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PREFACE

This volume comprises the proceedings of the second Symposium on Nonlinear Semigroups, Partial Differential Equations, and Attractors held at Howard University in Washington, D.C. on August 3-7, 1987. The proceedings of the first symposium, held two years earlier, was published as volume 1248 of this Lecture Notes Series. The present Symposium was made possible by grant support from the following funding agencies: U.S. Air Force Office of Scientific Research, U.S. Army Research Office, U.S. Department of Energy, National Aeronautics and Space Administration, U.S. National Science Foundation, and the U.S. Office of Naval Research.

The local support committee consisted of James A. Donaldson (Howard University), Lawrence C. Evans (University of Maryland), James Sandefur and Andrew Vogt (Georgetown University), and Michael C. Reed (Duke University) whom we thank for their helpful advice.

The Symposium brought together a total of 76 distinguished researchers in the Mathematical, Physical, and Engineering sciences working on analytical, topological, and numerical aspects of a large variety of nonlinear partial differential equations. This multidisciplinary character of the Symposium attendees brought about a productive exchange of ideas on various approaches to current problems in applied mathematics.

In the past twenty or so years, there has been an increased interest in the study of nonlinear models of physical, chemical, biological, and engineering systems. The evolution of new analytical and topological methods for the study of infinite dimensional systems concurrently with the advent of large-scale computers and efficient algorithms has served to further stimulate research on problems that were considered impossible to attack just a few years ago.

There are many problems in the natural sciences which are naturally formulated in terms of nonlinear partial differential equations. Over the years, new methods and special techniques have evolved for the study of nonlinear problems. In addition, there has been a great deal of recent activity devoted to the study of stochastic ("chaotic") solutions to nonlinear differential equations in cases where the "conventional wisdom" physics leads us to believe that only deterministic solutions exist. Many of these studies have been numerical and confined to either maps or ordinary differential equations, which are more easily analyzed than are partial differential equations. Recently however, various methods have been developed for the study of partial differential equations which, because of the complicated nature of these equations, are a valued addition to the mathematical sciences.

A general method that has been very effective in the treatment of large classes of nonlinear partial differential equations makes use of the theory of nonlinear semigroups. Given appropriate conditions, these semigroups generate solutions to nonlinear evolution equations which may have a compact global attractor with finite Hausdorff dimension. This type of analysis applies to numerous nonlinear partial differential equations. Most of the papers contained in the present collection are concerned with nonlinear semigroups.

A major contribution to the multidisciplinary character of the Symposium is the existence of the Large Space Structures Institute at Howard University. This is a special institute devoted to the study of physical, engineering, and mathematical problems that arise in the development of large structures (space-stations) to support life in space. It is a joint effort of the departments of mathematics and of electrical, mechanical, and civil engineering. One afternoon session of the Symposium was devoted to the presentation and general discussion of new classes of nonlinear problems that model certain components of these structures. The rationale was to introduce direct interaction among the symposium participants and some of the research engineers concerned with analyses of these types of problems. We feel that this interaction among scientists with varying backgrounds and interests gave the symposium a distinctive flavor and provided a unique cross-fertilization of ideas.

Tepper L. Gill
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Washington, D.C.
November 1988

Other contributions to the symposium:

M.E. Aluko, Controller-induced Bifurcations in a Distributed Particulate (crystallizer) non-isothermal System.

Anthony K. Amos, Nonlinear P.D.E. Issues for Space Structure Problems of Interest to AFOSR

Stuart Antman, Asymptotics of Quasilinear Equations of Viscoelasticity

Joel D. Avrin, The Semilinear Parabolic Equations of Electrophoretic Separation

Stavros A. Belbas, Parabolic Nonlinear Partial Differential Equations arising in Stochastic Game Theory

Melvyn S. Berger, Vortex motions in Mathematics and Fluids, their Bifurcation and Instabilities

Nam P. Bhatia, Separated Loops and an Extension of Sarkovskii's Theorem

Shui-Nee Chow, Bifurcation of Homoclinic Orbits

Michael G. Crandall, Hamilton-Jacobi Equations in Infinite Dimensions

Lawrence C. Evans, Hamilton-Jacobi Equations in Large Deviation

Tepper L. Gill, Time-Ordered Nonlinear Evolutions

Carlos Hardy, Generating Quantum Energy Bounds by the Moment method - a Linear Programming Approach

Christopher K.R.T. Jones, Behavior of the Nonlinear Wave Equation Near an Equilibrium Solution

Jack Lagnese, Infinite Horizon Linear-Quadratic Problems for Plates

John Mallet-Paret, Poincare-Bendixon Theory for Reaction Diffusion Equations

David W. McLaughlin, The Semiclassical Limit of a Nonlinear Schrodinger Equation

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STATE-SPACE FORMULATION FOR FUNCTIONAL DIFFERENTIAL EQUATIONS OF NEUTRAL-TYPE

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1. INTRODUCTION

In recent years various classes of functional differential equations (FDE) have been studied in the context of functional analytic semigroup theory (see e.g. [1], [2], [4], [10] and the references therein). The basic approach in this direction is to establish equivalence between the FDE and an abstract evolution equation (AEE) in some appropriate state space (i.e. space of initial data.) Furthermore if the associated AEE is well-posed, then the equivalence between the FDE and the AEE provides an excellent framework to study approximation techniques for systems governed by FDEs. Well-posedness is dependent on the choice of a state-space and the choice of an appropriate state-space is tied to the particular application. It was shown (see [1], [2] for retarded and [10], [12] for neutral functional differential equations) that certain classes of FDEs can be transformed into well-posed Cauchy problems in the product spaces $\mathbb{R}^n \times L_p$. The product space model also proved to be very useful

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in investigating a variety of control and identification problems for problems governed by FDEs ([2], [3], [5], [10], [12]).

In this paper we extend previous results concerning the well-posedness of FDEs on the product spaces $\mathbb{R}^n \times L_p$. In particular we develop general necessary and sufficient conditions for the well-posedness of neutral systems to include non-atomic neutral equations and certain classes of singular integro-differential equations.

2. WELL-POSEDNESS OF FDEs ON $\mathbb{R}^n \times L_p$

We consider the FDE of neutral-type

$$\frac{d}{dt} Dx_t = Lx_t + f(t) \tag{1}$$

with initial data

$$Dx_0(\cdot) = \eta;\ x_0(s) = \varphi(s), \quad -r \leq s < 0 \tag{2}$$

where D and L are linear $\mathbb{R}^n$-valued operators with domains $\mathscr{D}(D)$ and $\mathscr{D}(L)$ subspaces of the Lebesgue-measurable $\mathbb{R}^n$-valued functions on $[-r,0]$. We assume that $W^{1,p} \subseteq \mathscr{D}(D) \cap \mathscr{D}(L)$, $(\eta,\varphi) \in \mathbb{R}^n \times L_p([-r,0],\mathbb{R}^n)$(or shortly $\mathbb{R}^n \times L_p$), $f \in L_{p,loc}$, $1 \leq p < \infty$, $0 \leq r < \infty$ and n is a positive integer.

Define the linear operator $\mathscr{A}$ with domain

$$\mathscr{D}(\mathscr{A}) = \{(\eta.\varphi) \in \mathbb{R}^n \times L_p / \varphi \in W^{1,p}, D\varphi = \eta\} \tag{3}$$

by

$$\mathscr{A}(\eta,\varphi) = (L\varphi\ \dot{\varphi}) \tag{4}$$

and consider the AEE

$$\dot{z}(t) = \mathscr{A}z(t) + (f(t),0) \tag{5}$$

with

$$z(0) = z_0 = (\eta,\varphi). \tag{6}$$

The well-posedness of the FDE (1)-(2) and the AEE (5)-(6) has been studied extensively ([5], [10], [13]) assuming various continuity conditions on L and D. It is known (see [13]) that, if L and D belong to $\mathscr{B}(W^{1,p},\mathbb{R}^n)$, then the FDE (1)-(2) is well-posed if and only if the AEE (5)-(6) is well-posed (i.e., if $\mathscr{A}$ defined by (3)-(4) is the infinitesimal generator of a C_0-semigroup $\{S(t)\}_{t\geq 0}$ on $\mathbb{R}^n\times L_p$). It is also known (see [5]) that if $\mathscr{A}$ generates a C_0-semigroup on $\mathbb{R}^n\times L_p$, then it is necessary that i) $L\in\mathscr{B}(W^{1,p},\mathbb{R}^n)$ and $\bar{D}\in\mathscr{B}(W^{1,p},\mathbb{R}^n)$ and ii) $D\notin\mathscr{B}(L_p,\mathbb{R}^n)$. Concerning the sufficiency conditions for the well-posedness of the FDE (1)-(2) it is known that i^*) $L\in\mathscr{B}(W^{1,p},\mathbb{R}^n)$; ii^*), $D\in\mathscr{B}(C,\mathbb{R}^n)$ and D is atomic at zero imply well-posedness, but condition ii^*) above is not necessary (see Remarks 2 and 3).

<u>Remark 1</u>: Observe that if D is defined on $W^{1,p}$ by $D\varphi=\varphi(0)$ (i.e., when the FDE is retarded), then $L\in\mathscr{B}(W^{1,p},\mathbb{R}^n)$ is necessary and sufficient condition for the well-posedness of the FDE (1)-(2), because $D\in\mathscr{B}(W^{1,p},\mathbb{R}^n)$, $D\notin\mathscr{B}(L_p,\mathbb{R}^n)$, D has a bounded extension to C, and D is atomic at zero.

<u>Remark 2</u>: Consider the scalar FDE of the form (1) with $L\varphi\equiv 0$ and $D\varphi\equiv\int_{-r}^{0}\varphi(s)|s|^{-\alpha}ds$; $0<\alpha<1$. It can be shown (see [5], [9]) that the FDE is well-posed on $\mathbb{R}\times L_p$ if and only if $p<1/(1-\alpha)$. This example demonstrates that: I): $L\in\mathscr{B}(W^{1,p},\mathbb{R})$, $D\in\mathscr{B}(W^{1,p},\mathbb{R})$ and $D\notin\mathscr{B}(L_p,\mathbb{R})$ is not sufficient (i.e., consider $p=1/(1-\alpha)$), and II): $L\in\mathscr{B}(W^{1,p},\mathbb{R})$, $D\in\mathscr{B}(C,\mathbb{R})$ and D is atomic at zero is not necessary (i.e., consider $p<1/(1-\alpha)$) for the well-posedness of the FDE on $\mathbb{R}\times L_p$.

<u>Remark 3:</u> The authors studied (see [16]) a scalar equation of the form (1) with $L\varphi\equiv 0$ and $D\varphi\equiv\int_{-r}^{0}\dot{\varphi}(s)|s|^{-\alpha}ds$; $0<\alpha<1$, and established well-posedness of this equation on $\mathbb{R}\times L_p$ for $p>1/(1-\alpha)$. Since D does not have a bounded extension to C, this example implies, that $D\in\mathscr{B}(C,\mathbb{R})$ is not necessary for well-posedness.

<u>Remark 4:</u> Kappel and Zhang [9] considered the problem (1)-(2) in the space $C([-r,0],\mathbb{R})$ under the assumptions that D belongs to $\mathscr{B}(C,\mathbb{R})$ and $L\equiv 0$. They proved that the well-posedness of the FDE (1)-(2) in

the state space C implies that D is weakly atomic at zero.

Remark 5: At this point the most general necessary condition for well-posedness is given in [16]. Assuming only that $L \in \mathcal{B}(W^{1,p}, \mathbb{R}^n)$, $D \in \mathcal{B}(W^{1,p}, \mathbb{R}^n)$ and $D \notin \mathcal{B}(L_p, \mathbb{R}^n)$ it was shown that $\mathcal{A}$ defined by (3)-(4) is the generator of a C_0-semigroup only if the $n \times n$ matrix valued function $D(e^{\lambda \cdot} I) = [D(e^{\lambda \cdot} e_1) | D(e^{\lambda \cdot} e_2) | \ldots | D(e^{\lambda \cdot} e_n)]$ exhibits "certain" asymptotic behavior as $\lambda \to \infty$; $(\lambda \in \mathbb{R})$.

As the previous Remarks indicate it is not known if there is a set of conditions on L and D that are both necessary and sufficient for $\mathcal{A}$ to generate a C_0-semigroup on $\mathbb{R}^n \times L_p$.

In the next section we consider a relatively large class of nonatomic neutral equations (NNFDE)(i.e., D is not necessarily atomic at zero) and give conditions which imply the well-posedness of these equations on $\mathbb{R}^n \times L_p$ for certain values of p.

3. NONATOMIC NEUTRAL EQUATIONS (NNFDEs)

In this section we consider the class of neutral functional differential equations given by (1)-(2) and provide conditions on D and L that imply the well-posedness of these equations on the product spaces $\mathbb{R}^n \times L_p$. Our results extend the results of Burns, Herdman and Stech [5] in that we obtain the well-posedness of (1)-(2) without assuming that the operator D be atomic at zero.

Our approach is based on the fact that the FDE (1)-(2) is well-posed on $\mathbb{R}^n \times L_p$ provided that $\mathcal{A}$ defined by (3)-(4) is an infinitesimal generator of a C_0-semigroup on $\mathbb{R}^n \times L_p$. Thus, our main result will establish sufficient conditions on D and L implying that $\mathcal{A}$ generates a C_0-semigroup on $\mathbb{R}^n \times L_p$. We assume that $W^{1,p} \subseteq \mathcal{D}(D) \cap \mathcal{D}(L)$, where the operators L and D satisfy the following conditions:

(H1) The operator $D \in \mathcal{B}(C, \mathbb{R}^n)$ has representation

$$D\varphi = \int_{-r}^{0} [A d\beta(s) + d\mu(s)]\varphi(s) \tag{7}$$

where the $n \times n$ matrix functions μ, β and the nonsingular matrix A

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satisfy: i) μ is of bounded variation on $[-r,0]$, $\mu(0) = 0$, μ is left continuous on $[-r,0]$ and $\lim_{\epsilon\to0^+} \mathrm{Var}_{[-\epsilon,0]}\mu = 0$; ii) β is a diagonal matrix and there exists an integer k; $0 \le k \le n$, such that the entries, β_{ii}, satisfy $\beta_{ii}(s) = -\rho(-s)$ for $i \le k$, $\beta_{ii}(s) = -(-s)^{1-\alpha_i}/(1-\alpha_i)$ for $i > k$, where $\rho:[0,r] \to \mathbb{R}$, $\rho(0) = 0$, $\rho(s) = 1$ for $s > 0$ and the constants α_i; $i > k$, satisfy $0 < \alpha_i < 1$; iii) A has the block matrix form $A = \mathrm{diag}(A_{11}, A_{22})$ where A_{11} and A_{22} are $k \times k$ and $\ell \times \ell$ matrices, respectively, with $k + \ell = n$.

(H2) The operator $L \in \mathcal{B}(W^{1,p},\mathbb{R}^n)$ has representation

$$L\varphi = B\varphi(0) + \int_{-r}^{0} B(s)\varphi(s)ds \tag{8}$$

where B is a $n \times n$ constant matrix and $B(\cdot)$ is a $n \times n$ matrix valued function having column vectors in L_q, $\frac{1}{p} + \frac{1}{q} = 1$.

(H3) The $n \times n$ matrix valued function α defined on $[-r,0]$ by

$$\alpha(s) = \mu(s) - \int_{0}^{s} B(u)du$$

has the representation $\alpha(s) = [\alpha_1(s) \mid \alpha_2(s)]^T$ where α_1 and α_2 are $k \times n$ and $\ell \times n$ matrix valued functions, α_2 is absolutely continuous and $\dot{\alpha}_2$ is of bounded variation on $[-r,0]$.

Remark 6: If (H1) holds, then we may assume, without loss of generality, that $A = I$. In the event that the original nonsingular matrix A is not the identity matrix, one can multiply (1)-(2) by A^{-1}, introduce $\hat{\mu} = A^{-1}\mu$; $\hat{B} = A^{-1}B$, $\hat{B}(\cdot) = A^{-1}B(\cdot)$ and reduce the original problem to the case of $A = I$.

Remark 7: The operators L and D defined in (H1) and (H2) belong to $\mathcal{B}(W^{1,p},\mathbb{R}^n)$. In the case $k = n$ (i.e., $\ell = 0$), the operator D is atomic at zero and the sufficiency result of Burns, Herdman and Stech [5, Theorem 2.3] yields the well-posedness of (1)-(2) on $\mathbb{R}^n \times L_p$. The case $k = 0$, $\ell = n = 1$, $\mu(\cdot) \equiv 0$, $L \equiv 0$ and $f \equiv 0$ was also considered in [5] and well-posedness of (1)-(2) on $\mathbb{R}\times L_p$ was established for $1 \le p < 1/(1-\alpha_1)$.

In Theorem 1 below we establish the well-posedness of a large class of FDEs (1)-(2) on the product spaces $\mathbb{R}^n \times L_p$ for certain values of p.

<u>Theorem 1:</u> Let $\alpha_{min} = \min_{i>k} \{\alpha_i\}$, $1 \le p < 1/(1-\alpha_{min})$ and $D \in \mathcal{B}(C, \mathbb{R}^n)$, $L \in \mathcal{B}(W^{1,p}, \mathbb{R}^n)$ have representations (7), (8), respectively. If conditions H1) - H3) are satisfied, then the system

$$y(t) = \eta + \int_0^t (Lx_u + f(u))du, \quad t > 0$$

$$Dx_t = y(t) \text{ a.e. on } [0,\infty)$$

with initial condition

$$x_0(s) = \varphi(s) \text{ a.e. on } [-r,0]$$

has a unique solution $y(t) = y(t;\eta,\varphi,f)$, $x(t) = x(t;\eta,\varphi,f)$ defined on $[0,\infty)$ and $[-r,\infty]$, respectively such that $y(\cdot)$ is continuous and $x_t(\cdot) \in L_p$. Moreover, for $t_1 > 0$ the mapping $(\eta,\varphi,f) \to (y(\cdot;\eta,\varphi,f), x(\cdot;\eta,\varphi,f))$ from $\mathbb{R}^n \times L_p([0,t_1], \mathbb{R}^n)$ into $C([0,t_1], \mathbb{R}^n) \times L_p([-r,t_1], \mathbb{R}^n)$ is continuous.

<u>Proof</u>: First we note that to prove the theorem it is sufficient to consider the problem

$$Dx_t = \eta + \int_0^t [Lx_u + f(u)]du \quad \text{a.e. on } [0,\infty)$$

$$x(s) = \varphi(s) \qquad \text{a.e. on } [-r,0]. \tag{9}$$

Using the representations (7) and (8) and changing the order of integration of the integral involving $B(s)$, (9) becomes

$$\begin{aligned}&\int_{-r}^0 [d\beta(s) + d\mu(s)]x(t+s) - \int_{-r}^0 B(s)x(t+s)ds \\ &-B\int_0^t x(u)du = \eta - \int_{-r}^0 B(s)\varphi(s)ds + \int_0^t f(u)du.\end{aligned} \tag{10}$$

For $0 < t \leq r$ we can rewrite (10) as

$$\int_0^t [d\overline{\beta}(s) + d\overline{\gamma}(s)]x(t-s) = g(t), \tag{11}$$

where $\overline{\beta}(s) = -\beta(-s)$, $\overline{\gamma}(s) = -\gamma(-s)$, $\gamma(s) = \alpha(s) - Bs$ and

$$g(t) = \eta - \int_{-r}^{0} B(s)\varphi(s)ds + \int_0^t f(u)du - \int_{-r}^{-t} [d\beta(s) + d\alpha(s)]\varphi(t+s) . \tag{12}$$

Note that $\overline{\alpha}$, $\overline{\gamma} \in NBV([0,r],\mathbb{R}^{nxn})$, where $NBV([0,r],\mathbb{R}^{nxn})$ denotes the space of nxn matrix-valued functions which are of bounded variation on $[0,r]$, right continuous for $0 < s < r$, and take the value 0 at $s = 0$.

Define $h(\cdot) \in NBV([0,r],\mathbb{R}^{nxn})$ by $h(\cdot) = [h_{ij}(\cdot)]$, $1 \leq i,j \leq n$, where for all $1 \leq j \leq n$

$$h_{ij}(s) = \begin{cases} \overline{\gamma}_{ij}(s) & i \leq k, \\ \frac{d}{ds}\left[\frac{\sin\alpha_i\pi}{\pi} \int_0^s (s-u)^{\alpha_i - 1} \overline{\gamma}_{ij}(u)du\right] & i > k . \end{cases} \tag{13}$$

It can be shown (see [15] for details) that for $0 < t \leq r$, equation (11) is equivalent to

$$\int_0^t d\overline{\beta}(s)\, w(t-s) = g(t), \tag{14}$$

where

$$w(t) = x(t) + \int_0^t dh(u)x(t-u) . \tag{15}$$

Recall that (15) is a Volterra-Stieltjes integral equation. Our assumptions guarantee that $h \in NBV([0,r],\mathbb{R}^{nxn})$ and that h is continuous at 0 from the right, i.e.

$$\lim_{t\to 0^+} h(t) = 0 . \tag{16}$$

Note that (16) is a sufficient condition (see for example [12]) for the existence and uniqueness of the fundamental solution, $\zeta \in NBV([0,r],\mathbb{R}^{nxn})$, of equation (15) Moreover, if $x(\cdot)$ the unique solution of (15), then $X(\cdot)$ belongs to $L_p([0,r],\mathbb{R}^n)$ and has representation

$$x(t) = \int_0^t d\zeta(s)w(t-s). \tag{17}$$

Continuous dependence of x on w with respect to the L_p - norm is an immediate consequence of (17). In particular, for $0 < t_1 \leq r$, we have the estimate

$$||x||_{L_p([0,t_1],\ \mathbb{R}^n)} \leq Var_{[0,t_1]}(h)||w||_{L_p([0,t_1],\ \mathbb{R}^n)}. \tag{18}$$

Next we consider equation (14) in component form, i.e.

$$\int_0^t d\overline{\beta}_i(s)w_i(t-s) = g_i((t);\ 1 \leq i \leq n. \tag{19}$$

Using the special form of $\overline{\beta}(\cdot)$, equation (19) implies that

$$w_i(t) = g_i(t) \quad , \quad i \leq k \tag{20}$$

and

$$\int_0^t s^{-\alpha_i} w_i(t-s)ds = g_i(t) \quad , \quad i > k . \tag{21}$$

For $t \in (0,r]$ define G_i by

$$G_i(t) \equiv \int_0^t (t-s)^{\alpha_i - 1} g_i(s)ds \quad , \quad i > k \quad . \tag{22}$$

Note that if $(\eta,\varphi) \in \mathbb{R}^n x L_p$, $f \in L_p([0,r],\ \mathbb{R}^n)$ and $1 \leq p < 1/(1-\alpha_{min})$, then $g_i \in L_p([0,r],\mathbb{R})$, $1 \leq i \leq k$, and $G_i \in W^{1,p}([0,r],\mathbb{R})$, $i > k$ (see [5], [9] or [15] for details). Therefore, w_i, the ith component of the unique L_p solution of (14), is given by

$$w_i(t) = \begin{cases} g_i(t) & , \quad \text{for } i \leq k \\ \frac{d}{dt}\left[\frac{\sin\alpha_i\pi}{\pi} G_i(t)\right], & \text{for } i > k . \end{cases} \tag{23}$$

Moreover, there exists a nonnegative, increasing function $M \in C([0,r],R)$ such that

$$||w||_{L_p([0,t],\ \mathbb{R}^n)} \leq M(t)||(\eta,\varphi,f)||_{\mathbb{R}^n \times L_p \times L_p([0,r],\mathbb{R}^n)} \tag{24}$$

for $t \in [0,r]$ (see[15]). Substituting (23) into (17) we get a representation for the unique, L_p-solution to (9) for $0 \leq t \leq r$. Continuity of the mapping $(\eta,\varphi,f) \to (y(\cdot;\eta,\varphi,f),x(\cdot;\eta,\varphi,f))$ from $\mathbb{R}^n x L_p([0,t_1],\mathbb{R}^n)$ into $C([0,t_1],\mathbb{R}^n) x L_p([-r,t_1],\mathbb{R}^n)$ is an easy consequence of the estimates (18) and (24) for $0 < t_1 \leq r$. The "method of steps" is employed to extend the above results to $[0,+\infty)$. □

As an immediate consequence of Theorem 1 and the equivalence of the FDE (1)-(2) and the AEE (5)-(6) we have the following sufficiency result.

Theorem 2: If (H1)-(H3) hold, $1 \leq p < 1/(1-\alpha_{min})$, and D and L have the representations (7) and (8), then $\mathcal{A}$ defined by (3)-(4) is the infinitesimal generator of a C_0-semigroup on $\mathbb{R}^n x L_p$.

4. CONCLUSIONS:

We have extended earlier results concerning the well-posedness of FDEs on product spaces. In particular, we have presented sufficient conditions for the well-posedness of a large class of functional differential equations (NNFDE). This class contains the "standard" neutral and retarded functional differential equations and many weakly singular integro-differential equations. It appears that results in this paper can be applied to infinite delay problems by using proper weighting on the state-space.

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