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ELEMENTARY NONLINEAR ELECTRONIC CIRCUITS

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Prentice-Hall, Inc. Englewood Cliffs, N.J.

PRENTICE-HALL INTERNATIONAL, INC., *London*
PRENTICE-HALL OF AUSTRALIA, PTY, LTD., *Sydney*
PRENTICE-HALL OF CANADA, LTD., *Toronto*
PRENTICE-HALL OF INDIA (PRIVATE) LTD., *New Delhi*
PRENTICE-HALL OF JAPAN, INC., *Tokyo*

Library of Congress Catalog Card Number 66–27953
Printed in the United States of America
25941C

Current printing (last digit):
10 9 8 7 6 5 4 3 2 1

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PREFACE

A large number of excellent text books have been written on the subject of electronic circuits. Why, then, write another?

The answer is that in recent years books tend to emphasize the small signal, linear aspects of the circuits. In so doing they certainly have contributed to a clearer understanding of this phase of electronics, but we must face up to a fundamental truth about electronic devices: their behavior is *nonlinear*. If this fact is neglected, the consequences of nonlinearity, good or bad, cannot be considered, and worse—may, in fact, be forgotten. Yet many of these factors such as distortion, frequency translation and multiplication, and periodic signal amplitudes in oscillatory circuits are of primary importance in many applications. Some texts touch on these subjects by considering only the most gross features of nonlinearity through use of the piecewise linear technique.

It seems, then, that there is need for a companion text to this literature—a text which emphasizes the nonlinear character of electronic devices and its effect in some sample circuit configurations. This book has been written as such a supplementary text. As the title indicates, the methods used are elementary and teachable at the undergraduate level.

It is inevitable that a text so conceived will include much review material. Its inclusion, however, permits the establishment of a consistent method of approach and notation. With the latter, I have used current standards, despite the recent trend to depart from them.

ELEMENTARY NONLINEAR ELECTRONIC CIRCUITS

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Chapter ONE

ONE-PORT DEVICES

Electronic devices inherently exhibit rather complicated behavior: first, their behavior is nonlinear; second, it is frequency dependent because of capacitive and transit-time effects; third, it is corrupted by internally generated noise signals, and fourth, it may be influenced by environmental conditions such as temperature. It is therefore rather difficult to find some means for expressing all aspects of this behavior in a simplified form.

It is fortunate, however, that this is usually unnecessary since in any given application of an electronic device not *all* these aspects are equally important or relevant. For example, it is usually true that when the signals applied to the device are so large that nonlinearity must be considered, they are also so much larger than the internally generated noise signals that the latter may be neglected.

Thus in any given application we seek a simplified representation of only the cogent aspects of the device behavior, omitting those aspects which have little or no effect in the particular application. This process of neglecting irrelevant behavior and representing the important aspects in simplified form is known as *modeling*.

In this book we shall be concerned primarily with the nonlinearity of electronic devices and we shall use the measured terminal characteristic curves of a device as the basic means of describing this at low frequencies. Our attempts at modeling will then follow two paths: the first will be mathematical. Here we derive an equation giving an *approximate* fit to the device curves, either in a gross manner, or over only a limited range. The second path will lead to the arrangement of a set of linear circuit elements into a configuration whose response *approximates* the device curves. Notice, then, that we shall use three methods of approximating the performance of electronic devices—curves, equations, and circuit models. As might be expected all three methods of specification are equivalent for a particular electronic device only within the limits under which they are measured or derived.

From the point of view of modeling, we often face the problem of obtaining suitable device curves. This is because, at the present state of the art, it is impossible to manufacture a number of electronic devices which have *identical* characteristics even though they may have *similar* characteristics and hence may be grouped together under a certain identifying type number or designation.

A good example of this is afforded by Fig. 1-1 which displays the measured curves of the two sections of a 6AL5 twin-diode vacuum tube which was chosen at random. From the curves we may note, for example, that at $e = 1.6$ volts, the currents of the two sections, which presumably should be identical, differ by roughly 8 per cent. It is not feasible for device manufacturers to furnish curves with each diode; instead the behavior of such a device type is usually described by *average* curves (or average parameter values) which are made available by the manufacturer. It must be remem-

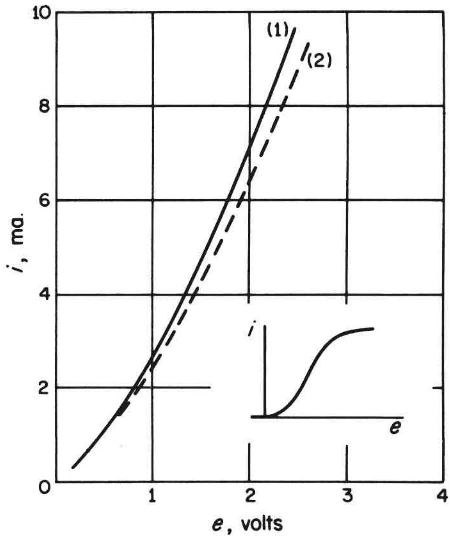

Fig. 1-1. Characteristic curves for the two sections of a 6AL5 vacuum twin diode. The inset shows a typical curve for e large enough for saturation and cutoff.

bered, however, that any particular sample of the type may differ considerably from these average specifications. In vacuum devices this deviation may run as high as ± 20 per cent and in semiconductor devices as much as ± 50 per cent. The latter are quite temperature sensitive and even greater deviations may occur when the operating temperature differs markedly from the published values. For these reasons we find that manufacturers are often reluctant to furnish even average curves for the semiconductor devices.

The analysis and design of electronic circuits, then, is not an exact business when we work from published average data. If a basic error of up to 50 per cent is present in specifying the device behavior, calculating circuit performance to 3 or 4 significant figures is pointless. In fact in many instances a chief concern of design is to minimize the effect of device parameter variation.

A. NONLINEARITY, NONLINEAR RESISTANCE

Since we shall be directing our primary attention to the subject of nonlinearity, some clarification of notation should be made. At low frequencies, electronic devices are basically resistive in nature, i.e., a curve (or family of curves) in the current-voltage plane defines the device operation. Now if a passive resistance element is linear, it may be described for all values of e and i by the equation $i = e/R$, where R is the linear resistance. The corresponding curve in the i vs e plane would be a straight line through the origin with a slope of $m = 1/R$. Notice that m has the dimension of conductance and can also be symbolized by G, the conductance of the passive element.

A nonlinear passive resistance, which we may represent by the symbol $\mathscr{R}$, will, then, depart from this ideal. Two examples are shown in Fig. 1-2. At (a) we see a curve which is nonlinear in the usual sense that it is a curved rather than a straight line. This sort of curve is said to display a *soft* nonlinearity.

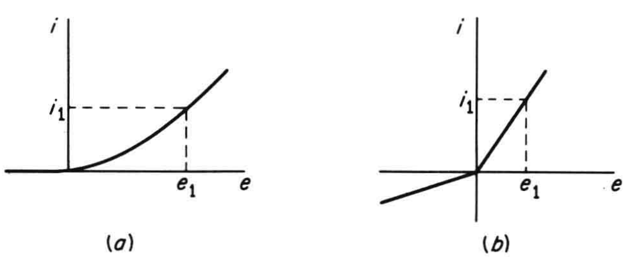

Fig. 1-2. Nonlinear resistance curves. (a) Soft nonlinearity. (b) Hard nonlinearity of piecewise linear form.

At (b) we observe a nonlinearity of a different type in that no curved region is present. The curve comprises two linear segments with a "break-point" or change of slope which, in this instance, lies at the origin. This type of curve is often called *piecewise linear*, abbreviated PWL, and is said to display a *hard* nonlinearity.

It is apparent that no single value of resistance, R, can be associated with the nonlinear resistance in Fig. 1-2. By convention we define two resistances as follows:

$$R_b = \text{static or total resistance}$$

$$= \frac{e}{i} \tag{1–1}$$

$$r_p = \text{dynamic, differential, or incremental resistance}$$

$$= \frac{de}{di} \tag{1–2}$$

It is apparent in Fig. 1-2(b) that $R_b = r_p$ at any point, and further that R_b is constant for positive e and constant at a different value for negative e. In Fig. 1-2(a), R_b and r_p vary from point to point and are never equal for positive e. For negative e, however, and at the origin, $R_b = r_p = \infty$. As we encounter other nonlinear characteristics that differ in some manner from those of Fig. 1-2, we shall observe other properties of the total and incremental resistances.

In the remainder of this chapter we shall consider the characteristics of some common one-port or two-terminal, electronic devices. Their basic behavior is described by a single $\mathscr{R}$ curve. We shall also investigate means for modeling these characteristics.

B. VACUUM DIODES

The characteristic curves of typical vacuum diodes have the form shown in Fig. 1-1 which is plotted for the 6AL5 twin diode. Shown at the inset is the curve for a larger range of positive e values. For large e where current is less dependent upon voltage the operation is said to be *saturated*. In this saturation region, which lies above the normal range of current and voltage, current is limited by emission from the cathode. For negative e, $i = 0$, i. e., the current is cut off. The reverse voltage region is often called the *cutoff* region.

B-1 POWER LAW

In trying to derive a mathematical model or equation to describe the device curve, we find it is helpful to replot the positive e data, say of curve (1) in