. How many feet are there in 3.0 meters? (1 m = 39.37 in, 12 in = 1 ft.) You may use a calculator.

____

10. Write each of the following statements as an algebraic equation:

(a) Bob is 8 years older than Joe was 3 years ago. (Use B and J to represent the present ages of Bob and Joe in years.)

____

(b) If Ed were 20 pounds heavier he would be twice as heavy as Alice.

____

11. Make a simple sketch which represents the following situation: a chair tipped back on its rear legs so that they make an angle θ with respect to the floor.

12. Sketch a freehand map which illustrates the following statement. Label the drawing and show the line of sight from girl to mountain.

Having started out from her camp (C), Joan (J) walked in a north-easterly direction until after a mile she spotted the summit of Old Baldy (B) lying about 2 miles directly south.

13. A platoon of six soldiers line up in order of height. Dan, Joan, and Carl are taller than Emily, Sam, and Henry. Joan is taller than Dan, but shorter than Carl, whereas Sam is taller than Henry who, in turn, is taller than Emily. In what order do they line up? (HINT: A diagram may be useful for finding the answer.)

ENDING TIME________ ____________

ANSWERS:

1. 1.56×10^4

2. 0.00234

3. (c)- around 90,000

4. 1.5

5. 7.0×10^3

6. 3.0×10^{-2}

7. (b)- about 22 pounds

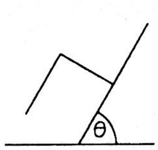
8. 22 mpg

9. 9.8 ft

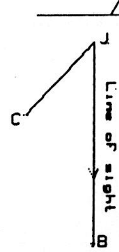
10. B = (J-3 yr) + 8yr

E + 20 lb = 2A

11.



12.



13. CJDSHE

Review 1 — Dealing with Numbers

If there is something which characterizes physics when compared with other fields of study, including its sister sciences, it is an extraordinary reliance on numbers to describe and verify conclusions. In this section we review important ways of writing numbers and how numbers can be efficiently combined to give answers with an appropriate degree of precision.

SCIENTIFIC NOTATION

Physics involves concepts which are described by numbers ranging from the unimaginably small to the astronomically large. The most convenient way to express numbers over a wide range is called "scientific notation."

In scientific notation numbers are represented by the product of a multiplying factor and a power of ten.

(A "power of ten" is the number 10 raised to an integer exponent.) Negative as well as positive multiplying factors and exponents may be used. Some examples of numbers written in scientific notation and their ordinary decimal equivalents are the following:

$$35.00 \times 10^6 = 35{,}000{,}000$$
$$-2.70 \times 10^3 = -2700$$
$$4.3 \times 10^{-2} = 0.043$$

Besides being a compact way of writing quantities over a wide range of values, scientific notation also gives some idea of how precisely a quantity is known. All the digits in the multiplying factor in front of the power of ten are considered *significant* — they convey meaningful information and are to be "taken seriously." More will be said about "significant figures" later on.

Translating into scientific notation and back again. The above examples illustrate the meaning of the power of ten: 10^6 is simply a way of writing 1,000,000, 10^3 means 1000, and 10^{-2} means 0.01.

Given a number in decimal form, to find the power ten in its scientific notation equivalent: the exponent is the number of places from where you wish to place the decimal point in the multiplying factor to where it lies in the decimal form.

The sign of the exponent depends on whether you count off places to the right (+) or left (–). How this procedure is used to work out one of the numerical examples above is discussed here:

Write 2700 in scientific notation, assuming 3 significant figures.

DISCUSSION: The multiplying factor is written with 3 digits to indicate three significant figures; a convenient value is 2.70 . (We could have also chosen 0.270, or 27.0, etc.; the power of ten simply would have to be different.) Counting off places to the *right* this way:

2 ⁽¹⁾ 7 ⁽²⁾ 0 ⁽³⁾ 0. → the exponent in the power of ten is +3.

Thus the scientific notation equivalent is 2.70×10^3.

Here is a second example in which the scientific notation equivalent contains a *negative* exponent in the power of ten:

Write 0.043 in scientific notation, assuming 2 significant figures.

DISCUSSION: Choose the multiplying factor to be 4.3. Then counting off to the *left* this way:

0 . 0 4 3 → the exponent is −2.

The equivalent is thus 4.3×10^{-2}.

To go from scientific notation to a decimal form the above procedure is reversed, as in this example:

Write 35.00×10^6 in decimal form.

DISCUSSION: Move the decimal point in the multiplying factor 6 places to the *right*, as follows.(In other words multiply by 1,000,000.)

3 5 . 0 0 0 0 0 0 → 35,000,000.

The final result is ambiguous about the number of significant figures.

CALCULATIONS AND ESTIMATES

Calculators. The use of electronic calculators is encouraged for finding numerical answers to physics problems, but this should be done only when appropriate and only with intelligence. Operating a calculator is not entirely error-free, but mistakes can often be caught if one has the habit of making quick estimates.

When your answer looks ridiculous it is wise to recheck your logic and your calculation!

Approximate answers. Scientific notation makes it simple to multiply and divide numbers to get an approximate answer without a calculator. As an example consider the following combination of three numbers:

$$\frac{(2700)(0.043)}{35{,}000{,}000} = ?$$

To get a quick estimate, the expression can be rewritten in scientific notation with each multiplying factor rounded off to a single digit. The calculation is then carried out using the usual rules for multiplying and dividing, as follows:

The multiplying factors are separately combined to find the multiplying factor in the answer. To find the exponent in the answer the individual exponents are added if there is a multiplication or subtracted if there is a division.

Thus for the combination of numbers given above

$$\frac{(2.70\times10^{3})(4.3\times10^{-2})}{3.500\times10^{7}} \simeq \frac{(3)(4)}{4}\times10^{(3-2-7)} = 3\times10^{-6}$$

(The symbol $\simeq$ means "approximately equal." The "slash marks" / indicate that the 4's divide exactly into one another to give a value of 1, i.e., they "cancel.")

Order of magnitude estimates. With practice, getting a sense of the size of an answer by doing rough calculations can become routine and often can be done in one's head. This is particularly true when only an "order of magnitude" answer is wanted. An order of magnitude estimate is usually considered one in which the exact value is rounded off to the nearest factor of ten. Thus an order of magnitude result for the above calculation is

$$\frac{(2700)(0.043)}{35{,}000{,}000} \sim (10^{3})(10^{-2})(10^{-7}) = 10^{-6}$$

(The symbol $\sim$ stands for "is order of magnitude of-.")

SIGNIFICANT FIGURES

The number of digits used to write out a number can tell us how precise the number is meant to be. For instance something described by the number 2.54 can be supposed to have an actual value between 2.535 and 2.545. But simply writing 2.54 means we are uncertain about just where in that range the actual value falls. Only three figures are "significant".

Calculations and significant figures. When 2.54 is multiplied or divided by a number which is more precisely known, the result is only meaningful to three significant figures. For example, the most accurate value we can give for the product (2.54)(3.213) is 8.16. It is not 8.16102, the number which a calculator might display.

> *When multiplying or dividing, the answer cannot be more precise than the least precise factor in the calculation; this usually means that the answer has the same number of digits as the factor with the least number of significant figures.*

(For numbers somewhat greater than a multiple of ten, an extra digit does not imply greater precision. For instance, the three digit number 105, is just about as precise as the two digit number 95.)

In any event, do not confuse the number of decimal places with the number of significant figures. The number 0.0254 has the same number of significant figures as does 2.54; both should be regarded as being known to about 1 part in 254.

> *It is bad form as well as incorrect to write down all the digits displayed by your calculator. Your answer should be rounded off to the appropriate number of significant figures.*

Exact numbers. There is an exception to the above rule about significant figures of products and quotients. Some numbers have, by implication, an unlimited accuracy, and therefore cannot set a limit on the number of significant figures in the answer. For example, suppose we wish to calculate the diameter of a circle, given its radius. If the radius is 3.4 inches then the correct calculation is:

diameter = (2)(3.4 inches) = 6.8 inches.